ALTERNATING CURRENT BRIDGE METHODS

FOR THE MEASUREMENT OF INDUCTANCE, CAPACITANCE, AND EFFECTIVE RESISTANCE AT LOW AND TELEPHONIC FREQUENCIES

A Theoretical and Practical Handbook
for the use of Advanced Students

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PREFACE

THE subject of A.C. Bridge Measurements has undergone considerable development during the past six years, not only in a continuously widening range of applications but also in the design of new apparatus and in the addition of important advances in fundamental theory. The Third Edition of this book, published in 1932, was an attempt to bring up to date the Second Edition of 1929 by an Appendix of Supplementary Notes. In preparing the present edition three objects have been kept in view: (i) to insert this supplementary matter into its proper place in the text; (ii) to give an account of developments to the end of 1937; and (iii) to expand certain matters that experience had shown to be in need of further explanation. The result has been a much enlarged and more fully illustrated volume, commensurate with the increased growth and importance of the subject.

On the theoretical side a short account has been given of circuit locus diagrams, in order to explain their use in discussing the sensitivity and balance convergence of bridge networks. Additions have also been made to the theory of bridges containing mutual inductance and to many minor questions. Finally, a complete classification of bridges, based on the work of Ferguson, has been included.

Bridge apparatus has recently been much developed, and a full account of modern tendencies has been given. In particular, the treatment of thermionic apparatus has been expanded in view of the wide use now made of oscillators, amplifiers, and detectors.

New bridges are numerous, and a large number of new applications of old methods have been made. These have been described, and at the same time the treatment of some of the older methods has been extended when recent developments have made this necessary. An outstanding instance is the versatile Schering bridge. An extended, and at the same time simplified, classification of frequency bridges has been given. The subject of shielding has, at the request of several correspondents, been more fully treated because of its importance in high-frequency bridges. Although radio-frequency bridges fall outside the declared scope of this book, it has been necessary to mention them briefly, since many bridges are now in use which are designed to work over an enormous frequency range, from the lower acoustic values up to the radio range.

I am particularly indebted to numerous correspondents all over the world who have made important suggestions, pointed out errors in earlier editions and drawn my attention to many developments that might otherwise have escaped my notice; I am very grateful to them for their assistance in making this book a reasonably complete account of its subject. I am also deeply indebted to many firms, at home and abroad, who have supplied particulars of new apparatus and material from which illustrations have been prepared; it has been possible to utilize only a small fraction of the information generously put at my disposal. In particular I tender thanks to the Cambridge Instrument Co.; General Radio Co. of Cambridge, Mass., and their British representative, Claude Lyons Ltd.; Muirhead & Co.; H. W. Sullivan Ltd.; H. Tinsley & Co.; and D. C. Gall, Esq. Numerous additional acknowledgments will be found in the text. I have also to thank my colleague, Dr. A. J. Small, for his valuable assistance with the reading of proofs. Finally, I would express my thanks to the Editorial and Composing staffs of Sir Isaac Pitman, Ltd. for their co-operation in a long and arduous task, and for their skill and patience in overcoming many difficulties.

GLASGOW.
April, 1938.

PREFACE TO FIRST EDITION

THE object of the author in preparing the present volume is to deal with the subject of Alternating Current Bridge Measurements of Inductance, Capacitance, and Effective Resistance at low and telephonic frequencies in a manner suited to the needs of the advanced student. The importance of such measurements in modern laboratory and test-room practice, in research work and in the training of students, would seem to be sufficient reason for the publication of a handbook dealing fairly completely with all the matters involved. As the book is intended primarily for practical use, every endeavour has been made to make clear the experimental side of the subject. At the same time an attempt has been made to provide a logical treatment of the theory underlying the use of a.c. bridge networks, since this is a matter which falls outside the scope of textbooks dealing with the theory of a.c.

The book is based on a course of lectures given for the past three years to third-year students of the City and Guilds (Engineering) College, amplified by the addition of material intended to make the volume useful to post-graduate workers and to others engaged on original research or accurate testing. The subject-matter is divided into five chapters, each dealing with some aspect of the theme. The object of Chapter I is to define the various quantities which are to be dealt with in the rest of the book; considerable attention has been paid to the discussion of electrostatic phenomena. In Chapter II the theory of a.c. is developed from the standpoint of the symbolic method, and an attempt is made to show the true relationship between the symbolic method and the more usual vector diagram and mathematical treatment of a.c. problems.

The apparatus required for bridge measurements is considered at some length in Chapter III, attention being chiefly directed to the explanation of the principles underlying the action of the various instruments rather than to a catalogue-like description of constructional details. The various bridge networks are classified in Chapter IV, the theory, uses, and full details of laboratory procedure being given in each case. With few exceptions, typical measurements have been made by all the methods described in this chapter, the results being included in the text as a guide to the student in carrying out his own experiments. Finally, the choice of the method suitable for a given measurement and the general precautions to be observed in laboratory practice are dealt with in Chapter V.

In preparing the book, full advantage has been taken of the information contained in original papers widely scattered throughout technical literature, complete references being given in the footnotes. In particular, the writings of Mr. A. Campbell and his associates at the National Physical Laboratory, of the papers published by the Washington Bureau of Standards, and by the Physikalische Technische Reichsanstalt have been drawn upon to a considerable extent. The manuscript of the book was in the hands of the printer before Mr. Campbell's informative articles in the *Dictionary of Applied Physics* were published, but reference has been made to them in revising the proofs for press.

In conclusion, thanks are due to many friends for help, advice and criticism during the preparation of the manuscript and proofs. In particular, the author wishes especially to thank Professor T. Mather, F.R.S., who not only suggested the preparation of the book, but who gave his unstinted help throughout. Mr. G. W. Sutton, B.Sc., has helped with the revision of the final proofs. Mr. S. Butterworth, M.Sc~~., of the~~ Admiralty Research Laboratory, has kindly read the sections of the book on which he is an acknowledged authority.

The diagrams were all specially prepared for the book by the author with the skilled assistance of Mr. M. G. Say, M.Sc., to whom thanks are accorded. The author also wishes to record his indebtedness to the various firms, at home and abroad, who have kindly supplied information during the preparation of Chapter III.

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July, 1923.

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ABBREVIATED TITLES FOR REFERENCES

Alta Freq.	Alta Frequenza
Amer. J. Sc.	The American Journal of Science and Arts
Annales de Ch. et Ph.	Annales de Chemie et de Physique
Annales des Postes, Télég. et Téléph.	Annales des Postes, Télégraphes et Téléphones
Ann. de l'École Normale	Annales scientifiques de l'École Normale Supérieure de Paris
Ann. der Phys.	Annalen der Physik und Chemie (1824–99), continued as Annalen der Physik
Arch. f. Elekt.	Archiv für Elektrotechnik
Arch. f. tech. Mess.	Archiv für technisches Messen
Assoc. Française p. l'avancement d. Sc.	Association Française pour l'avancement de Science
Beama J.	Journal of the British Electrical and Allied Manufacturers Association (continued as *World Power* from Jan., 1924, to June, 1937)
Bell Lab. Rec.	Bell Laboratories Record
Bell Syst. Tech. J.	The Bell System Technical Journal
Bull. Assoc. Ing. Él. Liège	Bulletin de l'Association des Ingénieurs Électriciens sortis de l'Institut Electrotechnique Montifiore, Liège
Bull. Bur. Stds.	Bulletin of the Bureau of Standards
Bur. Stds. Sci. Papers	Bureau of Standards, Scientific Papers, being Vols. 15 to 22 of *Bull. Bur. Stds.*
Bur. Stds. Journal of Res.	Bureau of Standards Journal of Research. continuing from July, 1928, *Bur. Stds. Sci, Papers*
Bull. Soc. Belge des Élecns.	Bulletin de la Société Belge des Électriciens
Bull. Soc. Franç. des Élecns.	Bulletin de la Société Française des Électriciens
Comptes Rendus.	Comptes Rendus hebdomadaires des séances de l'Académie des Sciences
Dist. of Elec.	Distribution of Electricity (publication of the W. T. Henley's Telegraph Works, Co. Ltd.)
Écl. Élect.	L'Éclairage Électrique
Elec. Eng.	Electrical Engineering
Elecn.	The Electrician
Elec. Rev.	The Electrical Review (continuation of *Tel. J. and Elec. Rev.*)
Elec. Times	Electrical Times
Elec. World	The Electrical World and Engineer
Elect. Comm.	Electrical Communication
Elect. J.	The Electric Journal
Elect. Lab. Min. Comm. Tokyo Res.	Researches of the Electrotechnical Laboratory of the Ministry of Communications, Tokyo.
Electrotechnics	Journal of the Electrical Engineering Society, Indian Institute of Science, Bangalore
Elekt. Wirts.	Elektrische Wirtschaft
Elekt. Zeits.	Elektrotechnische Zeitschrift

Engg.	Engineering
E.N.T.	Elektrische Nachrichten-Technik
Ericsson Tech.	Ericsson Technics (see also *L. M. Ericsson Rev.*)
E.u.M.	Elektrotechnik und Maschinenbau
Exp. W. and W. Eng.	Experimental Wireless and Wireless Engineer (Oct., 1923 to Aug., 1931)
Faraday House J.	Faraday House Journal
Gdes. Réseaux	Grandes Réseaux. International Conference on large H. T. Systems, Paris Papers
Gen. Elec. Rev.	General Electric Review
G. R. Exp.	General Radio Experimenter (publication of the General Radio Co., Cambridge, Mass.)
Jahrb. d. d. Tel.	Jahrbuch der drahtlosen Telegraphie
Journal Amer. Chem. Soc.	Journal of the American Chemical Society
Journal Amer. I.E.E.	Journal of the American Institute of Electrical Engineers; continued as *Elec. Eng.*
Journal Chem. Soc.	Journal of the Chemical Society
Journal Elect. Soc. Waseda	Journal of the Electrical Society of the University of Waseda
Journal F. Inst.	Journal of the Franklin Institute
Journal I.E.E.	Journal of the Institution of Electrical Engineers (continuation of *Journal S.T.E.*)
Journal I.E.E. Japan.	Journal of the Institution of Electrical Engineers of Japan
Journal Opt. Soc. Amer.	Journal of the Optical Society of America and Review of Scientific Instruments
Journal de Phys.	Journal de Physique Théorique et Appliquée
Journal P.O.E.E.	The Post Office Electrical Engineers Journal
Journal R.T.C.	Journal of the Royal Technical College of Glasgow
Journal Sci. Insts.	Journal of Scientific Instruments
Journal S.T.E.	Journal of the Society of Telegraph Engineers (continued as *Journal I.E.E.*)
Journal Tél.	Journal Télégraphique
L'Elettro.	L'Elettrotecnica
L. M. Ericsson Rev.	L. M. Ericsson Review (publication of Telefonaktiebolaget L. M. Ericsson)
Lum. Élect.	La Lumière Électrique
Mem. Acc. Tor.	Memoria della Reale Accademia delle Scienze di Torino
Messtech.	Messtechnik
M.V. Gaz.	Metropolitan-Vickers Gazette
Nature	Nature
N.P.L. Rep.	National Physical Laboratory Report
N.P.L. Researches	Collected Researches of the National Physical Laboratory
Nuovo Cimento	Il Nuovo Cimento, Giornale di Fisica, Chimica, e Storia Naturale
Phil. Mag.	The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science
Phil. Trans. Roy. Soc.	Philosophical Transactions of the Royal Society of London
Physics	Physics
Phys. Rev.	The Physical Review
Phys. Zeits.	Physikalische Zeitschrift
Proc. Camb. Phil. Soc.	Proceedings of the Cambridge Philosophical Society

Proc. Inst. Rad. Eng. .	Proceedings of the Institute of Radio Engineers
Proc. Nat. Acad. Sci. .	Proceedings of the National Academy of Sciences
Proc. Phys. Soc. . .	Proceedings of the Physical Society
Proc. Roy. Soc. . .	Proceedings of the Royal Society of London
Rad. Rev. . . .	The Radio Review
Rev. A.C.E.C. . .	Revue des Ateliers des Constructions Électriques de Charleroi
Rev. Gén. de l'Él. .	Revue Générale de l'Électricité
Rev. Sci. Insts. . .	Review of Scientific Instruments, continuing from 1930 the non-optical section of *Journal Opt. Soc. Amer.*
Schweiz. Elekt. Ver. Bull.	Bulletin der schweitzerische elektrotechnische Verein
Siemens Zeits. . .	Siemens Zeitschrift
Tech. Phys. U.S.S.R. .	Technical Physics of the U.S.S.R.
Tech. Rep. Tohoku Univ.	Technological Reports of the University of Tohoku
Tel. J. and Elec. Rev. .	The Telegraphic Journal and Electrical Review (after 1891 continued as *Elec. Rev.*)
Tel. u. Ferns. Tech. .	Telegraphisches-und Fernsprechos-Technik
Tijd. v. Elekt. . .	Tijdschrift voor elektrotechniek
Trans. Amer. I.E.E. .	Transactions of the American Institute of Electrical Engineers
Trans. Far. Soc.. .	Transactions of the Faraday Society
Trans. Roy. Soc. Edin.	Transactions of the Royal Society of Edinburgh
Verh. d. Deutsch. Phys. Ges.	Verhandlungen der Deutschen Physikalischen Gesellschaft
W. Eng.	Wireless Engineer (Jan., 1936 onward)
W. Eng. and Exp. W. .	Wireless Engineer and Experimental Wireless (Sept., 1931 to Dec., 1935)
World Power . .	World Power (see *Beama J.*)
Zeits. f. Inst. . .	Zeitschrift für Instrumentenkunde
Zeits. f. Phys. . .	Zeitschrift für Physik
Zeits. f. tech. Phys. .	Zeitschrift für technische Physik

ALTERNATING CURRENT BRIDGE METHODS

CHAPTER I

FUNDAMENTAL PRINCIPLES

1. Introductory. Among the applications of physics to modern electrical engineering practice, the recent developments in land and submarine telegraphy, in the telephonic transmission of speech, and in radio-telegraphy and telephony must be counted as of extreme importance. The enormous advances which have been made in these branches of electrical technology are largely the result of careful experimental researches, guided by a fairly complete mathematical and physical theory, in which the various electric and magnetic quantities involved have been accurately measured. It is to be expected, therefore, that further developments will be largely dependent upon the possession of methods of measurement which can be used in precise investigations to test the predictions or conclusions of theory.

In telegraphy, telephony, and radio-technology more or less complicated circuits are used, in which electric currents of a transient or of a periodic nature are caused to flow. It becomes, therefore, a matter of extreme importance to be able to measure not merely the currents and potential differences in the circuits, but also the various quantities, such as resistance, inductance, capacitance, etc., of which the circuits are composed. For this reason a large amount of work has been done with the object of devising methods by means of which these quantities may be accurately measured under actual working conditions, and it is the aim of the following chapters to discuss the theory and practical working of a most important class of methods—known as *Alternating Current Bridge Methods*—which have a very wide practical application.

In general, the principle underlying an alternating current bridge method may be briefly summarized in the following way. Let a piece of apparatus, of which the constants are to be measured, be inserted in a network of known conductors, e.g. a Wheatstone bridge. Two points in the network are connected to a source of alternating current; while two other points are "bridged" by some instrument capable of detecting alternating potential differences, such as a telephone. The constants of the known conductors are then adjusted until the two bridged points are at the same potential at every instant, this condition being indicated by a zero reading of the detecting instrument, e.g. silence in a telephone. From the known constants of the network the constants of the apparatus under test can then be calculated.

While it is theoretically possible to make use of the bridge principle with alternating current of any frequency, certain important experimental difficulties are encountered at high frequencies which impose limitations to the range of frequency over which the principle is easily applicable. It is quite easy to devise alternating current bridge methods which, without any extraordinary precautions being taken, will give accurate results at frequencies up to 500 or 600 cycles per second. With higher frequencies the experimental difficulties—due principally to the importance assumed by capacitance effects between the bridge network and its surroundings, and to skin-effects, etc., in the conductors as the frequency is raised—increase until, at the upper telephonic range of 2,000 to 3,000 cycles per second, extreme care and very special precautions become necessary to secure precise results. At frequencies above this limit the experimental troubles become very great indeed until, at the values used in radio-telegraphic measurements, the application of the bridge principle is a matter of extreme difficulty, and special modifications in the technique becomes necessary.

It is to be clearly understood, therefore, that the theories and methods to be developed in this book are intended primarily to apply to measurements carried out at frequencies not exceeding the higher acoustic values of 2,000 to 3,000 cycles per second, such as will be found in ordinary laboratory practice and in telephonic research. Measurements at radio-frequencies are not contemplated in this volume owing to their very special character, but indication will be given, where possible, to show how the methods applicable at low frequencies are modified in order that they may eventually be applied in radio-research. For the present, it may be stated that bridge measurements* at radio-frequencies present very great difficulties and that the subject is still undergoing development.

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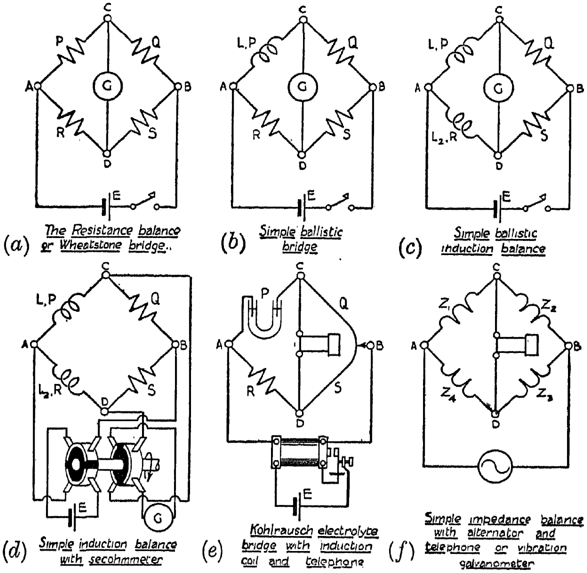
* See *Dictionary of Applied Physics*, Vol. 2, "Radio-Frequency Measurements." Also see page 550.

2. History of the Bridge Principle. The bridge principle was first introduced by S. H. Christie* in 1833, but was neglected until 1843, when Sir Charles Wheatstone† drew attention to Christie's idea and applied it to the comparison of resistances. In the well-known arrangement, shown in Fig. 1 (*a*), P is the resistance to be measured, Q and S are equal resistances, and R is an adjustable resistance or rheostat. A battery is connected across the points A and B, a galvanometer joining the points C and D. Clearly, the points C, D will be at the same potential and no current will flow through G if R be adjusted to be equal to P. The value of P is thus known in terms of the standard R.



(*a*) The Resistance balance or Wheatstone bridge.

(*b*) Simple ballistic bridge

(*c*) Simple ballistic induction balance

(*d*) Simple induction balance with secohmmeter

(*e*) Kohlrausch electrolyte bridge with induction coil and telephone

(*f*) Simple impedance balance with alternator and telephone or vibration galvanometer

FIG. 1.—ILLUSTRATING THE EVOLUTION OF THE SIMPLE IMPEDANCE BALANCE

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* S. H. Christie, "Experimental determination of the laws of magneto-electric induction," *Phil. Trans. Roy. Soc.*, Vol. 123, pp. 95-142 (1833).

† C. Wheatstone, "An account of several new instruments and processes for determining the constants of a voltaic circuit," *Phil. Trans. Roy. Soc.*, Vol. 133, pp. 303–327 (1843).

THE WHEATSTONE BRIDGE NETWORK. The arrangement was called by Wheatstone a *resistance balance*, because the resistances P, R are balanced against one another; the resistances Q, S are then referred to as the *arms* of the balance, by analogy with the process of weighing. With Q equal to S it will be seen that the greatest value of P which can be measured is limited by the maximum attainable value of R, and that the smallest measurable P is limited to the least value of R which is accurately readable. Werner von Siemens,[*] in 1848, increased the range of the balance for a given rheostat R by arranging Q and S to have values in any desired ratio. P and R are then compared on a balance which has unequal arms, so that $P = \frac{Q}{S} R$ by simple application of Ohm's law. The process of balancing is then analogous to the comparison of masses by means of a steelyard or by a chemical balance with unequal arms. In this case, Q and S are called the *ratio arms*, Q/S having any convenient value.

There is another simple way of regarding the resistance balance. The points A, C, B, D, A are joined successively by the resistances P, Q, S, R. The diagonal AB consists of a conductor containing the battery E, while CD includes a galvanometer G. The whole arrangement constitutes a network of conductors joining the four points in all possible ways. The points C, D, which are to be at the same potential when balance occurs, are *bridged* by the detecting instrument; CD is therefore referred to as the *bridge*. The whole network is then called a *bridge network*, of which P, Q, R, S, G, E are the *branches*. By a process of abbreviation, the network is often simply referred to as the *Wheatstone bridge*.[†]

The Wheatstone bridge is, then, a null method for the comparison of resistances, and, when suitably arranged, is capable of considerable sensitiveness and precision. Attempts were very soon made, therefore, to adapt the bridge principle to the measurement of other quantities, notably inductances and capacitances.

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[*] See C. W. Siemens, *Collected Scientific Works*, Vol. 2, pp. 126–127.

[†] In practice this nomenclature is frequently confused with that given in the preceding paragraph. By abbreviation the network is called a "bridge," confusion then arising from reference to P, Q, R, S as the "arms" of the bridge. The late Lord Rayleigh (*Journal I.E.E.*, Vol. 15, p. 29 (1886)), and Oliver Heaviside (*Electrical Papers*, Vol. 2, pp. 33 and 102 (1892)) have pointed out the absurdities arising from this mixing of notations. The confusion has entered the writings of French electricians, who refer to "le pont de Wheatstone." The Germans are more consistent, using "die Wheatstonesche Brücke" for the entire network, "die Brücke" for CD, and "der Zweig" (branch) for the conductors comprising the network. In this book any network in which the bridge principle is used will be called a ***bridge network***,

MAXWELL'S INDUCTION BALANCE. Maxwell,* in 1865, introduced the arrangement shown in Fig. 1 (*b*). Let the branch P of a Wheatstone bridge network be inductive, the bridge being balanced with steady current so that $SP = QR$. Then, on breaking or making the battery circuit, an electromotive force $-L\frac{di}{dt}$ will be induced in the branch AC when the current i therein is either diminishing or increasing. A quantity of electricity will thus be discharged through the galvanometer, since for transient currents the balance is not retained, producing a throw of the spot of light reflected from a mirror on the moving system. If the ballistic calibration of G be known, L is at once calculable; or, alternatively, the deflection of the galvanometer may be noted when the resistance of one of the branches is slightly increased, this deflection serving the purpose of a calibration when the period and damping of the instrument are known.

It is a simple step to convert this network into a null inductance balance. Let an adjustable inductive coil L_2 be put in the branch AD. Then, since the transient discharge from L_2 on making or breaking the battery circuit passes through the galvanometer in the reverse direction to that from L, a null condition can be arrived at by adjustment of L_2 until no ballistic kick is observed. The arrangement then forms a *ballistic bridge* or *induction balance*. Maxwell,† in his famous *Treatise*, describes several types of induction balance, many of them being in use with modern improvements at the present time (*see* Fig. 1 (*c*)).

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the various conductors of which the network is composed being its *branches*. Those branches which are adjusted to attain balance are called *balancing branches*; balance being attained when no current flows in the conductor, called the *bridge*, containing the detecting instrument which is bridged across two points in the network.

* J. C. Maxwell, "A dynamical theory of the electromagnetic field," *Phil. Trans. Roy. Soc.*, Vol. 155, pp. 459–512 (1865). For the practical application of the method, *see* Lord Rayleigh and A. Schuster, *Proc. Roy. Soc.*, Vol. 32, pp 104–141 (1881), and Lord Rayleigh, *Phil. Trans. Roy. Soc.*, Vol. 173, pp 661–697 (1882).

† J. C. Maxwell, *A Treatise on Electricity and Magnetism*, *see* Sects. 756, 757, 778 of Vol. 2, 1st. edn. (1873).

In a similar way, condensers can be balanced by the use of a bridge, or a condenser may be compared with an inductance, and so on. In all cases the principle is the same: the network is first adjusted with steady current until the ordinary Wheatstone balance of resistances is satisfied. Then, without disturbing this condition, adjustments are made until the transient ballistic throw on make or break of the battery circuit is reduced to zero. For absolute balance it is necessary, therefore, to fulfil *two* balance conditions, one providing for the steady current null condition—called the *resistance balance*—while the other accounts for the vanishing of the transient effects—called the *ballistic* or *induction balance.*

Attempts were then made to increase the sensitiveness of the ballistic balance so as to be comparable with that obtainable for the resistance balance. Consider a bridge network containing inductances or capacitances, supposing the bridge to be balanced for steady currents and very nearly balanced ballistically. Then, on making or breaking the battery circuit, the galvanometer will receive impulses in opposite directions, the resulting deflections being small. Suppose the battery to be reversed; then twice the swing will be obtained, but successive reversals of the battery will again give opposite impulses to the galvanometer. If, however, the connections of the galvanometer to the bridge be reversed after each reversal of the battery, the impulses will all be in the same direction. If the rate of reversal be sufficiently high, a steady deflection will be produced by the summation of the impulses given to the galvanometer moving system.

The Secohmmeter. This principle is embodied in the Secohmmeter of Ayrton and Perry,* illustrated in Fig. 1 (*d*), a similar device having been used by Brillouin in 1882. Two commutators are arranged on a shaft driven by hand gearing or by a small electric motor at a suitable speed. One commutator successively reverses and insulates the battery connections, while the other repeatedly changes the galvanometer leads. Resistance balance having been obtained with the commutators at rest, transient balance is secured by adjustment of the inductances or capacitances of the network with the commutators running. The apparatus can be applied to increase the sensitiveness of any type of induction balance.* The use of the secohmmeter with an induction balance really amounts to the employment of an alternating current in the network. The voltage wave applied to the terminals *A B* is rectangular, since the battery is reversed at each half-revolution, and produces a corresponding current; in order to detect the impulses due to successive half-waves, an ordinary ballistic galvanometer combined with a rectifying commutator is used. The secohmmeter bridge may, therefore, be looked upon as an early type of alternating current bridge.†

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* W. E. Ayrton and J. Perry, "Modes of measuring the coefficients of self and mutual induction," *Journal, S.T.E.*, Vol. 16, pp. 292–343 (1888); W. E. Sumpner, "The measurement of self-induction, mutual induction, and capacity," *Journal, S.T.E.*, Vol. 16, pp. 344–379 (1888).

USE OF THE TELEPHONE IN INDUCTION BALANCES. Measurements with the induction balance received a great impetus from the invention of the telephone by A. Graham Bell, in 1875, the high sensitivity of the instrument to small alternating currents leading many experimenters to use it to increase the sensitiveness of bridge methods. The early work of Lord Rayleigh, Heaviside, Hughes, Kohlrausch, and others established the use of the telephone in bridge networks with alternating currents. The source of current in these early methods was frequently a small induction coil, providing a secondary current of somewhat irregular wave-form and indefinite frequency; the detector was an ordinary Bell receiver. This arrangement is in use at the present day in a large amount of routine laboratory work where moderate accuracy is sufficient (*see* Fig. 1 (*e*)).

WIEN'S A.C. BRIDGE NETWORK. It is to Max Wien that the modern alternating current bridge is due. Following the lead of Oberbeck‡ he, in 1891, laid down the principles which are practically those used to-day. He supplied the network with alternating current of a definite steady frequency derived from an induction coil, the primary current of which was made and broken by a steadily vibrating wire. At a later date he used a small alternator and endeavoured to make the applied wave form approximately sinusoidal, so that tests could be carried out under precisely-known conditions. To increase still more the sensitiveness of the detector, he designed his "optical telephone," which was nothing more than a magnetic telephone of which the diaphragm could be tuned to be in resonance with the alternating current flowing in the telephone windings. By making use of the principle of resonance the amplitude of motion was greatly magnified, and was observed by reflecting a beam of light from a mirror operated by the moving diaphragm. This instrument is the forerunner of the "vibration galvanometers" used in modern practice. Using his apparatus Wien adapted a large number of old ballistic methods to work with alternating current, and introduced a number of new methods which are now standard practice (*see* Fig. 1 (*f*)).

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* *See*, for example, S. R. Milner, "The use of the secohmmeter for the measurement of combined resistances and capacities," *Phil. Mag.*, 6th series, Vol. 12, pp. 297–317 (1906).

† For a modern application of this principle, *see* p. 233.

‡ A. Oberbeck, *Ann. der Phys.*, Vol. 17, pp. 816–841 (1882); Max Wien, *Ann. der Phys.*, Vol. 42, pp. 593–621, Vol. 44, pp. 681–688, and pp. 689–712 (1891).

MODERN DEVELOPMENTS. From the time of Wien to the present day steady progress has been made in the development of the technique of alternating bridge measurements. In general, the modern methods make use of networks of impedances arranged either as a Wheatstone bridge or otherwise, as will be shown in Chapter IV; the source of current is chosen to have a steady frequency and a pure wave-form; the balance detector may be a highly sensitive telephone or some type of vibration galvanometer.

Since alternating current is used, the quantities measured in the bridge network are determined under the conditions in which they occur in practice, i.e. at the same frequency and with a known wave form. The quantities which occur in alternating current circuits, and which may be measured by a bridge method are capacitance, inductance, and effective resistance under the conditions of the test. It will be well at this point briefly to lay down what is meant by each of these terms—the remainder of this chapter being devoted to this purpose, though there will be no attempt at a complete account of the electrical theory involved. The reader is assumed to be familiar with the principles of alternating currents and to have clear ideas of the facts of electricity and magnetism.

Again, in dealing with the subject of this book it will be assumed that the sine-wave theory is adequate. This assumption is justified on two grounds: (i) that even if the currents in the network are not sinusoidal the sine-wave theory is sufficient since, by the application of Fourier's theorem, complex wave shapes can be treated by the summation of the results for a number of sine waves having frequencies in the proportions 1, 2, 3, etc.; (ii) since, in modern methods, the tuned detector is so widely used, the sine theory is sufficient, because the sensitiveness of such instruments is extremely small to currents of frequencies even slightly removed from that to which they are tuned.

3. Capacitance. One of the principal uses of alternating current bridges is for the measurement of capacitance; and one of the most important experimental troubles encountered in making bridge measurements is due to the capacitance effects between the bridge network and its surroundings. It becomes essential, therefore, to lay down exact definitions and clear ideas of electric phenomena. The reader is assumed to be acquainted with the simpler qualitative and quantitative laws of the electrostatic field.

4. Elementary Ideas. Consider a single conductor placed so that it is at a very great distance from all other conductors, and let it receive a charge. Assuming the potential of the conductor to be initially zero, let the electrostatic charge necessary to raise the potential to unity be C; the quantity C is then called the *capacity of the conductor*, i.e. the charge required to raise its potential to one electrostatic unit. To be consistent with the prevailing use of the terms "resistance," "inductance," etc., it is becoming the custom to refer to C as the *capacitance* of the conductor.

If the potential of the conductor be v electrostatic units, the charge upon it will be

$$q = Cv,$$

since the total charge is proportional to v. Now, in practice the electric field is, in general, due to charges distributed over a system of several conductors and the earth. It is important, therefore, to define the meaning of the term capacitance when more than one conductor is involved. This can be readily done by the aid of the theory of Maxwell* in its modified form due to Orlich.†

5. Theory of Charged Conductors. Consider a system of n conductors arranged in proximity to the earth as shown in Fig. 2. A clear idea of the meaning of capacitance as applied to such a system is best obtained by an examination of the distribution of the electric tubes of induction. A tube of induction is bounded by lines of force, a unit tube starting from a unit positive charge and terminating on an equal negative charge. The total number of unit tubes emitted from a given charged area is called the electric flux from the area.

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* J. C. Maxwell, *A Treatise on Electricity and Magnetism*, Vol. 1, 3rd edition, Sects. 87–90*b* (1892).

† E. Orlich, *Kapazität und Induktivität*, § 10, pp. 20–24 (1909).

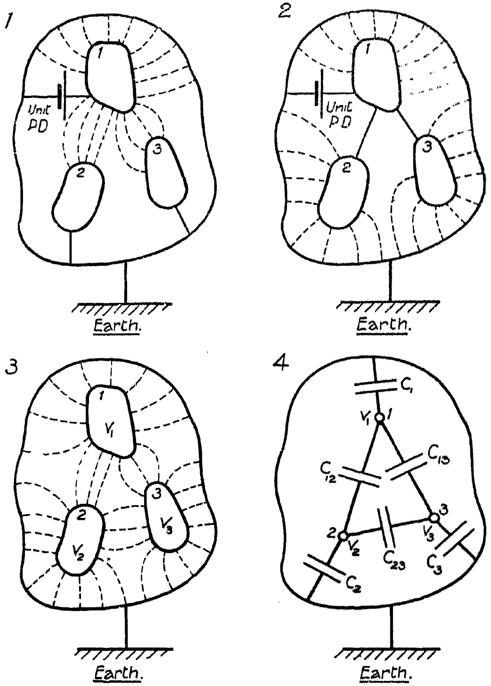

Fig. 2.—Illustrating the Partial Capacitances of a System of Conductors

Examining the third diagram in Fig. 2, the conductors 1, 2, 3 . . . *n* are shown raised to potentials $v_1, v_2, v_3 \ldots v_n$, the approximate paths of the tubes of induction being indicated by dotted lines. Consider such a conductor as 1, in the figure. The total flux leaving it may be divided into several portions represented by distinct groups of tubes of induction passing in certain directions. Firstly, there is the flux passing directly from the conductor to earth, the amount of it being proportional to the potential of the conductor. There remain $n-1$ groups of tubes of induction passing between 1 and each of the other conductors, the number of unit tubes in each group being proportional to the difference of potential between 1 and the conductor upon which the tubes terminate. A similar analysis can be made of the flux from the surface of each conductor in the field. It therefore remains to express the earth-fluxes and inter-fluxes mathematically.

Referring to the second diagram in Fig. 2, let all the conductors be connected together and raised to unit potential. Each conductor will be positively charged so that tubes of induction extend from each of them to earth. Let $c_1, c_2, c_3 \ldots c_n$ be the charges on each under these conditions. The quantities $c_1, c_2, c_3 \ldots c_n$ are called the *earth-capacitances* of the n conductors, and represent the flux passing from each conductor to earth when all are raised to unit potential. The earth-capacitances depend on the size and shape of the conductors, on their positions relative to earth and on the nature of the medium in which the conductors are situated.

Now let all the conductors be earthed, except the rth, which is raised to unit potential, as in the first diagram of Fig. 2. This conductor is, therefore, positively charged, and tubes of induction setting out from it will terminate on a negative charge distributed over the remaining earthed conductors and the earth. The flux will be made up of a part to earth starting from a charge c_r, together with parts passing from r to 1, r to 2, . . . r to s, . . . r to n. Denoting these fluxes by $c_{r1}, c_{r2}, \ldots c_{rs}, \ldots c_{rn}$, the charges from which they spring, the total charge on r is $c_r + c_{r1} + c_{r2} + \ldots c_{rs} \ldots + c_{rn}$. The quantity c_{rs} is called the *intercapacitance* between r and s; it represents the flux passing between r and s, when r is raised to unit potential, s and all other conductors being earthed. The intercapacitances are functions of the size, shapes, and relative positions of the conductors and the medium in which they are placed.

Now let the conductors be raised to potentials $v_1, v_2, v_3 \ldots v_r, v_s, \ldots v_n$, as in the third diagram of Fig. 2. Then the flux from r may be divided into n parts, viz., a part $c_r v_r$ passing from the conductor to earth, and parts $c_{r1}(v_r - v_1)$, $c_{r2}(v_r - v_2) \ldots c_{rs}(v_r - v_s) \ldots c_{rn}(v_r - v_n)$ passing between the conductor r and

1, 2, . . . s, . . . n respectively. These fluxes are called the partial fluxes from r, the coefficients c_r, c_{r1}, c_{r2}, etc., being called the *partial or component capacitances* of the conductor r (*Teilkapazitäten*). Clearly the flux leaving r and received upon s is equal in amount and opposite in direction to the flux which passes between s and r; i.e. $c_{rs}(v_r - v_s) = -c_{sr}(v_s - v_r)$ or $c_{sr} = c_{rs}$. The charges $q_1, q_2 \ldots q_r \ldots q_n$ upon the n conductors are then

$$q_1 = c_1v_1 + c_{12}(v_1 - v_2) + c_{13}(v_1 - v_3) + \ldots \quad c_{1n}(v_1 - v_n)$$
$$q_2 = c_2v_2 + c_{12}(v_2 - v_1) + c_{23}(v_2 - v_3) + \ldots \quad c_{2n}(v_2 - v_n)$$
$$\cdot$$
$$q_n = c_nv_n + c_{1n}(v_n - v_1) + c_{2n}(v_n - v_2) + \ldots \quad \text{etc.}$$

For n conductors there are clearly n earth-capacitances and $\frac{1}{2}n(n-1)$ intercapacitances, making a total of $\frac{1}{2}n(n+1)$ partial capacitances, as the student can easily verify.

SIMPLE CONDENSER. In practice a most important example is that of two conductors arranged so that their intercapacitance is very great compared with their earth-capacitances or their capacitances to other conductors. Such an arrangement is called a *simple condenser*, since all the electrostatic field is condensed within the space between the conductors and none exists outside. $C = c_{12}$ is then referred to as the *capacitance of the condenser*, and is understood to mean the charge which appears on one of the two conductors per unit potential difference between them.

Referring again to the general case of n conductors, if attention is directed merely to their potentials and charges, it will be obvious that the conductors may be replaced by n points maintained at the potentials v_1, v_2, . . . v_n, with simple condensers of capacitance equal to the respective partial capacitances joining the points to one another and to earth. Thus, in the case of the three conductors illustrated in the third diagram of Fig. 2, the system of charges thereon can be replaced by six simple condensers, three having capacitances c_{12}, c_{23}, c_{13} joining the points 1, 2, 3 cyclically to represent the intercapacitances; while three others, of capacitances c_1, c_2, c_3, represent the earth-capacitances. The points 1, 2, 3 have potentials v_1, v_2, v_3; the equivalent simple condensers being shown in the fourth diagram in Fig. 2.

In practice, condensers in which the intercapacitance is large and the earth-capacitances are small can be treated as simple *condenser*, represented very closely in practice by condensers with gaseous dielectrics.

When a condenser has a charge q upon one of its plates and therefore a charge $-q$ upon the other, and these charges change with time, the entry of an additional charge dq at the one terminal is accompanied by the entry of $-dq$ at the other terminal. This is equivalent to the transference of a quantity dq from one terminal to the other; the rate of passage of the quantity, dq/dt, is called the *displacement current* within the dielectric. Hence, when used in alternating current circuits a condenser is said to carry a displacement current and behaves as a conductor, though the current is in quadrature with the p.d. and not in phase therewith, as it would be in a simple resistance.

In all practical condensers with solid or liquid dielectrics the passage of the displacement current is accompanied by a dissipation of energy in the dielectric, the current and p.d. not being exactly in quadrature. Such condensers are referred to as *imperfect condensers*, and are considered at some length in Chapter III.

7. Dimensions and Units. It has been pointed out above that the partial capacitances of a system of conductors are a function of their sizes, shapes, and relative positions, and of the medium, called the dielectric, in which they are placed. Consider a simple condenser having a vacuum maintained between its electrodes, and let C_v be its capacitance. Now let some substance be introduced to occupy the space between the electrodes, C_ε being the new capacitance. Faraday then defined the ratio $\varepsilon = C_\varepsilon/C_v$ as the *specific inductive capacity* of the medium, the modern term being *dielectric constant* or *dielectric ratio*. Thus ε is conventionally unity for a vacuum, and C_v can be calculated from the form and dimensions of the electrodes.

Electric charges exert forces of attraction or repulsion on one another proportionate to the magnitudes of the charges and inversely as the square of the distance between them. The force is also inversely proportional to ε. Unit charge at a point repels a similar charge 1 cm. distant with a force of 1 dyne. If the work done in moving unit positive charge between two points in an electric field is 1 erg, unit electric potential-difference exists between the points. By definition, the capacitance of a conductor (or of a condenser) is the charge per unit potential difference. It remains now to consider the units in which capacitance is measured (i) in the absolute electrostatic system of units just defined, (ii) in the absolute and the practical electromagnetic units defined below.

Examining first the electrostatic system of units, let $[M]$, $[L]$, $[T]$ be the dimensional symbols for mass, length, and time, the units being the gramme, the centimetre, and the second. Then, since *Capacitance = charge/potential* and *potential = work/charge*, $[\mathit{Capacitance}] = [\mathit{charge}^2/\mathit{work}]$. Now from the law of inverse squares for electrostatic charges $[\mathit{charge}^2] = [\mathit{force} \times \mathit{length}^2 \times \varepsilon]$, so that $[C] = [\mathit{force} \times L^2 \times \varepsilon/(\mathit{force} \times L)] = [L\varepsilon]$. The dimensions of capacitance in the absolute electrostatic system of units are those of length; a capacitance is, therefore, specified in centimetres.

In most practical work the Practical electromagnetic system of units is used* for all measurements. In this system the unit of potential difference is the volt and that of electric charge the coulomb. The unit of capacitance is the *farad*, and is the capacitance of a condenser in which a p.d. of 1 volt produces a charge of 1 coulomb. In all practical work the farad is an inconveniently large unit, a working unit of a *microfarad* = μ F. = 10^{-6} farad being generally adopted. In dealing with many quantities arising in bridge measurements a still smaller unit is used, the micro-microfarad = $\mu\mu$ F. = 10^{-12} farad.

The volt is 10^8 absolute electromagnetic units of p.d. and the coulomb 10^{-1} absolute electromagnetic units of charge. Hence the farad is 10^{-9}, the μ F. 10^{-15} and the $\mu\mu$ F. 10^{-21} absolute e.m. units of capacitance.

The practical unit and the electrostatic unit of capacitance are simply related. One centimetre capacitance is equivalent to $1/v^2$ absolute electromagnetic units, v being the velocity of light, 3×10^{10} cm. per sec. Hence 1 cm. capacitance = $1/(9 \times 10^{20})$ e.m. units = $1/(9 \times 10^{11})$ farad. A convenient rule is 0·9 cm. = 1 $\mu\mu$ F., or, conversely, 1 cm. = 1·11 $\mu\mu$ F.

8. Inductance. The most important quantities measured in a.c. bridges are self and mutual inductances. The properties of a.c. circuits in which inductances are present are dealt with in Chapter II; it is proposed here to set down the simpler physical ideas underlying electromagnetic induction so that the later work may be adequately understood. At the same time, definitions and units applying to the various quantities involved will be given. The student is supposed to have a knowledge of the properties of the magnetic field of a current and of the general principles of electromagnetic induction.

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* More correctly, the units used in technical work are those of the International system wherein the current and resistance units are defined in terms of actual standards intended to represent the units of the Practical system to a high order of accuracy. Except in work of the highest precision, the differences between the International and the Practical units may be neglected.

9. Elementary Ideas. The principles and laws of electromagnetism are based upon Faraday's famous experiments on electromagnetic induction. Consider two closed circuits, one containing a battery and switch while the other includes a galvanometer, and suppose the circuits are placed so that they can influence one another. So long as the current in the first circuit remains steady the galvanometer in the second circuit is unaffected; if, however, the current be changed, an immediate indication is given by the galvanometer. For example, if the first circuit be broken the galvanometer gives a momentary deflection, indicating that a transient current has flowed in the second circuit in the same direction as the current just removed from the first. On the other hand, if the first circuit be closed again, so that a current is established in it, the galvanometer gives a transient deflection in the opposite direction, showing that the second circuit has been traversed momentarily by a current in the reverse direction to that which has been established in the first. The transient current induced in the second circuit by the changes of the current flowing in the first was said by Faraday to be due to *mutual induction.* He showed that, for the same strength of inducing current, the magnitude of the transient effect was the same no matter which of the circuits carried the current or which contained the galvanometer. Moreover, if one of the circuits be removed, Faraday showed that transient induction effects, formerly referred to as "extra-current," occur in the remaining circuit when a current is started, stopped, or otherwise changed in it. The effect is then said to be due to *self-induction.*

Induction phenomena are best approached from a consideration of the magnetic field produced by a current-carrying circuit. When a steady current flows in a circuit, a magnetic field is produced in surrounding space; this field can be mapped out by tubes of magnetic flux which are linked through the circuit producing them. A second circuit put into the field of the first will be linked by a certain number of tubes of flux, the total number of linkages being dependent on the size, shape, and position of the circuit. If now the strength of the current in the first circuit be changed, the strength of the magnetic field at every point will be altered, i.e. the linkages of flux with the circuits will be varied. Hence, when the flux linking the circuits is changed, Faraday's experiments show that electromagnetic induction takes place, transient currents appearing during the time that the linkages are changing. Observation shows that the induced currents are in such a direction as will tend to maintain the magnetic field in the same state as before the changes began, i.e. the induced currents tend to set up a magnetic field in such a direction as to oppose the changes. This statement is referred to as Lenz's law.

Considering now the inducing circuit, any attempt to alter the current in it, and thereby to modify its magnetic field, is immediately accompanied by transient induction effects tending to maintain the field in its original state. Hence the magnetic field of a circuit acts as a kind of electro-magnetic inertia to the current-producing effects, tending to delay the establishment of the current in the circuit and generally to retard any change in its strength. The appearance of these induction effects can be thought of as due to an electromotive force introduced into the circuit when the linkages of flux with it are changed; the direction of this induced e.m.f. is such as to oppose the growth of the current, and it is proportional in magnitude to the rate at which the current, and therefore its linkages, is changed. When the linkages, N, with the circuit are increasing (dN/dt positive), the induced e.m.f. opposes the increasing current; when N diminishes (dN/dt negative) the induced e.m.f. tends to keep the current flowing in its original direction. The e.m.f. of the battery is thus transiently increased by an e.m.f. of induction proportional to $-dN/dt$.

In a precisely similar way, if the current in a circuit be changing, the e.m.f. of induction appearing in a neighbouring circuit is proportional to the rate of diminution of the magnetic linkages threading it from the first circuit. This e.m.f. acts in addition to the e.m.f. of any battery or other source of current in the circuit.

By a suitable choice of units the induced e.m.f. can be expressed numerically. If a rate of change of one linkage per second produces one electromagnetic unit of e.m.f. (*see* p. 22), then, in general, $e = -dN/dt$.

10. Self and Mutual Inductance. If the circuits considered on page 18 were placed in air or other magnetically similar medium, the flux, and consequently the linkages, will be proportional to the strength of the current; this would not be the case were the circuits linked by magnetic material such as iron.

Consider now the case of a number of circuits, 1, 2, . . . m, . . . n, in a magnetically indifferent medium such as air. Let all the circuits be opened except n, in which one electromagnetic unit of current is caused to flow. The total number of linkages of flux with the circuit itself is then L_n and, in addition, the remaining $n - 1$ circuits have linkages M_{1n}, M_{2n}, . . . M_{mn}, . . . etc., due to flux passing through them from circuit n. Now let n be opened and unit current flow in m. The linkages of flux in m are L_m and in each of the others M_{1m}, M_{2m}, $M_{(m-1)}$, M_{nm}. Applying this process in turn to all the circuits, a number of linkage coefficients of these types are found, these having important properties.

Coefficients such as L_n are called *coefficients of self-induction*, or simply *self-inductances*. L_n denotes the number of linkages of flux with the circuit n when it is carrying unit current.

Coefficients such as M_{mn} are called *coefficients of mutual induction*, or *mutual inductances*. It follows from Faraday's experiments, or by deduction from energy considerations (Rayleigh's reciprocal theorem) that it is immaterial which of two circuits is treated as the inducing circuit and which as the induced; hence $M_{mn} = M_{nm}$ numerically. M_{mn} denotes the number of linkages of flux with the circuit m when unit current flows in n; reciprocally, it is equal to the linkages with n due to the field of unit current in m.

The self and mutual inductances are functions merely of the size, shape, and relative positions of the circuits when the medium in which they are placed has a constant permeability. When iron is present the inductances depend also on the strength of the current in the circuits, and are not constants dependent only on their geometrical properties. There are clearly n self-inductances and $\frac{1}{2}n(n-1)$ mutual inductances, making a total of $\frac{1}{2}n(n+1)$ coefficients.

Suppose now that the circuits are all closed and contain e.m.f.'s $e_1, \ldots e_n$ the resulting currents at any instant being $i_1, i_2, \ldots i_m, \ldots i_n$. Then the total linkages with each circuit will be the sum of its own self linkages and those due to all the other circuits, that is,

$$\begin{aligned} N_1 &= L_1 i_1 + M_{12} i_2 + M_{13} i_3 + \ldots M_{1n} i_n, \\ N_2 &= M_{12} i_1 + L_2 i_2 + M_{23} i_3 + \ldots M_{2n} i_n, \\ &\ldots \\ N_n &= M_{1n} i_1 + M_{2n} i_2 + M_{3n} i_3 + \ldots L_n i_n. \end{aligned}$$

The corresponding induced e.m.f.'s will be

$$-dN_1/dt, \; -dN_2/dt, \; \ldots \; -dN_n/dt.$$

By Ohm's law the total e.m.f. in each circuit must be equal to the resistance drop round it. If $R_1, R_2, \ldots R_m, \ldots R_n$ are the resistances of the circuits, the equations of equilibrium are

$$e_1 - \frac{dN_1}{dt} = R_1 i_1$$

$$e_2 - \frac{dN_2}{dt} = R_2 i_2$$

$$e_n - \frac{dN_n}{dt} = R_n i_n$$

from which the currents can readily be found when the applied electromotive forces are given. (For an example when there are six inducing circuits, *see* p. 70.) The equations apply no matter how the e.m.f.'s, and consequently the currents, may vary with time and hence apply directly to alternating quantities.

11. Dimensions and Units. Consider a current flowing in a wire bent into the form of a circle of 1 cm. radius, and let a free unit magnetic pole be imagined brought up to the centre of the circle. If the pole experiences a force of 2π dynes the current in the circle is said to be *one absolute electromagnetic unit.* When the current is of this strength, 1 absolute electromagnetic unit of quantity or electric charge passes any section of the circuit per second. If two points exist in the circuit such that 1 erg of work is necessary to transfer 1 e.m. unit of charge between them, the points are said to have a difference of potential of 1 e.m. unit.

If the free unit pole is allowed to move from any point in the magnetic field of a current it will trace out a closed curve, called a line of force, the tangent to which is in the direction of the field strength at every point on it. The lines of force passing through the boundary of any small closed area are said to bound a tube of force; the field can be thought of as mapped out by tubes of force. Although this conception is useful it leads to some trouble, chiefly due to the hypothetical nature of the exploring pole, when applied to the magnetic field within a magnetic material. As it is with the inductive effect of a magnetic field that one is concerned, a definition based on the inductive property is more appropriate.

Accepting the definitions of current and p.d. in the absolute system, a circuit in which absolute unit of current is produced by unit p.d. is said to have 1 absolute unit of resistance. Let a single loop having an area of 1 sq. cm. be placed at any point in the field; let the loop be connected to a ballistic galvanometer by twisted leads so that a circuit of 1 e.m. unit of resistance results. If the plane of the loop be turned about until, on making or breaking the current producing the magnetic field, the ballistic galvanometer gives a maximum deflection, the normal to the plane of the loop lies in the direction of the magnetic induction at the point. If 1 e.m. unit of quantity passes through the galvanometer when the exciting circuit is broken, the loop is traversed by unit induction. A line drawn through the field to show the direction of the induction by its tangent is called a line of induction. A tube bounded by lines of induction and containing unit induction, is called a unit tube of induction; the number of unit tubes crossing normally through unit area is called the normal induction or flux density. In air, the tubes of induction and tubes of force form identical systems; in iron, they have distinct significance.*

Each tube of induction may be regarded as enclosing a kind of circuital stream of magnetic flux linked with some part of the exciting circuit, and perhaps at some part of its path linking some portion of a neighbouring circuit. The number of linkages of flux with a circuit per unit current in it is the self-inductance of the circuit. Alternatively, when the linkages with a circuit change at the rate of 1 tube per second and 1 e.m. unit of e.m.f. is induced, the circuit has unit inductance. Similar definitions clearly apply to mutual inductance.

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* See C. C. Hawkins, *The Dynamo*, 6th edition, Vol. 1, p. 4 *et seq.*

From these definitions the dimensions of the absolute unit of inductance can easily be obtained. Remembering that free unit poles follow the inverse square law of attraction $[\mathit{pole}^2] = [\mathit{force} \times \mathit{length}^2]$. The field strength or force per unit pole at the centre of a circle carrying a current measured in absolute units is inversely proportional to the radius of the circle and directly proportional to the current, i.e. $[\mathit{current}] = [\mathit{force} \times \mathit{length}/\mathit{pole}] = [\sqrt{\mathit{force}}]$. Now the quantity of electricity induced in a circuit is proportional to the linkages of the tubes of induction and inversely as the resistance of the circuit, i.e. $[\mathit{quantity}] = [\mathit{current} \times \mathit{time}] = [\mathit{linkages}/\mathit{resistance}]$; thus

$$
\begin{aligned}
[\mathit{inductance}] &= [\mathit{linkages}/\mathit{current}] = [\mathit{resistance} \times \mathit{time}] \\
&= [\mathit{e.m.f.} \times \mathit{time}/\mathit{current}] = \left[\frac{\mathit{work}}{\mathit{quantity}} \times \frac{\mathit{time}}{\mathit{current}}\right] \\
&= \left[\frac{\mathit{force} \times \mathit{length}}{\mathit{current}^2}\right] = [\mathit{length}].
\end{aligned}
$$

Hence in the absolute electromagnetic system of units inductance has the dimensions of length, and is measured in centimetres.

Owing to the inconvenient size of the absolute units, decimal multiples and sub-multiples of them are chosen for use in practical work. Thus—

1 volt = 10^8 absolute e.m. units of p.d.
1 ampère = 10^{-1} ,, ,, ,, current
1 coulomb = 10^{-1} ,, ,, ,, quantity
1 ohm = 10^9 ,, ,, ,, resistance
1 henry = 10^9 ,, ,, ,, inductance

Hence 1 henry = 10^9 cm. Many inductances are expressed in millihenrys, mH. = 10^6 cm.; very small inductances are often measured in microhenrys, μH. = 10^3 cm.

Thus, a circuit has one henry inductance when there are 10^8 linkages of induction with the circuit per ampère. Alternatively, if the current in a circuit changes at the rate of 1 ampère per second, and an induced e.m.f. of 1 volt results, the circuit has an inductance of 1 henry.

12. Effective Resistance. If a direct current I ampères is passed through a circuit, heat is generated at a rate I^2R watts, where R is the resistance of the circuit in ohms as measured in a d.c. Wheatstone bridge. R is sometimes called the "*ohmic*" *resistance* of the circuit.

If now an alternating current of r.m.s. value I ampères flows in the circuit, heat is produced at a greater rate I^2R', where R' is the *effective resistance* of the circuit as measured in an a.c. bridge. The sources of additional loss in the circuit are numerous, and all arise from the pulsating nature of the magnetic and the electric fields around the circuit. Briefly may be enumerated (i) the so-called "skin-effect" in the conductor; (ii) losses due to eddy currents induced by transformer action in neighbouring masses of metal which lie in the magnetic field of the circuit; (iii) conduction and dielectric losses in the insulation of the circuit, and the effects of self-capacitance.

SKIN EFFECT. Of these sources of additional loss, one of the most important is the "skin-effect," the nature of which can be understood from a simple example. Consider a long straight conductor of circular section in which a current can be made to flow by application of a p.d. between its extremities. If the conductor carry a direct current, the density of which is uniformly distributed over the cross-section, then the strength of the magnetic field at a radius r from the centre of the wire is proportional to r within the conductor and to $1/r$ outside it.

Now let the conductor carry an alternating current and assume at first that the current density is uniform, so that the distribution of the magnetic field is the same as with a direct current. Examine two equal filaments, one at the centre and one at the circumference of the conductor. The former is linked by the entire alternating flux produced by the wire; the latter, on the other hand, is linked only by the flux outside the wire. A similar filament at a radius r is linked solely by the flux outside that radius. Hence, under the supposition of a uniform current density, i.e. equal currents in each similar filament of which the conductor is composed, the inner filaments have a greater inductance than the outer. It follows, therefore, that, since all filaments are subjected to the same p.d., the current density over the inner portion of the conductor must be less than over the outer. Apparently then, the current cannot be uniformly distributed and is crowded towards the periphery of the wire. At high frequencies it flows chiefly in a thin circumferential layer or skin, thus increasing the apparent resistance of the wire and the heating loss in it. The "skin-effect" is clearly greater at high frequencies and also when the conductor of which the circuit is composed is of appreciable cross-section. Hence standard coils are often wound with a conductor made up of a large number of separately insulated strands twisted together and connected in parallel; the skin-effect in each will be much reduced by making the strand sufficiently fine, and by arranging the strands in such a way that each occupies in turn, from point to point in the length of the conductor, all possible positions in the composite section. Stranding alone without plaiting is not in itself a cure for skin-effect.

The skin-effect will modify not only the effective resistance of a circuit but will also change its effective inductance, since the distribution of the magnetic field with a.c. is different from that obtained with d.c. owing to the current being no longer uniformly distributed over the cross-section of the conductors. Again, the insulation conductance and the self-capacitance of the circuit, acting as shunts across its terminals, will cause the effective resistance and inductance to change with frequency (*see* pp. 135 and 136). Hence, due to these secondary effects, the effective resistance and inductance of a circuit are functions of frequency, the change with frequency of the former being usually greater than that of the latter.

In general, whenever an alternating current flows in a circuit, secondary effects come into play and modify the resistance of the circuit from its d.c. value, at the same time producing an alteration of the inductance from the value which would be found by a ballistic measurement. The effects depend on the frequency of the current, and are of great importance at the higher frequencies. The field of utility of a.c. bridge methods, therefore, lies in the measurement of effective resistance, inductance, and capacitance under exactly known conditions of frequency and wave-form similar to those obtaining in actual practical a.c. working.

CHAPTER II

THE SYMBOLIC THEORY OF ALTERNATING CURRENTS AND THE APPLICATION OF IT TO ALTERNATING CURRENT BRIDGE NETWORKS

1. Introduction. In Chapter I it has been shown that measurements by means of an alternating current bridge method are made by arranging a network of conductors,* one of which is the piece of apparatus which is to be tested, and the others are suitable standards of resistance, inductance, capacitance, or combinations of these quantities; these conductors are referred to as the *branches* of the network. Two points in the network are then connected to a source of alternating potential difference, while a second pair of points are " bridged " by a conductor in which is contained a suitable detecting instrument. The constants of the various branches are then adjusted until the detector indicates that no current flows in the bridge conductor. The network is then said to be *balanced* and certain relations, called the *balance conditions,* will exist between the constants of the branches. From these balance conditions it is then possible to calculate the unknown constants of the apparatus under test.

The purpose of this chapter is to show, in a general way, how these conditions may be most easily determined. To this end, a brief account of the elements of the theory of alternating currents is first given, treated according to the symbolic vector method. Maxwell's theory of networks is then explained and combined with the symbolic method to demonstrate those properties of alternating current circuits which are of service in the present problem. Typical bridge networks can then be considered and the general conditions of balance found.

2. Elements of Vector Algebra.† A *vector quantity,* that is a quantity having magnitude, direction, and sense, can be represented graphically by a line, called a *vector*. The length of the line represents to scale the magnitude of the vector quantity; the inclination of the vector to some datum line gives the direction of the quantity; and an arrow-head drawn on the vector indicates the sense along the specified direction. In mathematical notation, vector quantities can be represented by the use of special type; thus a **Clarendon** or heavy-type letter will denote a vector, the magnitude of which is represented by the corresponding *Italic* letter. Thus, let **r** in Fig. 3 be a vector of magnitude r. Draw the axes of co-ordinates and let **x** and **y** be vectors of unit length measured along OX and OY respectively. Let the vector **r** make an angle ϕ with the axis OX; then, resolving **r** in the directions of the two axes, let **a** and **b** be the component vectors. Then if a be the number of times **x** is contained in **a** and b the number of times **b** contains **y**,

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* The term " conductor " is here employed in a general way to include not only ordinary metallic conductors but also condensers when used in alternating current circuits, *see* page 15.

† The author wishes to acknowledge helpful suggestions from Mr. F. M. Colebrook, B.Sc., during the preparation of this and subsequent sections. The student requiring further details concerning the general use of vector

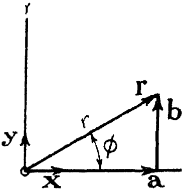

Fig. 3. — Graphical Representation of a Vector **r** in Terms of Unit Vectors **x** and **y**

$$\mathbf{a} = a\mathbf{x},$$

$$\mathbf{b} = b\mathbf{y},$$

are the components; and

$$\mathbf{r} = a\mathbf{x} + b\mathbf{y} \quad . \quad . \quad . \quad . \quad (1)$$

is the *vector equation* determining **r**. The sign " + " is understood to mean vector addition. This equation represents a vector of magnitude $r = \sqrt{a^2 + b^2}$ and direction $\phi = \arctan b/a$ relative to the axis OX, the sense being indicated by the arrow-head.

It will be apparent that the joint effect or sum of any number of vectors in a plane can be obtained by adding their component vectors parallel to OX and OY. The resultant

analysis in a.c. theory should refer to Mr. Colebrook's book, *Alternating Currents and Transients Treated by the Rotating Vector Method*, McGraw-Hill (1925).

For a treatment of vector algebra from the point of view of the electrical engineer, *see* W. G. Rhodes, *An Elementary Treatise on Alternating Currents*, Chaps. VII and VIII (1902). Also Oliver Heaviside, *Electrical Papers*, Vol. 2, pp. 4–5, for accurate definitions.

vector $\mathbf{r}$ obtained by adding n vectors of the type given by Equation (1) will be

$$\mathbf{r} = (\Sigma_1^n a_n)\mathbf{x} + (\Sigma_1^n b_n)\mathbf{y} = a\mathbf{x} + b\mathbf{y},$$

an equation which has an obvious geometrical interpretation illustrated in Fig. 4. The extension of this process to the subtraction of vectors is obvious; the sense of the vector to be subtracted is reversed and the rule for addition applied.

When the product of two vectors is desired, it will be clear that, as the vectors have not only magnitude but also direction the nature of the product becomes a matter for definition. It is found that there are two types of product to which definite physical meaning can be given, called the *scalar product* and the *vector product* respectively, these being defined in the following way—

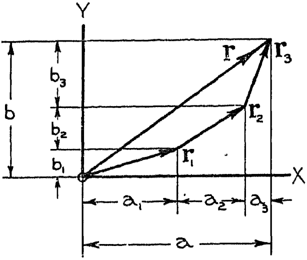

Fig. 4.—The Addition of Vectors

(i) The *scalar* or *dot product* of two vectors $\mathbf{r}$ and $\mathbf{r}'$ is defined as the product of their magnitudes and the cosine of the angle α between their directions, i.e.

$$\mathbf{r}\cdot\mathbf{r}' = rr' \cos \alpha.$$

The result is a scalar quantity. Note that $\mathbf{r}\cdot\mathbf{r}' = \mathbf{r}'\cdot\mathbf{r}$.

(ii) The *vector* or *cross product* of two vectors $\mathbf{r}$ and $\mathbf{r}'$ is defined as the product of their magnitudes and the sine of the angle α between their directions, i.e.

$$\mathbf{r} \times \mathbf{r}' = (rr' \sin \alpha)\mathbf{z}.$$

The result is a vector perpendicular both to $\mathbf{r}$ and $\mathbf{r}'$, $\mathbf{z}$ being unit vector in that direction. The sense of $\mathbf{z}$ relative to that of the other vectors is such that when $\mathbf{r}$ is turned to coincide with $\mathbf{r}'$ the positive direction of $\mathbf{z}$ is related to the rotation in the same way as the translation and rotation of an ordinary right-handed screw. Note that $\mathbf{r} \times \mathbf{r}' = -\mathbf{r}' \times \mathbf{r}$.

In order to appreciate the meaning of these definitions, consider the two products of a vector of force $\mathbf{F}$, with a vector of linear displacement $\mathbf{d}$. The dot product clearly represents mechanical work expended by the action of the force F through distance $d \cos \alpha$; it is a scalar quantity. On the other hand, the cross product is the couple produced by the force F acting at a lever arm $d \sin \alpha$; this is clearly a vector quantity having the direction of the axis of the couple, i.e. perpendicular both to **F** and to **d**.

The theory of alternating currents to be developed in a later section is directly deduced from the idea of scalar product of two vectors. In particular the scalar product of a vector **r** with the unit vector **x** in Fig. 3 is

$$\mathbf{r}\cdot\mathbf{x} = r \cos \phi, \qquad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

a theorem of considerable utility in later work.

3. The Operator j. It is now necessary to introduce the idea of an operation into the preceding vector algebra. As a preliminary, consider the simpler conception of a scalar operator embodied, for example, in the symbol 2. According to the method of interpretation the symbol 2 can denote either (i) a magnitude of two units; or (ii) if written in the form 2(1) may be read to signify the arithmetic *operation* of doubling unit magnitude. In the latter interpretation the symbol is used as a notation for scalar operation.

When this idea is applied to a vector quantity an extension of the principle is necessary, since direction as well as magnitude must be taken into account. The result of applying a scalar operation to a vector is simply to multiply the magnitude of the vector without affecting the direction of it. Now let the symbol $j\,(\;)$ denote the operation of rotating a vector through a counterclockwise angle of $\pi/2$ without alteration to its magnitude. Then, according to this definition,*

$$\begin{aligned} j\mathbf{x} &= \mathbf{y}, \\ j(j\mathbf{x}) &= j\mathbf{y} = -\mathbf{x}, \\ j[j(j\mathbf{x})] &= j(-\mathbf{x}) = -j\mathbf{x}, \end{aligned}$$

and so on, as illustrated by Fig. 5. Hence denoting successive operations with j upon any vector operand by powers of j

$$\begin{aligned} j\,(\quad) &= j\,(\quad), \\ j^2(\quad) &= -(\quad), \\ j^3(\quad) &= -j(\quad), \end{aligned}$$

and so on. From this it follows that the arithmetic relation existing between the results of successive operations with j

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* The bracket following j, in which the vector operand is contained, may be omitted in cases where there can be no ambiguity concerning the operand to which j is applied.

is, when expressed by powers of j, the same as that connecting the number $\sqrt{-1}$ with its powers.* Hence in the evaluation of expressions, the arithmetic effect of operation with j can be treated as equivalent to multiplication by $\sqrt{-1}$.

It is now possible to re-write Equation (1) in the operational notation just described. Since $j\mathbf{x} = \mathbf{y}$, Equation (1) becomes

$$\mathbf{r} = (a + jb)\mathbf{x}, \qquad \qquad (3)$$

illustrated by Fig. 6. The expression $(a + jb)$ is called a *complex operator*, and has the following properties when it acts on a vector operand. Referring to the above equation, and to Fig. 6, the interpretation of the equation is seen to be this; take a vector $a\mathbf{x}$ along OX and to the extremity of it attach a vector $b\mathbf{x}$. Operate with j upon the latter, i.e. turn it through $\pi/2$; then the joint effect of $a\mathbf{x}$ and $jb\mathbf{x}$ is a vector $\mathbf{r}$.

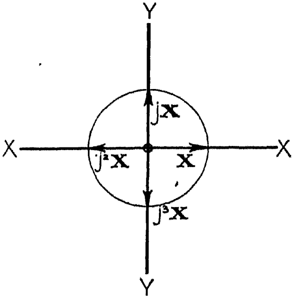

FIG. 5.—THE OPERATOR j

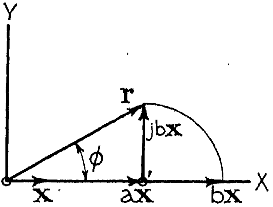

FIG. 6.—GRAPHICAL REPRESENTATION OF A VECTOR $\mathbf{r}$ IN TERMS OF UNIT VECTOR $\mathbf{x}$ AND THE OPERATOR j

This operation may be looked at in another way. The vector **r** has a magnitude $r = \sqrt{a^2+b^2}$ and a direction $\phi = \arctan b/a$ relative to **x**. Hence the effect of $(a + jb)$ on **x** is to multiply the magnitude of it by r and to rotate it through an angle ϕ. The operation consists, therefore, partly of an extending or *tensor* element and partly of a rotatory or *versor* effect. The two equivalent ways of regarding a complex operation may be symbolically represented by

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* This is by no means the same as the statement, so often made in text-books, that $j \equiv \sqrt{-1}$. The symbol j denotes the geometric *operation* of rotating a vector through $\pi/2$ radians; the quantity $\sqrt{-1}$ is an imaginary *numeric*. It is looseness of statement to describe these very different conceptions as identical; the real meaning of j and its relationship to $\sqrt{-1}$ is that given in the text. *See* T. R. Lyle, "Currents in branched and mutually influencing circuits produced by harmonically varying electromotive forces," *Elecn.*, Vol. 41, pp. 816–818 (1898), and Vol. 42, pp. 72–74, 148–151 (1899).

$$(a + jb) = \sqrt{a^2+b^2}\,\underline{/\arctan b/a} = r\underline{/\phi} = r\,(\cos\phi + j\sin\phi) \quad (4)$$

Moreover, it is clear that the arguments just applied to the operations on unit vector apply with equal force to any other vector operand.

4. Alternating Currents as Vectors.* The current in a circuit is a physical quantity which has at every instant a definite magnitude and flows in a certain sense round the circuit; it is possible in virtue of these properties to represent it by a vector, and a similar argument will apply to the electromotive force to which the current is due.

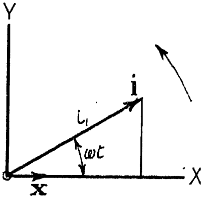

FIG. 7.—REPRESENTATION OF AN ALTERNATING CURRENT BY MEANS OF A ROTATING VECTOR

In particular, if the current is such that its strength at any instant is a harmonic function of time, it is referred to as an *alternating current.* The simplest type of alternating current, with which it is proposed exclusively to deal, can be represented as

$$i = i_1 \cos \omega t,$$

where i is the instantaneous value of the current at time t, i_1 is its maximum value, and ω is 2π times the frequency of alternation.

Now consider a vector **i**, of magnitude i_1 rotating about a point O with angular velocity ω, as shown in Fig. 7. Take the scalar product of **i** with **x**, as in Equation (2), then

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* *See* T. R. Lyle, *loc. cit.*; W. G. Rhodes, *loc. cit.*; W. E. Sumpner, "The vector properties of alternating currents and other periodic quantities," *Proc. Roy. Soc.*, Vol. 61, pp. 465–478 (1897); C. V. Drysdale, *The Foundations of Alternate Current Theory*, Chap. XIII (1910). The symbolic treatment of alternating currents is exclusively adopted in the writings of Steinmetz; *see*, for example, *Theory and Calculation of Alternating Current Phenomena*, 1st edition (1897).

Also E. Orlich, *Kapazität und Induktivität*, Sec. 35, pp. 98–107 (1909).

$$\mathbf{i}\cdot\mathbf{x} = i_1 \cos \omega t = i \tag{5}$$

Hence the harmonic vector **i** will represent completely the current i, since the result of taking its scalar product with **x** gives the magnitude of its OX component, i.e. the instantaneous value of the current. This provides at once a means of transforming the well-known scalar equations of the theory of alternating currents into vector notation, a proceeding which results in considerable analytical simplification and in greater clarity of statement.

In many alternating current problems the differential coefficients of i with respect to t are involved; it will now be shown how these may be expressed in symbolic notation. From Equation (5),

$$\frac{di}{dt} = \omega i_1 \cos\left(\omega t + \frac{\pi}{2}\right. \qquad \frac{d}{dt}(\mathbf{i}\cdot\mathbf{x}) = \mathbf{x}\cdot\frac{d\mathbf{i}}{dt},$$

since **x** is not a function of time. Now operate on Equation (5) with j, thus,

$$\mathbf{x}\cdot j\mathbf{i} = i_1 \cos\left(\omega t + \frac{\pi}{2}\right);$$

multiply by ω,

$$\mathbf{x}\cdot j\omega\mathbf{i} = \omega i_1 \cos\left(\omega t + \frac{\pi}{2}\right.$$

From this and the above result of direct differentiation

$$\frac{d\mathbf{i}}{dt} \quad j\omega\mathbf{i}.$$

In general,

$$\frac{d^n\mathbf{i}}{dt^n} = (j\omega)^n\mathbf{i}, \tag{5a}$$

by an obvious extension of the above process. Hence the result of successive differentiations of a harmonic vector is the same as that obtained by successive operations with $j\omega$.

As an example of the use of these principles consider the case of a coil of inductance L and resistance R to which an alternating potential difference is applied. If e be the instantaneous value of the potential difference and i that of the current, the scalar equation for the coil is

$$e = L\frac{di}{dt} + Ri.$$

Now let $\mathbf{e}$ be the vector of potential difference and $\mathbf{i}$ that of current, then $e = \mathbf{e}\cdot\mathbf{x}$ and $i = \mathbf{i}\cdot\mathbf{x}$. Making use of Equation 5 (a) the scalar equation becomes

$$\mathbf{e}\cdot\mathbf{x} = j\omega L\mathbf{i}\cdot\mathbf{x} + R\mathbf{i}\cdot\mathbf{x}$$

or simply

$$\mathbf{e} = (R + j\omega L)\mathbf{i}$$

is the vector equation.

From this example it is apparent that the rule to convert a given scalar equation, which involves harmonic functions of time, into vector notation is to replace the scalar symbols by the vector symbols and to substitute successive operations with $j\omega$ for the differential coefficients.

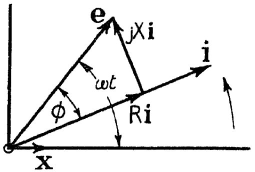


Fig. 8.—Graphical Representation of Ohm's Law for Harmonic Vectors

5. The Vector Ohm's Law. The particular problem worked out above is capable of important generalization. Take the case of a circuit having resistance R and reactance X, $\mathbf{e}$ being the vector representing the potential difference applied to it and $\mathbf{i}$ that denoting the current flowing into it. Then, as illustrated in Fig. 8, the vector $\mathbf{e}$ is composed of two components, one, $R\mathbf{i}$, in phase with $\mathbf{i}$, and a second, $jX\mathbf{i}$, perpendicular to $\mathbf{i}$. The vector equation is

$$\mathbf{e} = (R + jX)\mathbf{i} = z\mathbf{i},$$

the symbol z denoting the *impedance operator* of the circuit.

Examining this equation, it will be seen that the *form* is the same as that representing Ohm's law for pure resistances, $e = Ri$. It may be said, therefore, to express *Ohm's law for harmonic vectors*. In virtue of this fact it appears that any result proved for direct currents will hold for alternating currents if vector symbols are used in place of scalar symbols and impedance operators* are substituted for resistances.

6. The Operator z. The impedance operator z is a complex operator which can be expressed in another way by means of Equation (4), that is,

$$z = R + jX = \sqrt{R^2 + X^2}\,\underline{/\arctan \frac{X}{R}} = Z\,\underline{/\phi} = Z(\cos\phi + j\sin\phi)$$

The quantity $Z = \sqrt{R^2 + X^2}$ is called the *impedance* of the circuit.

Making use of this result in the preceding paragraph gives

$$\mathbf{e} = (R + jX)\mathbf{i} = z\mathbf{i} = (Z\,\underline{/\phi})\mathbf{i}, \quad . \quad . \quad . \quad (6)$$

which shows that the vector of potential difference is obtained by multiplying the current vector by Z and advancing its phase by ϕ. Hence the vector $\mathbf{e}$ is Z times the size of the vector $\mathbf{i}$ and, when ϕ is positive, is ahead of the latter in phase; a result in agreement with the vector diagram of the circuit and concisely represented by Equation (6).

In order to carry out calculations on electric circuits in which alternating currents are flowing, it is necessary to determine z. A few simple examples will now be considered and applied to circuits commonly met with in bridge networks.

1. *Pure Resistance.* In this case the current and potential difference are obviously in phase, so that if R be the resistance.

$$z = R = Z, \text{ and } \phi = 0.$$

2. *Inductive Coil.* Let L be the inductance and R the resistance of the coil, then the scalar equation is

$$e = L\frac{di}{dt} + Ri,$$

which in vector form is

$$\mathbf{e} = (R + j\omega L)\mathbf{i}$$

by the principles on page 32.

Thus,

$$z = R + j\omega L$$

$$Z = \sqrt{R^2 + \omega^2 L^2}$$

$$\phi = \arctan \omega L/R.$$

---

* For the general discussion of operators applied to the theory of electric circuits, *see* O. Heaviside, "On resistance and conductance operators, and their derivatives, inductance, and permittance, especially in connection with electric and magnetic energy," *Electrical Papers*, Vol. 2, pp. 355–374 (1892). The operators considered in this and the next paragraphs are special forms of the resistance and conductance operators of Heaviside when sinusoidal operands are assumed.

3. *Condenser.* Suppose q to be the charge in the condenser at any instant, then if C is the capacitance,

$$e = \frac{q}{C},$$

is the scalar equation.

Differentiating, $$\frac{de}{dt} = \frac{1}{C}\frac{dq}{dt} = \frac{1}{C}i.$$

In vector notation

$$j\omega\mathbf{e} = \frac{\mathbf{i}}{C}$$

$$\mathbf{e} = \frac{\mathbf{i}}{j\omega C} = \frac{-j}{\omega C}\mathbf{i}.$$

For this case, therefore,

$$-\frac{}{\omega C}$$

$$Z = \frac{1}{\omega C},$$

and $$\phi = \arctan(-\infty) = -\frac{\pi}{2},$$

so that the potential difference lags behind the current by a quarter of a period.

4. *Impedances in Series.* Consider the case of a number of impedances in series, $\mathbf{e}$ being the applied harmonic potential difference and $\mathbf{i}$ the corresponding vector of current. Then

$$\mathbf{e} = z_1\mathbf{i} + z_2\mathbf{i} + z_3\mathbf{i} + \ldots \text{ etc.} = z\mathbf{i},$$

if z is the impedance operator of the circuit. From this

$$z = z_1 + z_2 + z_3 + \ldots \text{ etc.},$$

so that the operators of impedances in series are added in the same way as resistances are added in direct current circuits. (*See* Fig. 9.)

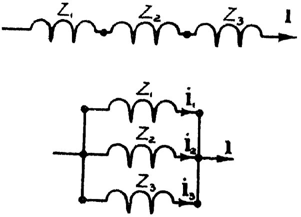


FIG. 9.—IMPEDANCES IN SERIES AND IN PARALLEL

5. *Impedances in Parallel.* When impedances are connected in parallel, as in Fig. 9,

$$\mathbf{e} = z_1\mathbf{i}_1 = z_2\mathbf{i}_2 = z_3\mathbf{i}_3 \ldots \text{ etc.},$$

so that $$\mathbf{i} = \mathbf{i}_1 + \mathbf{i}_2 + \mathbf{i}_3 + \ldots = \mathbf{e}\left(\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} + \ldots\right) = \frac{\mathbf{e}}{z}.$$

Then, $$\frac{1}{z} = \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} + \ldots$$

so that the operators of impedances in parallel are combined by the rule which applies to resistances in parallel.

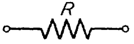
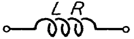
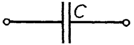
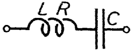
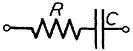
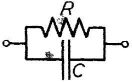
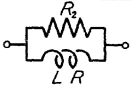
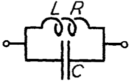
Circuit	z	$1/z$	Z	ϕ
	R	$1/R$	R	0
	$R+j\omega L$	$\frac{R-j\omega L}{R^2+\omega^2L^2}$	$\sqrt{R^2+\omega^2L^2}$	$\arctan\frac{\omega L}{R}$
	$-j/\omega C$	$j\omega C$	$1/\omega C$	$-\pi/2$
	$R+j[\omega L-(1/\omega C)]$	$\frac{R-j[\omega L-(1/\omega C)]}{R^2+[\omega L-(1/\omega C)]^2}$	$\sqrt{R^2+[\omega L-(1/\omega C)]^2}$	$\arctan\frac{\omega L-(1/\omega C)}{R}$
	$R-(j/\omega C)$	$\frac{R+(j/\omega C)}{R^2+(1/\omega^2C^2)}$	$\frac{1}{\omega C}\sqrt{1+\omega^2C^2R^2}$	$-\arctan(1/\omega CR)$
	$\frac{R(1-j\omega CR)}{1+\omega^2C^2R^2}$	$(1/R)+j\omega C$	$\frac{R}{\sqrt{1+\omega^2C^2R^2}}$	$-\arctan\omega CR$
	$\frac{R_2\{[R(R+R_2)+\omega^2L^2]+j\omega LR_2\}}{(R+R_2)^2+\omega^2L^2}$	$\frac{[R(R+R_2)+\omega^2L^2]-j\omega LR_2}{R_2(R^2+\omega^2L^2)}$	$R_2\sqrt{\frac{R^2+\omega^2L^2}{(R+R_2)^2+\omega^2L^2}}$	$\arctan\frac{\omega LR_2}{R(R+R_2)+\omega^2L^2}$
	$\frac{R+j\omega[L(1-\omega^2CL)-CR^2]}{(1-\omega^2CL)^2+\omega^2C^2R^2}$	$\frac{R-j\omega[L(1-\omega^2CL)-CR^2]}{R^2+\omega^2L^2}$	$\sqrt{\frac{R^2+\omega^2L^2}{(1-\omega^2CL)^2+\omega^2C^2R^2}}$	$\arctan\frac{\omega[L(1-\omega^2CL)-CR^2]}{R}$

FIG. 10.—TABLE OF IMPEDANCE AND ADMITTANCE OPERATORS

By the use of the five examples worked out above, it is easy to calculate the impedance operator, and thence the impedance and phase angle, of any given circuit. In Fig. 10* a number of circuits of common occurrence in alternating current bridge networks are shown, together with the appropriate values of z, Z, and ϕ. Frequent reference will be made in subsequent chapters to the results tabulated in this diagram.

7. The Operator $\frac{1}{z}$. In alternating current calculations it is usually required to find the current in terms of the applied potential difference. Transposing the vector equation gives

$$\mathbf{i} = \frac{1}{z}\mathbf{e} = y\mathbf{e},$$

so that it is necessary to interpret the operation $\frac{1}{z}$, called the *admittance operator*.

Now,

$$y = \frac{1}{z} = \frac{1}{R+jX} = \frac{R-jX}{(R+jX)(R-jX)} = \frac{R-jX}{R^2+X^2}.$$

Writing $G = R/(R^2+X^2) = R/Z^2$ and $B = -X/(R^2+X^2) = -X/Z^2$, where G is defined as the *conductance* and B as the *susceptance* of the circuit,

$$y = G + jB = \sqrt{G^2+B^2}\,\underline{/\arctan\frac{B}{G}} = Y\,\underline{/-\phi} = \frac{1}{Z}\,\underline{/-\phi},$$

since $\qquad Y = \sqrt{G^2+B^2} = 1/\sqrt{R^2+X^2} = 1/Z$

Note that B is negative for a coil (X positive) and positive for a condenser (X negative).

Then $$\mathbf{i} = y\mathbf{e} = \frac{1}{z}\mathbf{e} = \left(\frac{1}{Z}\,\underline{/-\phi}\right)\mathbf{e} \tag{6a}$$

The quantity $Y = \frac{1}{Z}$ is called the *admittance* of the circuit.

Equation (6*a*) shows that the current vector is obtained from the potential difference vector by multiplying the latter by the admittance and retarding its phase by an angle ϕ. For example, suppose $e = e_1 \cos \omega t = \mathbf{e}\cdot\mathbf{x}$ as shown in Fig. 8; then, from Equation (6a),

---

* The student to whom the subject is new is advised to verify as an exercise the results tabulated in this diagram.

$$\mathbf{i}\cdot\mathbf{x} = \left(\frac{1}{Z}\underline{/-\phi}\right.\ \mathbf{e}\cdot\mathbf{x}$$

which is
$$i = \left(\frac{1}{Z}\underline{/-\phi}\right).e_1 \cos \omega t = \frac{e_1}{Z}\cos(\omega t - \phi),$$

in agreement with the geometry of the diagram.

8. Impedance and Admittance Operator Loci. Let i_1 be the maximum value of a harmonic current and $\mathbf{1}$ a unit vector drawn in the direction of the harmonic vector $\mathbf{i}$; then Equation (6) can be written

$$\mathbf{e} = z\mathbf{i} = zi_1\mathbf{1} = (R + jX)i_1\mathbf{1},$$

so that it is only necessary to deal with the meaning of the vector $z\mathbf{1}$. This vector is drawn in Fig. 11 together with its components, using the current vector as a datum axis.

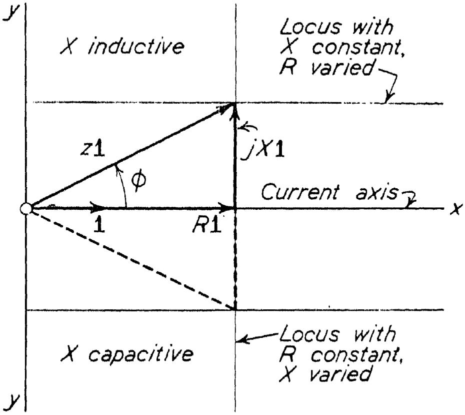


FIG. 11.—IMPEDANCE OPERATOR LOCI

Now let *R be given a fixed value while X is varied*; then the locus of the extremity of $z\mathbf{1}$ is a straight line through the end of $R\mathbf{1}$ and perpendicular to it. The value of X can run from $-\infty$ to $+\infty$; negative values of X refer to a capacitive circuit and positive values to an inductive circuit. When X is zero the circuit is either in resonance or it is a pure resistance. In the circuits considered in this book R is essentially a positive quantity, energy being absorbed by the circuit and converted into heat.* The loci for various values of R are lines parallel to yOy in the half-plane to the right thereof; a linear scale for R can be marked off along Ox.

When *X is given a fixed value while R is varied* the loci of the extremity of $z\mathbf{1}$ are a family of straight lines parallel to $\mathbf{1}$ through the end of $jX\mathbf{1}$ and extending from yOy ($R = 0$) to the right. A linear scale for X can be marked along yOy, positive values above Ox (inductive circuits) and negative values below it (capacitive circuits).

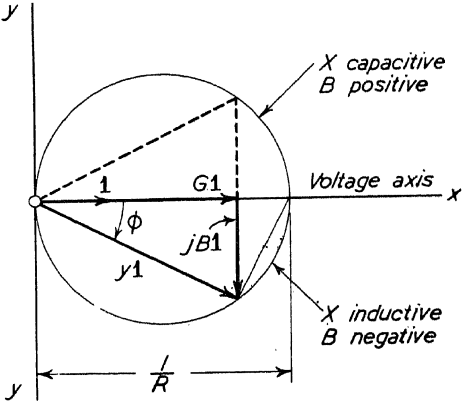


FIG. 12.—ADMITTANCE OPERATOR LOCUS; R CONSTANT, X VARIED

In a similar way, if e_1 is the maximum value of a harmonic voltage and $\mathbf{1}$ is a unit vector drawn in the direction of the vector $\mathbf{e}$, Equation (6a) becomes

$$\mathbf{i} = ye_1\mathbf{1} = \frac{1}{z}e_1\mathbf{1} = \frac{1}{R + jX}\,e_1\mathbf{1}$$

The inverse operation $1/z$ will now be interpreted.

With *R a given fixed value and X varied* the locus of the end of $y\mathbf{1}$ can be found in the following way.

---

* In the general case all energy-absorbing circuits, e.g. motors, are equivalent to a positive R and have loci to the right of yOy. Energy-generating circuits are equivalent to a negative R and have their loci on the left half-plane.

Since $\cos\phi = R/Z$ and $\sin\phi = X/Z$,

$$G = \cos^2\phi/R \text{ and } B = -\sin\phi\cos\phi/R,$$

hence $$y = G + jB = \frac{1}{R}(\cos^2\phi - j\sin\phi\cos\phi).$$

In Fig. 12 taking the voltage vector as the datum, mark off a length to represent $1/R$ and on this as diameter draw a circle; then it is easy to verify that the geometry of Fig. 12 satisfies this equation. The locus of $y\mathbf{1}$ is, therefore, a circle with its diameter $1/R$ along Ox. The family of loci for various values of R are circles with the origin as a common extremity of their diameters; the circle shrinks to a point at O when R is infinite. With the usual case of R a positive quantity the loci are confined again to the half-plane on the right of yOy.

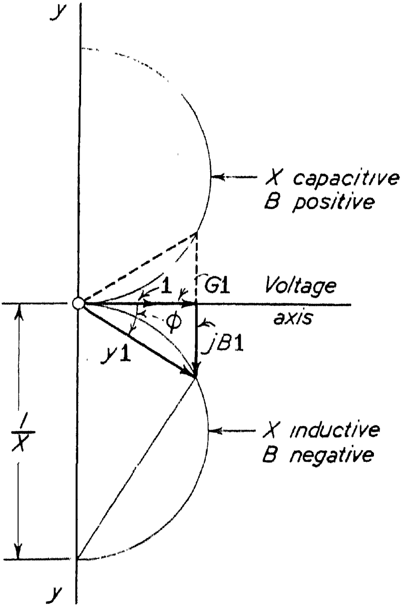


FIG. 13.—ADMITTANCE OPERATOR LOCUS; X CONSTANT, R VARIED

To find the locus *when X has a fixed value and R is varied*; from above

$$G = \cos\phi \sin\phi/X \text{ and } B = -\sin^2\phi/X,$$

so that

$$y = G + jB = \frac{1}{X}(\cos\phi\sin\phi - j\sin^2\phi)$$

This is easily seen to be satisfied by the geometry of Fig. 13. Hence the locus of $y\mathbf{1}$ for positive values of R is a semicircle with its diameter $1/X$ on the axis yOy, one end of the diameter being at the origin; the lower semicircle applies when X is positive and the upper one when X is negative.

In general the locus of any expression of the form $(a + jb)\mathbf{1}$ is a straight line when either a or b are varied, while that of the expression $\mathbf{1}/(a + jb)$ under the same conditions is a circle through the origin. The operators $z = a+jb$ and $y = 1/(a+jb) = 1/z$ are inverse, and the process of passing from one to the other is called *inversion*. Now if

$$r = \sqrt{(a^2 + b^2)} \text{ and } \phi = \arctan b/a,$$

$$z = a + jb = r\, \underline{/\phi}$$

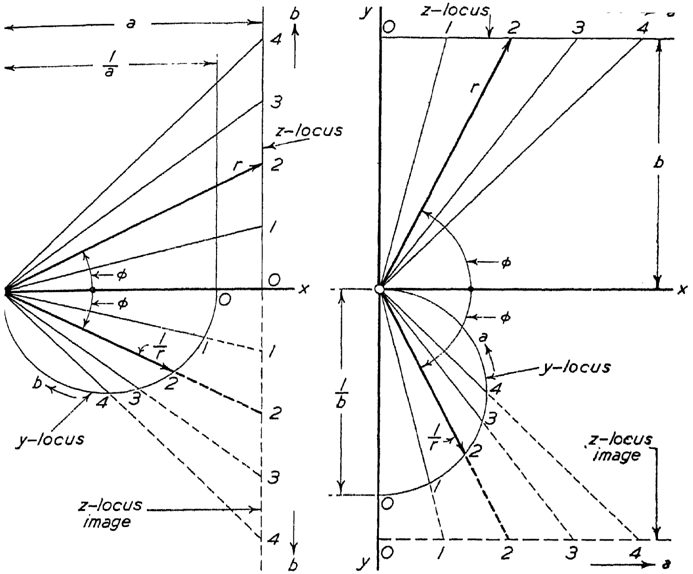
and $$y = 1/z = 1/(a + jb) = (1/r)\, \underline{/-\phi}$$

Hence any z-locus is turned into a y-locus by taking the reciprocal of the radius vector r from the origin, and reflecting through an equal angle ϕ on the opposite side of the axis Ox. Since the z-loci are lines and the y-loci circles, lines invert into circles, and conversely; the origin is the *centre of inversion*. These relations are shown for the loci $a =$ constant and $b =$ constant in Figs. 14 (a) and (b) respectively. Only those parts of the diagrams are drawn which relate to positive quantities; the completion for negative values can easily be made by the reader.

9. The General Circular Locus. In the preceding section a and b are real numbers. Consider now an operator of the form $\alpha_0 + \alpha_1\xi$ where α_0 and α_1 are given constant complex operators and ξ is a scalar variable quantity. Then if $\mathbf{1}$ is a unit vector along Ox it is required to find the locus of the vector

$$(\alpha_0 + \alpha_1\xi)\mathbf{1} \equiv \alpha_0\mathbf{1} + \xi\alpha_1\mathbf{1}$$

as ξ varies. As Fig. 15 shows, $\alpha_1\mathbf{1}$ is a vector through the origin equal to the size of α_1 times $\mathbf{1}$, and turned through the phase angle of α_1 from



a constant, b varied.

(a)

b constant, a varied.

(b)

FIG. 14.—INVERSION OF IMPEDANCE AND ADMITTANCE OPERATOR LOCI

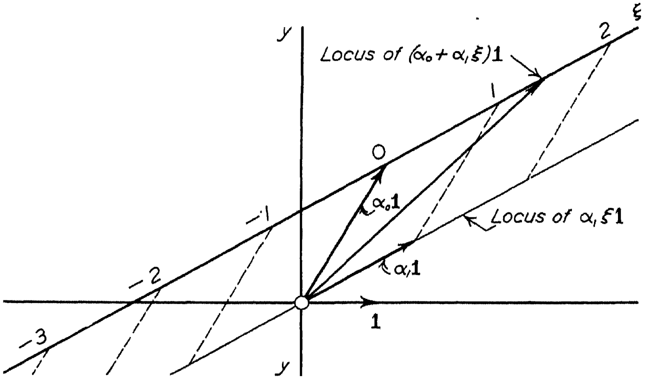


FIG. 15.—THE LINEAR LOCUS $(\alpha_0 + \alpha_1\xi)1$

Ox. Then $\xi\alpha_1\mathbf{1}$ is simply the linear extension of $\alpha_1\mathbf{1}$. Likewise, $\alpha_0\mathbf{1}$ is a constant vector. Its addition to $\xi\alpha_1\mathbf{1}$ shifts the locus to a line parallel to $\alpha_1\mathbf{1}$ passing through the end of $\alpha_0\mathbf{1}$, this end being the point where $\xi = 0$. Hence the expression $(\alpha_0 + \alpha_1\xi)\mathbf{1}$ is a straight line inclined to the axis Ox and not, in general, passing through O. The line passes through the origin either when α_0 and α_1 have the same phase angle, or when α_0 is zero.

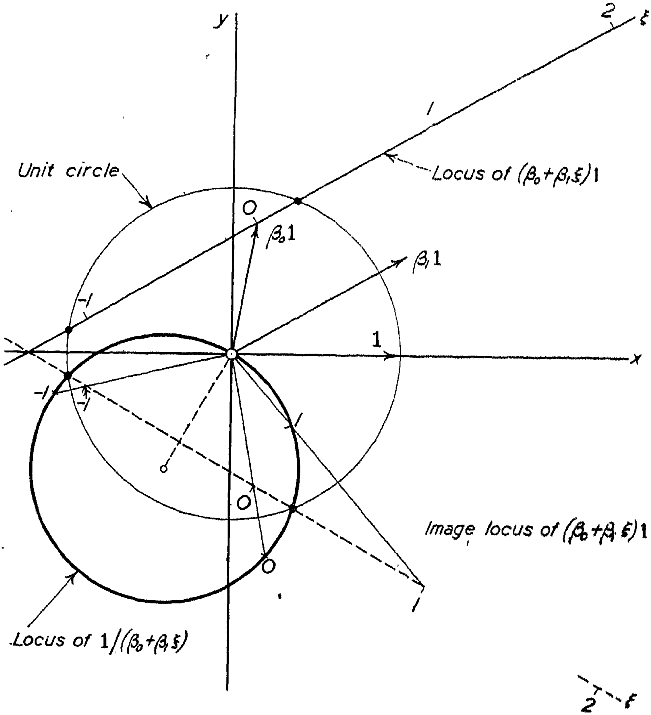

y
2
ξ
1
Unit circle
Locus of $(\beta_0+\beta_1\xi)\mathbf{1}$
O
$\beta_0\mathbf{1}$
$\beta_1\mathbf{1}$
-1
1
x
-1
-1
1
O
O
Image locus of $(\beta_0+\beta_1\xi)\mathbf{1}$
1
Locus of $\mathbf{1}/(\beta_0+\beta_1\xi)$
2
ξ

FIG. 16.—THE CIRCULAR LOCUS $\mathbf{1}/(\beta_0 + \beta_1\xi)$

In a similar way the expression

$$(\beta_0 + \beta_1\xi)\mathbf{1} \equiv \beta_0\mathbf{1} + \xi\beta_1\mathbf{1}$$

where β_0 and β_1 are constant complex operators, can be interpreted, and represents the linear locus drawn in Fig. 16. Taking the reciprocal operation, $\mathbf{1}/(\beta_0 + \beta_1\xi)$, the considerations of Section 8 show that the inverse locus is a circle, which passes through the origin since the vector vanishes for $\xi = \infty$. Hence the inverse figure of any straight line with respect to the origin as centre of inversion is a circle through the origin. By drawing the line $(\beta_0 + \beta_1\xi)\mathbf{1}$ and reflecting it in the axis Ox the angle component of the inversion is performed. A vector of unit length is unaltered in magnitude by inversion. A circle drawn through the end of $\mathbf{1}$ cuts the linear locus $(\beta_0 + \beta_1\xi)\mathbf{1}$ in two points, and also cuts the image line in two points that are the inverse of the first pair with respect to the origin. These two inverse points and the origin fix the circular locus $\mathbf{1}/(\beta_0 + \beta_1\xi)$; corresponding points on the line and circle are shown, and the origin corresponds with $\xi = \pm\infty$.

Now examine an expression of the form

$$\mathbf{p} = \frac{\alpha_0 + \alpha_1\xi}{\beta_0 + \beta_1\xi}\mathbf{1}$$

By division,

$$\mathbf{p} = \left[\frac{\alpha_1}{\beta_1} + \left(\alpha_0 - \frac{\beta_0}{\beta_1}\alpha_1\right) \cdot \frac{1}{\beta_0 + \beta_1\xi}\right]\mathbf{1}$$

The term $(\alpha_1/\beta_1)\mathbf{1}$ is simply a constant vector and corresponds with a mere addition of a constant vector to the following term. The preceding paragraph has shown that $\mathbf{1}/(\beta_0 + \beta_1\xi)$ is a circle. Since the factor in round brackets is a constant complex operator, its effect is to multiply the size of the circle by the magnitude of this operator, and to rotate the circle through the phase angle of the operator. Hence an expression such as $\mathbf{p}$ represents in its most general form a circular locus not passing through the origin. It is to be noted that since the operator $(\alpha_0 + \alpha_1\xi)/(\beta_0 + \beta_1\xi)$ is homogeneous in its numerator and denominator its inverse operator results in a circular locus also; hence the inverse of a circle is, in general, a second circle. Operators of this class are known as *bilinear operators* and have a wide range of application in the theory of alternating current circuits.* In many circuit problems it is possible to express the current or voltage in part of a network by a symbolic equation resembling $\mathbf{p}$, when one circuit parameter is varied; the range of variation of the dependent current or voltage is then confined to a circular locus and can easily be studied graphically, thus avoiding much tedious and lengthy algebra and arithmetical computation. Applications to the theory of unbalanced bridge networks will be given in Sections 22 and 23 and to their balance convergence on p. 299.

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* The best general treatments of loci in a.c. theory are given by O. Bloch, *Die Ortskurven der graphischen Wechselstromtechnik* (Rascher, 1917) and by A. Blondel, *Les Courants Alternatifs* (Baillière, 1933). See also, M. Schenkel, "Geometrische Oerter an Wechselstromdiagramm," *Elekt. Zeits.*, vol. 22, pp. 1043–1045 (1901); W. O. Schumann, "Zur Theorie der Kreisdiagramme," *Arch f. Elekt.*, vol. II, pp. 140–146 (1922); G. Hauffe, "Zur Theorie der allgemeinen Ortskurven," *E.u.M.*, vol. 48, pp. 57–58 (1930); A.C. Seletzky, "Circular loci of currents and voltages in a general network," *Journal F. Inst.*, vol. 222, pp. 197–209 (1936); H. Kafka, "Die Ermittlung von orthogonalen Kreisscharennetzen für die Darstellung vektorieller Zusammenhänge," *Arch. f. Elekt.*, vol. 30, pp. 712–728 (1936); A. Hazeltine, "Current loci in the general a.c. network," *Elec. Eng.*, vol. 56, pp. 325–330 (1937). For the treatment of a bilinear operator as a transformation of a line into a circle see G. Holzmüller, *Einführung in die Theorie der isogonalen Verwandschaften und der conformen Abbildungen* (Teubner, 1882).

The diameter of the locus and the position of its centre can be found from expressions that are given in the literature cited, but these are somewhat complicated and necessitate considerable laborious arithmetic. An indirect method is therefore preferred and consists in locating three points on the circle; the latter is then easily drawn. Put $\xi = 0$,

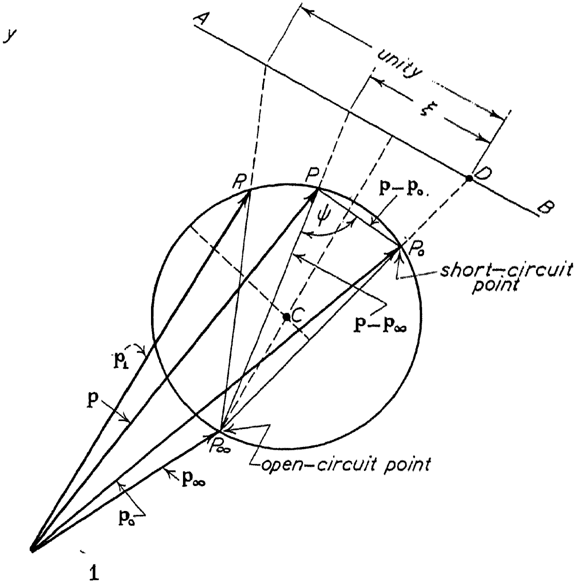

FIG. 17.—THE GENERAL CIRCULAR LOCUS

corresponding in an electrical network to reducing a variable resistance or reactance to zero; then the *short-circuit point* is

$$\mathbf{p}_0 = (\alpha_0/\beta_0)\mathbf{1}$$

With $\xi = \infty$, corresponding to an infinite resistance or reactance, the *open-circuit point* is

$$\mathbf{p}_\infty = (\alpha_1/\beta_1)\mathbf{1}$$

Any other convenient point will determine the circle and also settle the scale of ξ; very often that corresponding to $\xi = 1$ is useful. These relations are shown in Fig. 17, from which it is easily seen that the point

P_∞ is the origin of Fig. 14 or Fig. 16 displaced by the vector $\mathbf{p}_\infty$. Draw the diameter $P_\infty C$, produce it, and take any line AB perpendicular to it; join $P_\infty P_0$ and produce it to cut AB in D. Then D is a fixed point corresponding to $\xi = 0$. By comparison with Fig. 16, a linear scale of ξ marked off along AB from D will relate points on the circle to the variation of the parameter ξ; the scale is fixed by the value of ξ belonging to the third point used to construct the circle, e.g. unity for P_1.

Using the above expressions for $\mathbf{p}_0$ and $\mathbf{p}_\infty$ in that for $\mathbf{p}$,

$$\mathbf{p}-\mathbf{p}_0 = \frac{\alpha_1\beta_0-\alpha_0\beta_1}{\beta_0(\beta_0+\beta_1\xi)}\,\xi\mathbf{1} \text{ and } \mathbf{p}-\mathbf{p}_\infty = \frac{\alpha_0\beta_1-\alpha_1\beta_0}{\beta_1(\beta_0+\beta_1\xi)}\,\mathbf{1},$$

so that

$$\frac{\mathbf{p}-\mathbf{p}_0}{\mathbf{p}-\mathbf{p}_\infty} \quad -\xi\frac{\beta_1}{\beta_0}$$

The phase angle between these two vectors is that of $-\beta_1/\beta_0$ and is constant, since β_1 and β_0 are constant; call this angle ψ. Also $P_\infty P_0$ is a fixed distance; hence the locus of P is a circle which has the equation

$$x^2+y^2-a^2 = 2ay\cot\psi$$

when the origin is taken at the foot of the perpendicular from C upon $P_\infty P_0$ and $a = \frac{1}{2}P_\infty P_0$. Hence it is easy to show* that the radius of the circle is $a/\sin\psi$ where

$$\psi = \arg.\,(\beta_1-\beta_0) = \arg.\,\beta_1 - \arg.\,\beta_0 = \text{diff. of phase angles of operators}$$

and that the centre is at $(0,\ a\cot\psi)$. To find a, note that

$$\mathbf{p}_\infty-\mathbf{p}_0 = \frac{\alpha_1\beta_0-\alpha_0\beta_1}{\beta_1\beta_0}\,\mathbf{1}$$

so that

$$a = \tfrac{1}{2}P_\infty P_0 = \tfrac{1}{2}\ \frac{\alpha_1\beta_0-\alpha_0\beta_1}{\beta_1\beta_0}$$

10. The Impedance of Parallel Wires. In the circuits contemplated on page 35 the resistances, inductances, and capacitances involved are supposed to be localized between definite points in the networks. However, there are many cases in practice where this supposition is not strictly applicable, a most important example being a circuit composed of two similar parallel wires joined together at one end. It will be clear that the inductance and capacitance of such a pair of wires is uniformly distributed along their length, and cannot be correctly represented as localized between any pair of points chosen along the wires. Such an arrangement of two wires is very often used in alternating bridge practice as a standard resistance (*see* p. 118), and the results now to be obtained can also be taken as a first approximation to the impedance operator of an ordinary "bifilar" resistance coil (*see* p. 92).

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* See for example, C. Smith, *Elementary Treatise on Conic Sections*, p. 69, ex. 8 (1906)

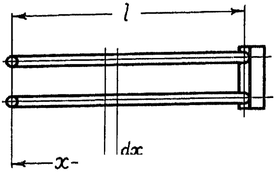

FIG. 18.—ILLUSTRATING THE IMPEDANCE OF PARALLEL WIRES

Two similar, parallel wires of equal length l, connected together at one end and supplied with a sinusoidal voltage $\mathbf{e}_0$ at the other, can be treated analytically in the same way as a transmission line* short-circuited at the distant end. Let ρ, λ, and κ be the resistance, inductance, and capacitance of the wires per unit length. Then, at a distance x from the terminals (Fig. 18), where the voltage between the wires is $\mathbf{e}$ and the current $\mathbf{i}$, the equations of equilibrium are

$$-(\rho + j\omega\lambda)dx.\mathbf{i} = d\mathbf{e} \text{ and}$$
$$-j\omega\kappa\, dx\,.\,\mathbf{e} = d\mathbf{i}\,;$$

$$\therefore \quad \frac{d\mathbf{e}}{dx} = -(\rho + j\omega\lambda)\mathbf{i} \text{ and } \frac{d\mathbf{i}}{dx} = -j\omega\kappa\mathbf{e}.$$

Eliminating $\mathbf{i}$,

$$\frac{d^2\mathbf{e}}{dx^2} = j\omega\kappa(\rho + j\omega\lambda)\mathbf{e} = a^2\mathbf{e},$$

whence

$$\mathbf{e} = \mathbf{A} \sinh ax + \mathbf{B} \cosh ax,$$

where $\mathbf{A}$ and $\mathbf{B}$ are constant vectors to be determined as follows:—

Now when $x = l$, $\mathbf{e} = 0$, so that $\mathbf{B} = -\mathbf{A} \tanh al$ and

$$\mathbf{e} = \mathbf{A}(\sinh ax - \tanh al \cosh ax).$$

Also when $x = 0$, $\mathbf{e} = \mathbf{e}_0$ and so $\mathbf{e}_0 = -\mathbf{A} \tanh al$.

From the first equilibrium equation,

$$\mathbf{i} = -\frac{1}{(\rho + j\omega\lambda)} \cdot \frac{d\mathbf{e}}{dx} \qquad \frac{\mathbf{A}a}{(\rho + j\omega\lambda)}(\cosh ax - \tanh al \sinh ax).$$

At the terminals $x = 0$ and $\mathbf{i} = \mathbf{i}_0$, or $\mathbf{i}_0 = -\mathbf{A}\frac{}{(\rho + j\omega\lambda)}$. The impedance operator is then

$$\tanh al = \sqrt{\frac{\rho + j\omega\lambda}{j\omega\kappa}} \cdot \tanh \sqrt{j\omega\kappa(\rho \quad j\omega\lambda)}\, l.$$

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* For a full treatment of the transmission problem, *see* J. A. Fleming, *The Propagation of Electric Currents in Telephone and Telegraph Conductors*, p. 68, pp. 84–85 (1911). *See* also E. Orlich, *Kapazität und Induktivität*, Sec. 41, pp. 126–133 (1909), and F. M. Colebrook, *op. cit.*, Chap. V.

Let R, L, C_1 be the total distributed resistance, inductance, and capacitance of the wires, then

$$z = \sqrt{\frac{R + j\omega L}{j\omega C_1}} \cdot \tanh \sqrt{j\omega C_1(R + j\omega L)}.$$

In resistances constructed in this manner, L and C_1 are both small quantities, such that their powers and products can be neglected when compared with unity. Expanding the hyperbolic tangent in series* gives

$$z = (R + j\omega L)\left[1 - \frac{1}{3}\{j\omega C_1(R + j\omega L)\} + \frac{2}{15}\{j\omega C_1(R+j\omega L)\}^2 \;\ldots\right],$$

or

$$\doteqdot R + j\omega\left(L - \frac{C_1 R^2}{3}\right),$$

the practical importance of which is discussed on pages 121 and 122.

11. Maxwell's Theory of Networks. In the preceding sections attention has been confined to the consideration of the current flowing in a single conductor. It is now necessary to develop some general principles which will enable the currents in a network of conductors to be calculated. For this purpose let an examination be made of a network composed of resistances carrying direct currents; the results obtained can then be readily generalized, so that they are applicable to a network of impedances in which alternating currents are flowing.

Let Fig. 19 represent a network of resistances supplied with current from a battery of electromotive force E. In this particular example 6 conductors join 4 points in such a way that 3 closed meshes are formed, 3 conductors meeting at each point. In general, if there are c conductors arranged to form a network by joining p points, the number of meshes will be $c - (p - 1)$. Each of the conductors will carry a definite current, as indicated by the arrows drawn on the resistances in the diagram, the magnitude of which can be found by the following procedure. Apply Ohm's law to each mesh in turn, writing down $c-p+1$ equations connecting the total resistance drop round each mesh with the electromotive force of any battery included in it. Then to $p-1$ of the points of junction apply Kirchhoff's first rule, making the algebraic sum of the currents which meet at each such point equal to zero;

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* *See* G. Greenhill, *Differential and Integral Calculus*, 2nd edition, pp. 238–39, p. 233, ex. (i), and p. 234, ex. (viii) (1891).

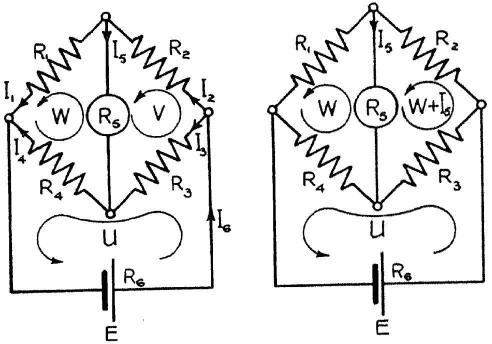

FIG. 19.—MAXWELL'S THEORY OF CYCLIC CURRENTS IN NETWORKS

an additional $p-1$ equations will thus be obtained. From the total c equations the current in any conductor can be found by elimination of all currents except the one desired, making use of the ordinary algebraic processes. In the example illustrated in Fig. 19, application of Ohm's law to the three meshes gives

$$E = R_6I_6 + R_3I_3 + R_4I_4,$$
$$0 = R_2I_2 + R_5I_5 - R_3I_3,$$
$$0 = R_1I_1 - R_4I_4 - R_5I_5;$$

using Kirchhoff's first rule at three branch-points gives

$$0 = I_6 - I_3 - I_2,$$
$$0 = I_2 - I_5 - I_1,$$
$$0 = I_1 - I_6 + I_4.$$

Hence, sufficient equations to determine any one of the c currents can be found by taking the $c-p+1$ mesh equations and only $p-1$ equations from the points of junction.

The reader will appreciate that, although the method just described is direct and obvious, the algebra involved in solving the c equations for any given current will be long and tedious, especially if the network be composed of a large number of conductors. Much of the complexity may be avoided by making use of Maxwell's* artifice of hypothetical *cyclic currents.* In this, each mesh is supposed to carry a current of different strength, these cyclic currents being independent and having the same direction of circulation. The current in any conductor will then be equal to the difference between the cyclic currents in the two meshes of which the conductor forms the common side. For example, in Fig. 19 the circulating currents in the meshes are u, v, and w; the current in a conductor such as R_5 will be $I_5 = v - w$, by Maxwell's artifice. All that is necessary is now to make a circuit of each mesh in turn and to apply Ohm's law to it. There will then be as many equations as there are meshes or cyclic currents, namely, $c - (p - 1)$, which will be sufficient to determine the cyclic currents completely. Then, by taking the difference between appropriate pairs of mesh currents the true current in any conductor will be found. It is clear that the algebra involved in this method of calculation is of a more modest nature, since the number of equations to be solved is reduced from c to $c - (p - 1)$.

In the example shown in Fig. 19, the application of Maxwell's artifice reduces the number of independent equations from six to three, as follows—

$$\begin{aligned} (R_3 + R_4 + R_6)u - R_3v - R_4w &= E, \\ -R_3u + (R_2 + R_3 + R_5)v - R_5w &= 0, \\ -R_4u - R_5v + (R_1 + R_4 + R_5)w &= 0. \end{aligned}$$

It is now easy to solve for u, v, and w and thence to find any desired current from the differences between appropriate pairs of them.

It is usually desired to find the current in one particular conductor of the network, such as R_5 in Fig. 19. The algebra can be further simplified by writing for the two cyclic currents bounded by R_5, w and $w + I_5$ respectively; I_5 is the true current in the resistance R_5. The necessity for solving the above equations for w and v and the ultimate subtraction of them is avoided, I_5 being found directly Applying this simplification gives, on substitution of $v = w + I_5$,

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* J. C. Maxwell, *A Treatise on Electricity and Magnetism,* Vol. 1, 3rd edition; Sec. 282*b*, pp. 406–7; Sec. 347, pp. 475–7 (1892).

$$\begin{aligned} -R_3I_5 - (R_3 + R_4)w + (R_3 + R_4 + R_6)u &= E, \\ (R_2 + R_3 + R_5)I_5 + (R_2 + R_3)w - R_3u &= 0, \\ -R_5I_5 + (R_1 + R_4)w - R_4u &= 0 ; \end{aligned}$$

solving these equations for I_5 by the usual method of determinants,*

$$I_5 = \frac{\begin{vmatrix} E & -(R_3 + R_4) & (R_3 + R_4 + R_6) \\ 0 & (R_2 + R_3) & -R_3 \\ 0 & (R_1 + R_4) & -R_4 \end{vmatrix}}{\begin{vmatrix} -R_3 & -(R_3 + R_4) & (R_3 + R_4 + R_6) \\ (R_2 + R_3 + R_5) & (R_2 + R_3) & -R_3 \\ -R_5 & (R_1 + R_4) & -R_4 \end{vmatrix}}$$

Evaluating the numerator, and writing the symbol Δ for the denominator gives

$$I_5 = \frac{(R_1R_3 - R_2R_4)}{\Delta} E \quad . \quad . \quad . \quad . \quad (7)$$

The network of resistances drawn in Fig. 19 is the much used Wheatstone bridge, R_5 including a galvanometer. When the resistances of the four balancing branches, R_1, R_2, R_3, and R_4 are arranged to make $I_5 = 0$, the galvanometer remains undeflected and the bridge is said to be *balanced.* From Equation (7) the condition for balance is seen to be

$$R_1R_3 = R_2R_4.$$

The principles which have been deduced in this section and applied to a typical network were, as pointed out earlier, originally due to Maxwell. Professor J. A. Fleming† has developed them in some detail in an important paper to which the reader is referred for further information ; sufficient has been said, however, to enable the balance conditions of a network to be deduced and to allow of the determination of its sensitivity as a method of measurement.

12. Application of Maxwell's Theory of Networks to Alternating Currents. It has been shown on page 32 that Ohm's law for direct currents has an exact analogue in the theory of harmonic alternating currents. It follows, therefore, that Maxwell's theory of networks outlined for direct currents in a previous paragraph may be applied to networks of impedances carrying alternating currents by putting symbols for harmonic vectors in place of the symbols for direct currents, and replacing resistances by impedance operators.

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* *See* H. S. Hall and S. R. Knight, *Higher Algebra*, 4th edition, Chap. 33, pp. 409–428 (1903).

† J. A. Fleming, "Problems on the distribution of electric currents in networks of conductors treated by the method of Maxwell," *Proc. Phys. Soc.*, Vol. 7, pp. 215–255 (1886). *See* also O. Heaviside, *Electrical Papers*, Vol. 1, pp. 412–415 (1892).

Professor Fleming, in the paper mentioned above, has treated certain cases where the current is interrupted; the application of the principle to harmonic vectors has been made by many writers, notably by Professor Lyle* and R. Appleyard.† In the succeeding sections of this chapter Maxwell's theory will be combined with the symbolic vector notation and applied to deduce the properties of various alternating current bridge networks.

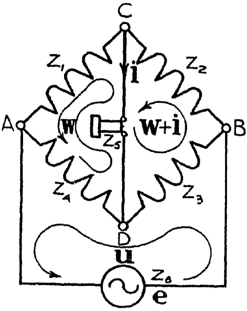

FIG. 20.—THE WHEATSTONE OR FOUR-BRANCH IMPEDANCE NETWORK

13. The Four-branch Impedance Network.

Let a Wheatstone bridge network‡ be constructed by arranging four impedances, as shown in Fig. 20, having operators z_1, z_2, z_3, and z_4. Let an alternator, having a sinusoidal electromotive force represented by the vector **e**, be applied to the points *AB*; and across the points *CD* connect a suitable detector. Then, if z_5 be the operator for the detector, z_6 for the alternator, and **u**, **w** + **i**, **w** be the three mesh currents, Fig. 20 is the alternating current analogue of Fig. 19. Hence from Equation (7) the current **i** in the detector is given by

$$\mathbf{i} = \frac{z_1 z_3 - z_2 z_4}{\Delta} \cdot \mathbf{e},$$

where

$$\Delta = \begin{vmatrix} -z_3 & -(z_3 + z_4) & (z_3 + z_4 + z_6) \\ (z_2 + z_3 + z_5) & (z_2 + z_3) & -z_3 \\ -z_5 & (z_1 + z_4) & -z_4 \end{vmatrix} \quad . \quad (7a)$$

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* T. R. Lyle, *loc. cit.*

† R. Appleyard, "The solution of network problems by determinants," *Proc. Phys. Soc.*, Vol. 24, pp. 201–209 (1912).

‡ T. R. Lyle, *loc. cit.*
O. Heaviside, *Electrical Papers*, Vol. 2, pp. 102–106, pp. 256–277 (1892).

If the bridge is to be balanced there must be no current in the detector at any time, i.e. $\mathbf{i} = 0$, whence

$$z_1 z_3 = z_2 z_4, \tag{8}$$

is the symbolic condition for balance which must be satisfied by the branch impedance operators.

This condition can be written in a different form ; remembering that $z = R + jX$, in general, gives

$$(R_1 + jX_1)(R_3 + jX_3)\mathbf{r} = (R_2 + jX_2)(R_4 + jX_4)\mathbf{r}, \quad . \quad (8a)$$

allowing each side to operate on an arbitrary vector $\mathbf{r}$. Again, since $z = R + jX = Z\,\underline{/\phi}$ the vector equation becomes

$$(Z_1\,\underline{/\phi_1})(Z_3\,\underline{/\phi_3})\mathbf{r} = (Z_2\,\underline{/\phi_2})(Z_4\,\underline{/\phi_4})\mathbf{r}.$$

Now the result of the operation $Z_1\,\underline{/\phi_1}$ on a vector is to multiply it by Z_1 and to advance its phase by ϕ_1 ; a further operation $Z_3\,\underline{/\phi_3}$ therefore multiplies the result of the first operation by Z_3 and advances the phase by a further angle ϕ_3. The total effect is equivalent to an operation $Z_1 Z_3 / \phi_1 + \phi_3$ on the original vector. A similar argument applies to the operations on the right-hand side, so that

$$(Z_1 Z_3\,\underline{/\phi_1 + \phi_3})\mathbf{r} = (Z_2 Z_4\,/\phi_2 + \phi_4)\mathbf{r}. \quad . \quad . \quad (8b)$$

Now when two vectors are equal they are of equal magnitude and are coincident in phase, hence

$$\left.\begin{aligned} Z_1 Z_3 &= Z_2 Z_4 \\ \text{and} \qquad \phi_1 + \phi_3 &= \phi_2 + \phi_4, \end{aligned}\right. \tag{8c}$$

a result which has an important graphical meaning, to be noticed in the following section. The results given in these equations show that *two* adjustments of the branch impedances will be necessary to satisfy the balance conditions, which is sufficiently obvious when it is remembered that, in dealing with the alternating quantities, phase as well as magnitude must be taken into account. The relations existing at balance between the branch constants are simply found by treating the operations in Equation (8*a*) algebraically, thus

$$\begin{aligned} &[\,(R_1R_3 - X_1X_3) + j\,(X_1R_3 + X_3R_1)\,]\mathbf{r} \\ &= [\,(R_2R_4 - X_2X_4) + j\,(X_2R_4 + X_4R_2)\,]\mathbf{r} \end{aligned}$$

Now when two vectors are equal their components are equal, so that

$$\begin{aligned} (R_1R_3 - R_2R_4) &= (X_1X_3 - X_2X_4), \\ \text{and}\quad (X_1R_3 - X_2R_4) &= (X_4R_2 - X_3R_1), \end{aligned} \qquad (8d)$$

are the necessary conditions to be fulfilled by the branch resistances and reactances.

In particular, if the resistances be arranged to satisfy the ordinary Wheatstone balance condition

$$R_1R_3 = R_2R_4$$

and the first of Equations (8*d*) gives

$$X_1X_3 = X_2X_4$$

also; which is a similar Wheatstone relation among the reactances. Using these relations to eliminate R_4 and X_4 from the second of Equations (8*d*) leads to

$$X_2/X_3 = R_2/R_3$$

But $$X_2/X_3 = X_1/X_4 \text{ and } R_2/R_3 = R_1/R_4$$

so that the complete relation between the resistances and reactances in this case is

$$R_1/R_4 = R_2/R_3 = X_1/X_4 = X_2/X_3$$

which is equivalent to

$$X_1/R_1 = X_4/R_4 \text{ and } X_2/R_2 = X_3/R_{\underset{3}{4}}; \qquad (8e)$$

i.e. the phase angles of z_1 and z_4 are equal, as also are those of z_2 and z_3.

Many other arrangements of the four-branch impedance network are possible and will be classified on p. 291.

14. The Vector Diagram of the Balanced Four-branch Impedance Network. The four-branch bridge can be easily treated by graphical methods* when balance is secured. Since **i** is zero in the detector, suppose $\mathbf{i}_C$ and $\mathbf{i}_D$ to be the currents in the branches *ACB* and *ADB* respectively; these currents are represented by vectors in the diagram (Fig. 21) lagging behind the vector **e** which now represents the potential difference applied to the points *A*, *B*. Since the bridge is balanced, *C* and *D* are always at the same potential, represented in magnitude

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[*] *See* Alfred Fraenckel, *Theorie der Wechselströme*, 2nd edition, Sec. 24, pp. 59–62 (1921). For a variety of special cases *see* H. H. Poole, "On the use of vector methods in the derivation of the formulae used in inductance and capacity measurements," *Phil. Mag.*, 6th series, Vol. 40, pp. 793–809 (1920). Also Chap. IV of this book.

and phase by coincident points C,D in Fig. 21. Then the remainder of the diagram is constructed by remembering that

$$\mathbf{e}_{AC} = \mathbf{e}_{AD} = z_1\mathbf{i}_C = z_4\mathbf{i}_D,$$
$$\mathbf{e}_{CB} = \mathbf{e}_{DB} = z_2\mathbf{i}_C = z_3\mathbf{i}_D,$$
$$\mathbf{e} = \mathbf{e}_{AC} + \mathbf{e}_{CB} = \mathbf{e}_{AD} + \mathbf{e}_{DB}.$$

The currents are found in magnitude and phase from

$$\mathbf{i}_C = \mathbf{e}/(z_1 + z_2)$$
$$\mathbf{i}_D = \mathbf{e}/(z_3 + z_4)$$

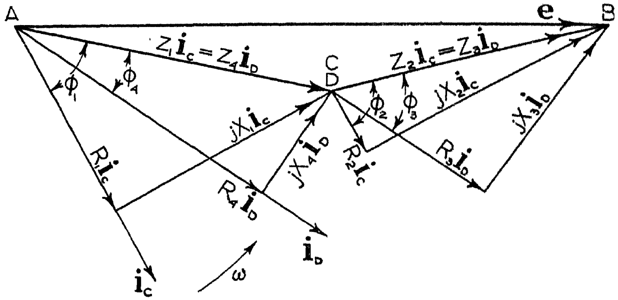

FIG. 21.—VECTOR DIAGRAM FOR THE BALANCED FOUR-BRANCH IMPEDANCE NETWORK OF FIG. 20

Since the vectors AC and AD are equal in magnitude and coincident in phase, and since also $CB = DB$ it follows from the geometry of the diagram that

$$\frac{Z_1}{Z_4} = \frac{Z_2}{Z_3},$$

and also $$\phi_1 - \phi_4 = \phi_2 - \phi_3\,;$$

hence the balance conditions are

$$Z_1Z_3 = Z_2Z_4$$

and $$\phi_1 + \phi_3 = \phi_2 + \phi_4,$$

in agreement with Equation (8*c*) on page 52.

15. The Anderson Impedance Network.

The four-branch impedance bridge, the general theory of which has been discussed in previous paragraphs, includes a very large number of the bridge networks used in practice. There are, however, a number of bridges in common use wherein more than four balancing branches are employed. It is proposed to show in this section how to find the balance conditions for a network with six balancing branches. The general theory of this network was first given by S. Butterworth,* and it contains as special cases many common bridges, including Anderson's well-known method. It will be referred to as the Anderson network on this account.

When the balance condition of a bridge is required, it is not necessary to find the absolute value of **i**, the current in the detector, in terms of **e**; it will be sufficient if its value in terms of some other current in the network be determined. Accordingly, let **i** be expressed in terms of the current in the alternator, **u**. Suppose that there are m meshes in the network. Assume a cyclic current in each mesh, and then write down $m-1$ equations for the potential drop round each mesh. The equation omitted is that for the mesh which contains the alternator. Transfer the terms involving **u** to the right-hand side and solve the equations for **i** in terms of **u**. The solution will consist of the ratio of two determinants, so that if **i** is to be zero, the determinant in the numerator must vanish. Now this determinant is formed of the coefficients of **u** and those of all other cyclic currents except the coefficients of **i**; hence, to find the balance condition, write down this determinant and equate it to zero.

Consider first a simple case, that of the four-branch network of Fig. 20. The mesh equations for the meshes *CBD* and *ACD* are

$$(z_2 + z_3 + z_5)\mathbf{i} + (z_2 + z_3)\mathbf{w} = z_3\mathbf{u}$$
$$-z_5\mathbf{i} + (z_1 + z_4)\mathbf{w} = z_4\mathbf{u}.$$

The numerator determinant in the solution for **i** in terms of **u** is

$$\begin{matrix} z_3 & z_2 + z_3 \\ z_4 & z_1 + z_4 \end{matrix}$$

which must vanish if **i** is to be zero. Writing equal to zero and evaluating gives

$$z_1z_3 = z_2z_4$$

for balance, as proved above.

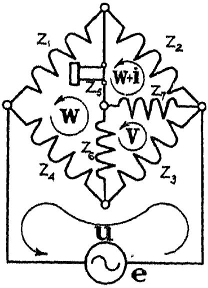

FIG. 22.—THE ANDERSON OR SIX-BRANCH IMPEDANCE NETWORK

Now examine the Anderson network of Fig. 22. Omitting the equation for the alternator mesh, the equations for the three remaining meshes are—

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* S. Butterworth, "On the vibration galvanometer and its application to inductance bridges," *Proc. Phys. Soc.*, Vol. 24, pp. 75–94 (1912).

$$\begin{aligned} -z_7\mathbf{i} + (z_3 + z_6 + z_7)\mathbf{v} - (z_6 + z_7)\mathbf{w} &= z_3\mathbf{u}, \\ -z_5\mathbf{i} - z_6\mathbf{v} + (z_1 + z_4 + z_6)\mathbf{w} &= z_4\mathbf{u}, \\ (z_2 + z_5 + z_7)\mathbf{i} - z_7\mathbf{v} + (z_2 + z_7)\mathbf{w} &= 0\,; \end{aligned}$$

the numerator determinant, equated to zero, is

$$\begin{vmatrix} z_3 & (z_3 + z_6 + z_7) & -(z_6 + z_7) \\ z_4 & -z_6 & (z_1 + z_4 + z_6) \\ 0 & -z_7 & (z_2 + z_7) \end{vmatrix} = 0.$$

Evaluating and collecting terms

$$z_7(z_1z_3 - z_2z_4) = z_2\left\{z_6(z_3 + z_4) + z_3z_4\right\} \quad . \quad . \quad (9)$$

is the condition that the Anderson network be balanced. It should be observed that when $z_6 = 0$ and $z_7 = \infty$ the network reduces to the four-branch type, Equation (9) becoming $z_1z_3 = z_2z_4$, the usual balance relation of Equation (8).

16. Star-Mesh Transformations. The methods given in the preceding sections for dealing with problems connected with a.c. bridge networks are usually sufficient for most purposes. In certain cases, particularly when a large number of meshes become involved, the algebraical work can be much reduced by transformation of the network into one of a simpler type. For this purpose the well-known star-mesh transformations introduced by Kennelly* are frequently of service and will be briefly dealt with here in their application to bridge networks.

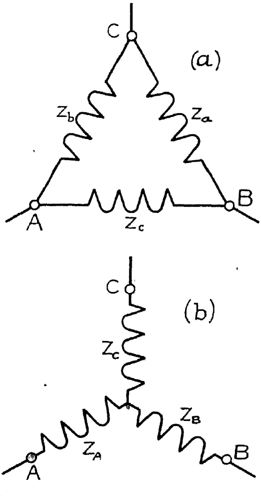

FIG. 23.—EQUIVALENT STAR AND MESH CONNECTIONS OF IMPEDANCES

In the first instance, consider three terminals in a network, *A*, *B*, *C*, joined by a simple mesh of impedances the operators for which are z_a, z_b, z_c, as in Fig. 23(*a*). It is required to replace this mesh by a simple star-connected system of three impedances, z_A, z_B, z_C, which shall be equivalent, with respect to the external circuit joined to A, B, C, to the original mesh (*see* Fig. 23(b)). Let it be supposed that measurements are made of the impedance existing between successive pairs of terminals both for the star and for the mesh arrangements. Then, since the two are to be equivalent,

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* A. E. Kennelly, "The equivalence of triangles and three-pointed stars in conducting networks," *Elec. World*, Vol. 34, pp. 413-414 (1899).

$$\text{across } A \text{ and } C, \quad z_A + z_C = \frac{z_b(z_a + z_c)}{z_a + z_b + z_c},$$

$$\text{across } C \text{ and } B, \quad z_C + z_B = \frac{z_a(z_b + z_c)}{z_a + z_b + z_c},$$

$$\text{across } B \text{ and } A, \quad z_B + z_A = \frac{z_c(z_a + z_b)}{z_a + z_b + z_c}.$$

Solving for the impedance operators of the star in terms of those of the mesh,

$$\begin{aligned} z_A &= \frac{z_b\, z_c}{z_a + z_b + z_c}, \\ z_B &= \frac{z_a\, z_c}{z_a + z_b + z_c}, \\ z_C &= \frac{z_a\, z_b}{z_a + z_b + z_c}. \end{aligned} \qquad . \quad (10a)$$

It is sometimes more convenient to express this result in terms of the admittance operator $y = 1/z$; making this substitution in Equation (10a) gives

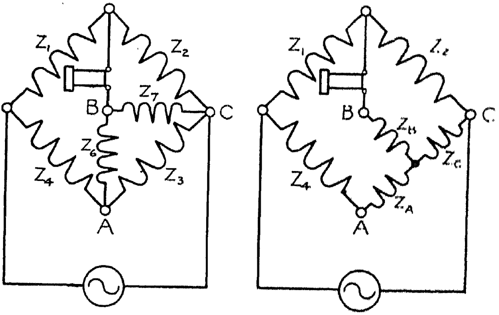
$$\left.\begin{aligned} y_A &= y_b\, y_c \left(\frac{1}{y_a} + \frac{1}{y_b} + \frac{1}{y_c}\right), \\ y_B &= y_a\, y_c \left(\frac{1}{y_a} + \frac{1}{y_b} + \frac{1}{y_c}\right. \\ y_C &= y_a\, y_b \left(\frac{1}{y_a} + \frac{1}{y_b} + \frac{1}{y_c}\right). \end{aligned}\right\} \qquad (10b)$$

Suppose that it is desired to replace a star-connected system of impedances by an equivalent mesh, the converse of the preceding problem. Then it is easy to show by a similar method that

$$\left.\begin{aligned} & z_B z_C\left(\frac{1}{z_A}+\frac{1}{z_B}+\frac{1}{z_C}\right), \\ z_b &= z_A z_C\left(\frac{1}{z_A}+\frac{1}{z_B}+\frac{1}{z_C}\right), \\ z_c &= z_A z_B\left(\frac{1}{z_A}+\frac{1}{z_B}+\frac{1}{z_C}\right), \end{aligned}\right\} \qquad (11a)$$

is the relation between the impedance operators in the two systems; while the admittance operators are connected by the relations

$$\begin{aligned} y_a &= \frac{y_B y_C}{y_A + y_B + y_C}, \\ y_b &= \frac{y_A y_C}{y_A + y_B + y_C}, \\ y_c &= \frac{y_A y_B}{y_A + y_B + y_C}. \end{aligned} \qquad (11b)$$



(a) (b)

FIG. 24.—MESH-STAR TRANSFORMATION OF THE ANDERSON NETWORK

As an example of the mesh-star transformation consider the Anderson network of Fig. 22, which is re-drawn in Fig. 24(*a*). This network differs from the simple four-branch or Wheatstone network by the additional mesh formed by the impedances z_3, z_6, z_7 joining the points *A*, *B*, *C* as shown in the diagram.

Transform this mesh into its equivalent star by means of Equation (10*a*), then

$$z_A = \frac{z_3 z_6}{z_3 + z_6 + z_7},$$

$$z_B = \frac{z_6 z_7}{z_3 + z_6 + z_7},$$

$$z_C = \frac{z_3 z_7}{z_3 + z_6 + z_7}.$$

Referring to the transformed network shown in Fig. 24(*b*), z_B lies in the detector circuit and does not, therefore, enter into the balance equation. The remaining impedances form a four-branch network $z_1, z_2, z_C, z_4 + z_A$, so that, for balance,

$$z_1 z_C = z_2(z_4 + z_A);$$

that is $z_1 z_3 z_7$

$$= z_2[z_4(z_3 + z_6 + z_7) + z_3 z_6],$$

on substituting the values of z_A and z_C. Re-arranging terms,

$$z_7(z_1 z_3 - z_2 z_4) = z_2\{z_6(z_3 + z_4) + z_3 z_4\}$$

which is identical with Equation (9).

A further important application of the transformation is provided by the calculation of the detector current in a Wheatstone network, a problem which has already been considered in Sections 9 and 11 by the use of Maxwell's mesh equations. Referring to Fig. 25(*a*), the network is shown in its usual form; convert the mesh *ACD* into its equivalent star as shown in Fig. 25(*b*). The total impedance of the network is then easily seen to be

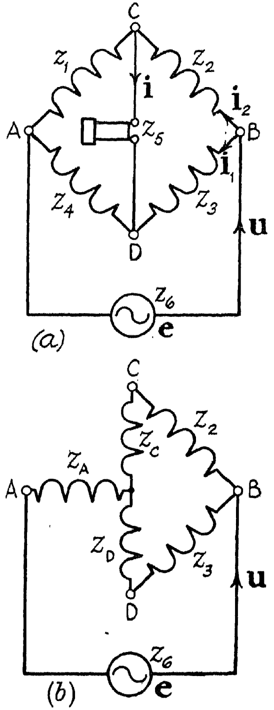


FIG. 25
MESH-STAR TRANSFORMATION OF THE WHEATSTONE NETWORK

$$z = z_6 + z_A + \frac{(z_2 + z_C)(z_3 + z_D)}{z_2 + z_3 + z_C + z_D},$$

and the current delivered by the source is $\mathbf{u} = \mathbf{e}/z$. The currents in the branches BC and BD are, therefore,

$$\mathbf{i}_2 = \frac{z_3 + z_D}{z_2 + z_3 + z_C + z_D}\,\mathbf{u},$$

$$\mathbf{i}_3 = \frac{z_2 + z_C}{z_2 + z_3 + z_C + z_D}\,\mathbf{u},$$

by the ordinary principles of parallel circuits. The potential drops down BC and BD will be $z_2\mathbf{i}_2$ and $z_3\mathbf{i}_3$, so that the potential difference between C and D is $z_3\mathbf{i}_3 - z_2\mathbf{i}_2$. The current through the detector will therefore be

$$\begin{aligned}\mathbf{i} &= \frac{z_3\mathbf{i}_3 - z_2\mathbf{i}_2}{z_5}\\ &= \frac{z_3(z_2 + z_C) - z_2(z_3 + z_D)}{z_5(z_2 + z_3 + z_C + z_D)} \cdot \frac{\mathbf{e}}{z}.\end{aligned}$$

Now, from Equation (10a) the star impedances are,

$$z_A = z_1z_4/S,\ z_C = z_1z_5/S,\ z_D = z_4z_5/S,\ \text{where } S = z_1 + z_4 + z_5.$$

Substituting these values in the expressions for $\mathbf{i}$ and z makes

$$\begin{aligned}\mathbf{i} &= \frac{(z_1z_3 - z_2z_4)\,\mathbf{e}}{\{(z_6S+z_1z_4)[(z_2+z_4)S+z_5(z_1+z_4)]+(z_2S+z_1z_5)(z_3S+z_4z_5)\}/S}\\ &= \frac{z_1z_3 - z_2z_4}{\Delta}\cdot\mathbf{e}.\end{aligned}$$

The denominator is easily simplified and becomes

$$\begin{aligned}\Delta = z_5z_6(z_1 + z_2 + z_3 + z_4) &+ z_5(z_1 + z_2)(z_3 + z_4)\\ &+ z_6(z_1 + z_4)(z_2 + z_3) + [z_1z_2(z_3 + z_4) + z_3z_4(z_1 + z_2)],\end{aligned}$$

which is readily shown to be in agreement with the evaluation of the determinant in Equation (7a), page 51.

Suppose now that the source and detector are interchanged; then it is easy to show by redrawing the bridge diagram of Fig. 20 that the new network is equivalent to the first with z_2 and z_4 transposed. Making this change, the detector current is

$$\mathbf{i}_c = \frac{z_1z_3 - z_2z_4}{\Delta_c}\,\mathbf{e},$$

where

$$\begin{aligned}z_5z_6(z_1 + z_2 + z_3 + z_4) &+ z_5(z_1 + z_4)(z_2 + z_3)\\ &+ z_6(z_1 + z_2)(z_3 + z_4) + [z_1z_2(z_3 + z_4) + z_3z_4(z_1 + z_2)]\end{aligned}$$

Hence, interchanging the positions of source and detector has no effect on the balance condition $z_1z_3 - z_2z_4 = 0$. The two networks are said to be *conjugate*. The interchange may, however, have a considerable effect on the detector current, an important matter when a bridge is arranged for greatest sensitivity to a given out-of-balance setting. Clearly $\mathbf{i}_c$ will exceed $\mathbf{i}$ if Δ_c is less than Δ, i.e. if $\Delta - \Delta_c$ is positive. This difference is easily shown to be

$$\Delta - \Delta_c = (z_5 - z_6)(z_1 - z_3)(z_4 - z_2)$$

Now let z_1 be greater than z_3 and z_4 greater than z_2; then if Δ is to exceed Δ_c, z_5 must exceed z_6. Conversely, if Δ_c is to exceed Δ, z_6 must be larger than z_5. This leads to the well-known rule that the greater sensitiveness is obtained by that arrangement which ensures that the greater of z_5 and z_6 lies in the diagonal connecting the junction of the two greater impedances (z_1 and z_4) with the junction of the two smaller impedances (z_2 and z_3).

Further examples of the use of the star-mesh transformation will be found in Appendix I.

Generalizations of Rosen and Russell. The transformations just given are capable of important generalization. Consider n points in a network; then these points may be connected by impedances in three special ways, as shown in Fig. 26 for the case of $n = 4$. In the first method n impedances are joined in star to a common point S, their other ends being connected to the n terminals. In the second method n impedances are connected successively to the points 1, 2; 2, 3; etc., until a closed mesh is formed. In the third method the n points are connected in pairs in every possible way by $n(n-1)/2$ impedances.

Rosen* shows that any star of n rays may be replaced by an equivalent pair-connected system or complete multilateral figure of $n(n-1)/2$ impedances. Let $y_1, y_2, y_3, \ldots y_p, y_q, \ldots y_n$ be the admittance operators for the n branches of the star; let y_{pq} be the admittance operator for the branch joining the points p, q in the pair-connected system. Then

$$y_{pq} = y_p y_q / \Sigma y,$$

where $\Sigma y = y_1 + y_2 + \ldots y_n$. The corresponding relation for impedance operators will be

$$z_{pq} = z_p z_q \sum \frac{1}{z},$$

where

$$\sum \frac{1}{z} = \frac{1}{z_1} + \frac{1}{z_2} +$$

These expressions enable any star to be replaced by a pair-connected system. In the particular case of $n = 3$, the pair-connection becomes identical with a simple mesh connection of three impedances and the

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[*] A. Rosen, "A new network theorem," *Journal I.E.E.*, Vol. 62, pp. 916–918 (1924). G. A. Campbell, *Trans. Amer. I.E.E.*, Vol. 30, p. 873 (1911).

general expressions reduce to Equations (11). The case of $n = 4$ is of particular interest in connection with earth capacitance effects in bridges and will be referred to again on page 541; the equivalent pair-connection has six impedances.

Russell* has given a formal proof of Rosen's theorem, and has derived its converse. His results may be expressed in the following three statements: (i) A star-connected system of n impedances can always be replaced by a pair-connected system of $n(n-1)/2$ impedances, as Rosen has shown; (ii) when n is greater than 3, a mesh-connected system can never be replaced by a star-connected system; (iii) in some cases a pair-connected system can be replaced by a star-connected system, but the restrictions are so severe that this converse theorem is of little use except when $n = 3$, which is Kennelly's theorem. The mesh-star transformation with $n = 3$, and the star-pair transformation with $n = 4$ are the cases of most frequent occurrence in a.c. bridge theory.

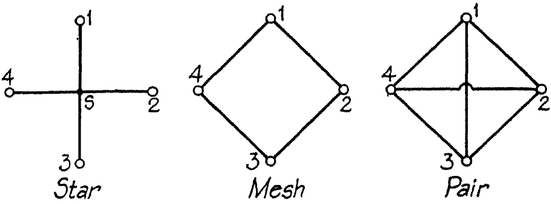

Fig. 26.—Star, Mesh, and Pair Connections to n Terminals ($n = 4$)

17. The Kelvin Double Bridge. The Wheatstone network is not very suitable for the measurement of very low impedances, e.g. the residual inductance of low value four-terminal resistances. Some experimenters have adapted for this purpose the Kelvin double bridge used in the analogous d.c. measurement of low resistances, as shown in Fig. 27(a). In this diagram z_3 and z_4 are four-terminal impedances; z_1, z_2, z_5, z_6 are the balancing impedances of the network; z_7 is the impedance of the connector joining z_3 to z_4. Using Equation (10a), transform the mesh ABC into its equivalent star, as shown in Fig. 27(b). The transformed network is of simple four-branch or Wheatstone type, the branches being z_1, z_2, $z_3 + z_B$, $z_4 + z_A$; z_C goes in series with the detector and has no effect on the balance condition, although it has an important influence on the sensitiveness. Substituting in Equation (8), balance is secured if

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* A. Russell, "Star and pair connections in networks," *Faraday House J.*, Vol. 20, pp. 86–90 (1927). Also see J. Riordan, "La transformation triangle-étoile pour des éléments de circuits généraux," *Rev. Gén. de l'Él.*, vol. 38, pp. 401–404 (1935) for the discussion of certain anomalous cases.

$$z_1(z_3 + z_B) = z_2(z_4 + z_A)$$

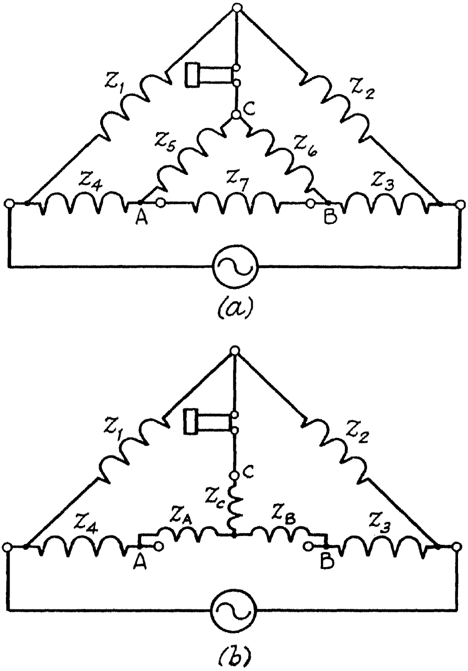

FIG. 27.—THE KELVIN DOUBLE BRIDGE

Now, from Equation (10*a*), $z_A = z_5z_7/(z_5 + z_6 + z_7)$ and

$$z_B = z_6z_7/(z_5 + z_6 + z_7);$$

inserting these in the balance condition gives

$$(z_5 + z_6 + z_7)\,(z_1z_3 - z_2z_4) + z_7(z_1z_6 - z_2z_5) = 0 \quad . \quad . \quad (12)$$

In order to eliminate the effect of z_7 the following technique is adopted. Suitable adjustments of the branch impedances are made until balance is secured whether z_7 is closed or open, balance in the latter circumstance involving

$$z_1(z_3 + z_6) = z_2(z_4 + z_5),$$

or

$$(z_1z_3 - z_2z_4) = -(z_1z_6 - z_2z_5).$$

Substitute, in Equation (12),

$$(z_1z_3 - z_2z_4)(z_5 + z_6) = 0;$$

thus either $(z_5 + z_6) = 0$, which is not possible unless one be a resistance-free coil and the other a condenser; or,

$$(z_1z_3 - z_2z_4) = 0,$$

involving also

$$(z_1z_6 - z_2z_5) = 0.$$

Hence balance can be secured and the effect of z_7 eliminated if

$$z_1/z_2 = z_4/z_3 = z_5/z_6 \quad . \quad . \quad . \quad . \quad (12a)$$

exactly as in the bridge when used on d.c. with resistances. Further practical details will be discussed in Chapter IV, page 409.

18. Alternative Treatment of Circuit Problems. Thévenin's Theorem.

Attention must now be directed to an alternative method of dealing with circuit problems which, like the star-mesh transformations worked out in Section 14, gives a more direct solution that can be obtained by simple application of Kirchhoff's rules. The method is based upon a theorem, usually attributed to Thévenin* and occasionally reintroduced by other writers, applied with great success to bridge networks by Wenner† and to distribution problems by Genkin.‡

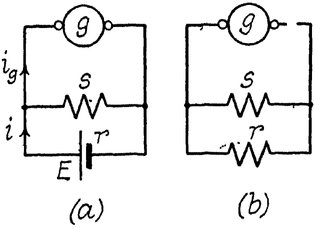

FIG. 28.—ILLUSTRATING THÉVENIN'S THEOREM

The theorem may be stated as

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[*] L. Thévenin, "Sur un nouveau théorème d'électricité dynamique," *Comptes Rendus*, Vol. 97, pp. 159–161 (1883). The original statement appears to be due to H. Helmholtz, *Ann. der Phys.*, Vol. 89, pp. 211–233, 353–377 (1853), *vide* H. Wigge, "Einige Folgerungen aus dem Ableitungssatz von Helmholtz (Theorem de Thévenin), *Arch. f. Elekt.*, Vol. 30, pp. 754–759 (1936). Further general discussions are given by J. G. Brainerd, *Proc. Inst. Rad. Eng.*, Vol. 81, pp. 1050–1054 (1933) and by A. J. van den Moersche, *Rev. Gén. de l'Él.*, Vol. 38, pp. 259–263 (1935); *Bull. Soc. Belge des Elecns.*, Vol. 51, pp. 335–342 (1935).

[†] F. Wenner, "A principle governing the distribution of current in systems of linear conductors," *Proc. Phys. Soc.*, Vol. 39, pp. 124–144 (1927).

[‡] V. Genkin, "Contribution à l'étude des réseaux maillés," *Rev. Gén. de l'Él.*, Vol. 24, pp. 659–672 (1928).

follows: In a system of impedances the current in any branch is that which would result should an electromotive force, equal to the potential difference which would appear across the break were the branch opened, be introduced into the branch and all other electromotive forces be removed. Wenner gives a verbal deductive proof of the theorem; an analytical proof is provided by Genkin. It will serve our present purpose if a simple problem be worked out in order to indicate some of the advantages of the method. Consider the case of a galvanometer of resistance g shunted by a resistance s, the combination being supplied with current by a battery of e.m.f. E and internal resistance r, as in Fig. 28(a). By the usual principles,

$$E\Big/\left(r + \frac{gs}{g+s}\right) = i_g(g+s)/s,$$

so that
$$i_g = \frac{Es}{(g+s)r + gs} = \frac{Es}{(r+s)} \cdot \frac{r+s}{g(r+s)+sr},$$

which may be written

$$i_g = \frac{\dfrac{Es}{(r+s)}}{g + \dfrac{sr}{r+s}}$$

Now apply Thévenin's theorem; cutting the galvanometer branch, the p.d. at the break will be $Es/(r+s)$. Suppressing E, as in Fig. 28(b), the resistance measured at the break will be $g + \frac{sr}{r+s}$. If a cell or e.m.f. $Es/(r+s)$ be put into the break to act upon this resistance it is clear that the current flowing in g will be i_g, exactly as proved directly from Ohm's law.

More complete networks can be built up by adding further meshes and an inductive proof of the general theorem may thus be established, as has been pointed out by R. Appleyard in the discussion on Wenner's paper, and extended to include a.c. impedance networks. It suffices, therefore, to find the current in any branch of a network (i) to suppose the branch cut; (ii) to express the potential difference between the terminals of the cut; (iii) to express the total impedance operator made up of the branch itself and of the other branches connected to it, all electromotive forces being suppressed; (iv) to divide the result of (ii) by that of (iii). Wenner treats a great variety of problems, including the calculation of the detector current in (i) the Wheatstone network; (ii) the Anderson network; and (iii) the Hughes induction balance. The reader is referred to the original paper for full details of the working, which is very much shorter and more direct than the use of simultaneous equations; use is conveniently made of the mesh-star transformation* in conjunction with Thévenin's theorem.

* An extension of Thévenin's theorem has been given by A. T. Starr, "A new theorem for active networks," *Journal I.E.E.*, Vol. 73, pp. 303–308 (1933). Combining it with Kennelly's star-mesh theorem he shows that any three-terminal network can be replaced by a star or mesh of certain voltages and impedances.

19. Mutual Inductance in Bridge Networks. In all the bridge networks which have been considered above, the balancing branches have consisted of independent impedances. There is, however, an important class of networks in which certain pairs of branches are arranged to react mutually on one another; an examination of the effects of mutual inductance is, therefore, necessary.

If a circuit carries a current i and M is its coefficient of mutual induction with respect to a second circuit, the electromotive force which must be applied to the latter to balance

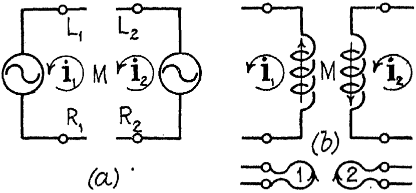

Fig. 29.—Illustrating the Relative Signs of Self and Mutual Induction Effects

the effect of mutual inductance is $M\frac{di}{dt}$, or in vector notation, $j\omega M\mathbf{i}$ for sinusoidally varying quantities.

Now, although this expression represents the effect of mutual inductance in a general way, it is not sufficiently explicit as a statement of all the physical facts, since, as is obvious, M can be a positive or a negative quantity. It is necessary, therefore, to adopt a convention in order to make determinate the sign of mutual induction effects.

Consider two linear circuits in the same plane, as shown in Fig. 29(a), M being the mutual inductance between them. The positive direction of the cyclic currents in the two circuits is conventionally taken as counter-clockwise, so that positive flux will, by Ampère's rule, pass normally out of the plane of the paper toward the reader. Then, in accordance with the expressions on page 20, the electromotive force applied to circuit 1 will be

$$e_1 = R_1 i_1 + L\frac{di_1}{dt} + M\frac{di_2}{dt},$$

or

$$\mathbf{e} = R_1\mathbf{i}_1 + j\omega L\mathbf{i}_1 + j\omega M\mathbf{i}_2;$$

R_1 and L_1 are necessarily positive, while M can have either a positive or a negative sign. For the two circuits as shown if i_1 be increased, so that di_1/dt is positive, the back e.m.f. of self-induction in circuit 1 will act in opposition to i_1. If in a similar way i_2 be increasing, di_2/dt is also positive; but the flux set up by current i_2 through circuit 1 is opposite in sign to the self-flux due to i_1; hence the sign of M must be opposite to that of L_1 and is, therefore, negative. With this convention for the cyclic currents and the adoption of the voltage equation stated above, the terms L_1di_1/dt and Mdi_2/dt are, for this pair of circuits, in opposition and M is negative.

If the positions of the circuits be altered until they are superposed one upon the other, it is clear that the mutual flux and the self fluxes have now the same sign, namely, positive through the circuits; hence L_1di_1/dt and Mdi_2/dt are of the same sign for superposed linear circuits and M is positive.

These two simple problems are special cases of the following rule: The sign of M is considered to be positive when positive current in one circuit causes positive flux to link the other circuit.* The convention, therefore, lies in the choice of positive directions for the currents in the circuits in order that the directions of positive flux may be determined.

In drawing circuit diagrams of bridge networks it is very usual to show mutual inductances as concentrated between two coils with parallel axes and the same sense of winding, as in Fig. 29(*b*). Taking counter-clockwise cyclic currents as positive in the two meshes in which the coils are situated, the positive directions of the currents *round the coils* are counter-clockwise in 1 and clockwise in 2, as shown in the plan view;

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* This convention is in agreement with that adopted by the Bureau of Standards, but is contrary to that used by several workers at the N.P.L. It is, of course, a matter of no consequence whether the convention here used or the converse is adopted, provided that the one chosen is strictly adhered to. It may be pointed out, however, that the one given in the text makes, for example, the self-inductance of a coil equal to the sum of the self-inductance of two parts of the coil plus twice the mutual inductance between the parts. This is physically reasonable since the whole effect of the windings is to circulate flux through the coil; whereas the converse method would substitute a negative mutual effect in place of the positive one postulated by the present convention and which is physically preferable. In this connection see *Exp. W. and W. Eng.*, Vol. 6, pp. 233–234 (1929). Further discussion of conventions are given by R. R. Nimmo and H. D. Poole, "The significance of the sign of a mutual inductance," *Proc. Camb. Phil. Soc.*, Vol. 28, pp. 531–537 (1932); J. Greig, "Mutual inductance," *W. Eng.*, Vol. 13, pp. 362–364 (1936) and correspondence following this article.

the directions of positive fluxes through the coils will be as shown by the arrows. It will be seen that the influence of coil 2 on coil 1 is to produce a mutual flux passing through coil 1 in the same direction as its self flux, i.e. M assists L_1 and is positive. In the same way it can be seen that M assists L_2. Thus for two similarly wound parallel coils arranged as shown, M is positive, and in the theory of bridge networks this is one of the most useful forms in which the rule may be stated.

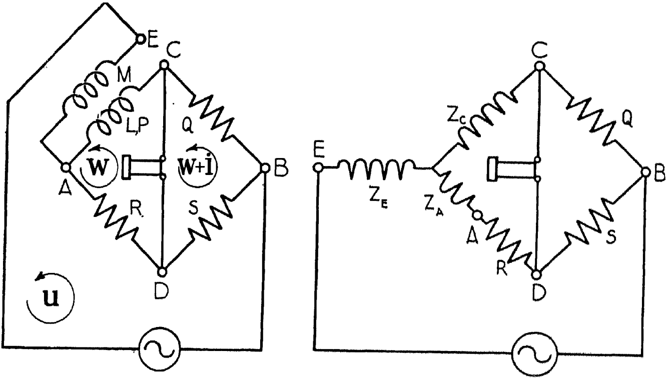

FIG. 30.—EXAMPLE OF MUTUAL INDUCTANCE IN A BRIDGE NETWORK

To show the application of this rule, consider the case of Maxwell's method for comparison of a mutual inductance and a self-inductance. The network is shown in Fig. 30, the cyclic currents being **u**, **w** + **i**, and **w**, **i** being the detector current vector; it is required to find the condition for **i** to be zero. The coils AE and AC are parallel and similarly wound; they carry mesh currents **u** and **w** in positive directions. Hence, in agreement with Fig. 29(b), M is positive. The mesh equations for ACD and CBD are, if z_5 be the detector impedance,

$$(P + R + z_5 + j\omega L)\mathbf{w} - z_5(\mathbf{w} + \mathbf{i}) + (j\omega M - R)\mathbf{u} = 0,$$
$$(Q + S + z_5)(\mathbf{w} + \mathbf{i}) - z_5\mathbf{w} - S\mathbf{u} = 0\,;$$

re-arranging, gives

$$-z_5\mathbf{i} + (P + R + j\omega L)\mathbf{w} = (R - j\omega M)\mathbf{u},$$
$$(Q + S + z_5)\mathbf{i} + (Q + S)\mathbf{w} = S\mathbf{u}.$$

By the principles on page 55, **i** will be zero when

$$\begin{vmatrix} R - j\omega M & P + R + j\omega L \\ S & Q + S \end{vmatrix} = 0,$$

that is, when $SP = QR$

$$L = -\left(1 + \frac{Q}{S}\right) M,$$

which are the required balance conditions; hence it is necessary to make M negative to secure balance. This is physically obvious; since Q, S and R are all resistances the branch AC must simulate one when balance occurs. Hence the self inductance L must be annulled by the mutual inductance M.

20. Transformation of Networks containing Mutual Inductance. Network problems involving mutual inductance can often be much simplified by the use of certain transformations due to Professor G. A. Campbell,* and developed in England by S. Butterworth,† one of these being noted here.

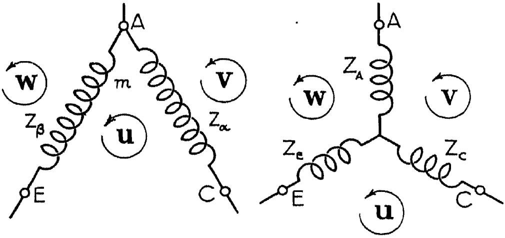

FIG. 31.—TRANSFORMATION OF TWO COILS WITH MUTUAL INDUCTANCE INTO A STAR-CONNECTED ARRANGEMENT OF THREE IMPEDANCES

Let two impedances, having operators z_α and z_β form branches of a network, $m = j\omega M$ being the mutual operator between them, as shown in Fig. 31. Then, making use of the rule of the preceding paragraph, the equations for the electromotive forces acting in the meshes containing **v** and **w** will be

$$\mathbf{e}_V = z_\alpha(\mathbf{v} - \mathbf{u}) + m(\mathbf{w} - \mathbf{u}),$$

$$\mathbf{e}_W = z_\beta(\mathbf{w} - \mathbf{u}) + m(\mathbf{v} - \mathbf{u}).$$

Now suppose that the two coils be removed and replaced by a star-connected system of three impedances z_A, z_C, z_E, such that the electromotive forces and the currents remain unaltered. The equations now are

$$\mathbf{e}_V = (z_A + z_C)\mathbf{v} - z_C\mathbf{u} - z_A\mathbf{w},$$

$$\mathbf{e}_W = (z_A + z_E)\mathbf{w} - z_E\mathbf{u} - z_A\mathbf{v}.$$

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* G. A. Campbell, "Cisoidal oscillations," *Trans. Amer. I.E.E.*, Vol. 30, Part 2, pp. 873–913 (1911).

† S. Butterworth, "Capacity and eddy current effects in inductometers," *Proc. Phys. Soc.*, Vol. 33, p. 314 (1921).

Comparing the coefficients,

$$z_A = -m,$$
$$z_C = m + z_\alpha,$$
$$z_E = m + z_\beta.$$

Hence a pair of mutually influencing impedances can be replaced by a star-connected system of three impedances without mutual inductance.

Apply this device to the example worked out in the last section. The transformed circuit is shown in Fig. 30; $z_\alpha = P + j\omega L$, $z_\beta =$ operator of mutual inductance secondary, $m = j\omega M$; then, from above,

$$z_A = -j\omega M.$$
$$z_C = P + j\omega(L + M).$$
$$z_E = j\omega M + z_\beta.$$

The impedance z_E is in the alternator branch; hence the transformed network is a four-branch impedance bridge with $z_1 = z_C$, $z_2 = Q$, $z_3 = S$, $z_4 = R + z_A$. For balance $z_1z_3 = z_2z_4$, so that

$$S\{P + j\omega(L + M)\} = Q(R - j\omega M),$$

giving

$$SP = QR$$

and

$$L = -\left(1 + \frac{Q}{S}\right)M$$

as the balance conditions, in agreement with the direct method.

21. The Generalized Wheatstone Network. By the use of the principles established above, it is possible to generalize the theory of the Wheatstone network discussed on page 51 to include the effect of mutual inductance between any pair of branches, and thereby to find the general condition for balance of a large class of bridge networks in which mutual inductance is used to attain the null condition. This generalization was first made by Heaviside* in a series of papers published in 1886–7 and will be given here with such modification as is necessary to adapt it to the methods and notation of this book.

Consider the Wheatstone network shown in Fig. 20 and assume that there is, in addition to the impedance operators shown in the six branches, mutual inductance between every pair of branches in the network. In setting up the equations the convention of p. 67 is adopted. Counter-clockwise cyclic currents are put in each of the three meshes to indicate the positive cyclic current directions. Each mutual inductance is in turn imagined to consist of a pair of neighbouring coils wound in the same sense and placed with parallel axes as in Fig. 29. It is then easy to see by an inspection of the sense of a given current whether its mutual inductive effect on a second current is to be regarded as plus or minus in the mesh containing the second. Let the mutual inductances be denoted by double subscripts; thus M_{12} is the mutual inductance between branches 1 and 2, M_{56} between 5 and 6, and so on. It should be remembered that $M_{12} = M_{21}$, and in general $M_{\mathrm{AB}} = M_{\mathrm{BA}}$. Let the symbol m_{AB} denote the mutual inductance operator $j\omega M_{\mathrm{AB}}$; then the equations for the meshes *BCD* and *ADC* are

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* Oliver Heaviside, *Electrical Papers*, Vol. 2, pp. 33–38, pp. 106–115, and particularly pp. 284–286; (1892).

$$\begin{aligned}&(z_2 + z_3 + z_5)\mathbf{i} + (z_2 + z_3)\mathbf{w} - z_3\mathbf{u} + [2(m_{23} + m_{25} + m_{35})\mathbf{i}\\&+ (m_{12} + m_{13} - m_{15} + 2m_{23} + m_{24} + m_{25} + m_{34} + m_{35} - m_{45})\mathbf{w}\\&- (m_{23} + m_{24} - m_{26} + m_{34} + m_{35} + m_{36} - m_{45} - m_{56})\mathbf{u}] = 0,\end{aligned}$$

$$\begin{aligned}&- z_5\mathbf{i} + (z_1 + z_4)\mathbf{w} - z_4\mathbf{u} + [(m_{12} + m_{13} - m_{15} + m_{24} - m_{25} + m_{34}\\&\qquad\qquad\qquad\qquad\qquad\qquad - m_{35} - m_{45})\mathbf{i}\\&+(m_{12} + m_{13} + 2m_{14} + m_{15} + m_{24} - m_{25} + m_{34} - m_{35} + m_{45})\mathbf{w}\\&- (m_{13} + m_{14} - m_{16} + m_{34} - m_{35} + m_{45} + m_{46} + m_{56})\mathbf{u}] = 0.\end{aligned}$$

Collecting coefficients, the balance determinant can be written as

$$\begin{matrix} z_3 + \alpha & z_2 + z_3 + \beta \\ z_4 + \gamma & z_1 + z_4 + \delta \end{matrix} = 0,$$

where

$$\alpha = m_{23} + m_{24} - m_{26} + m_{34} + m_{35} + m_{36} - m_{45} - m_{56}$$

$$\beta = m_{12} + m_{13} - m_{15} + 2m_{23} + m_{24} + m_{25} + m_{34} + m_{35} - m_{45}$$

$$\gamma = m_{13} + m_{14} - m_{16} + m_{34} - m_{35} + m_{45} + m_{46} + m_{56}$$

$$\delta = m_{12} + m_{13} + 2m_{14} + m_{15} + m_{24} - m_{25} + m_{34} - m_{35} + m_{45}$$

so that no current flows in the detector when

$$(z_1z_3 - z_2z_4) + \alpha(z_1 + z_4 + \delta) - \gamma(z_2 + z_3 + \beta) + z_3\delta - z_4\beta = 0.$$

As an example, consider again the network shown in Fig. 30. Put $z_1 = P + j\omega L$, $z_2 = Q$, $z_3 = S$, $z_4 = R$; make all mutual inductance operators zero except $m_{16} = j\omega M$.

Then, $\alpha = 0$, $\beta = 0$, $\gamma = -j\omega M$, $\delta = 0$; so that

$$S(P + j\omega L) - QR + j\omega M(Q + S) = 0.$$

Separating the components, gives for balance,

$$SP = QR,$$

$$L = -\left(1 + \frac{Q}{S}\right)M,$$

as found in the previous paragraphs.

The general theory can be put into a symmetrical form in the way indicated by Potthoff.* By analogy with the balance condition

$$z_1 z_3 = z_2 z_4$$

for a Wheatstone network without mutual inductances, write

$$(z_1 + \zeta_1)(z_3 + \zeta_3) = (z_2 + \zeta_2)(z_4 + \zeta_4),$$

where ζ_1, ζ_2, ζ_3, ζ_4 are functions of the mutual inductances. Expanding both sides and comparing with the solution on p. 71 gives

$$\zeta_1 = \delta - \gamma,\ \zeta_2 = \beta - \alpha,\ \zeta_3 = \alpha,\ \zeta_4 = \gamma,$$

and $\qquad \zeta_1\zeta_3 - \zeta_2\zeta_4 = \alpha\delta - \beta\gamma,$

this last expression being an obvious identity. The desired functions can now be written out in the form

$$\zeta_1 = m_{12} + m_{14} + m_{15} + m_{16} - m_{25} - m_{46} + m_{24} - m_{56},$$
$$\zeta_2 = m_{21} + m_{23} + m_{25} + m_{26} - m_{15} - m_{36} + m_{13} + m_{56},$$
$$\zeta_3 = m_{32} + m_{34} + m_{35} + m_{36} - m_{45} - m_{26} + m_{24} - m_{56},$$
$$\zeta_4 = m_{41} + m_{43} + m_{45} + m_{46} - m_{35} - m_{16} + m_{13} + m_{56},$$

which can be given a simple interpretation. Each of the branches z_1, z_2, z_3, z_4 is connected to two others and to the source and detector; for example, z_1 is joined to z_2 and z_4, which are on opposite sides of the Wheatstone quadrilateral, and to z_5 and z_6; see Fig. 20. Any such group of five impedances will have ten possible mutual inductances; it will be noted, however, that each of the above functions involves only eight coefficients. The first four terms consist of the mutual inductances between the balancing branch under consideration and the four impedances joined to its ends; e.g. for z_1 these terms are m_{12}, m_{14}, m_{15}, m_{16}. These may be called the four direct mutual inductances between a balancing branch and those connected directly to it. The fifth and sixth terms are the mutual inductances between each pair of impedances at the ends of the chosen branch; e.g. for z_1 these are m_{25} and m_{46}. These may be called the end mutual inductances for a given balancing branch. The seventh term is the mutual inductance between the two balancing branches at the ends of the chosen branch; for z_1 this will be m_{24}. Since these branches are opposite sides of the bridge, this term may be called the cross mutual inductance. Finally the eighth term is the mutual inductance between source and detector, the diagonal mutual inductance. The two terms omitted are the mutual inductances between a diagonal arm and the balancing branch at the opposite end of the chosen branch; for z_1 these would be z_{26} and z_{45}. In general, branch z_n $(n = 1, 2, 3, 4)$ has branches z_{n+1} and z_{n-1} joined to its terminals (z_0 being interpreted as z_4) in addition to the detector z_5 and source z_6. Then

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* K. Potthoff, "Die Gleichgewichtsbedingung von Wechselstrombrücken mit Gegeninduktivitäten," *Elekt. Zeits.*, Vol. 58, pp. 793–794 (1937).

$$\zeta_n = (\text{sum of four direct mutuals}) - (\text{sum of two end mutuals}) + (\text{cross mutual}) + (-)^n(\text{diagonal mutual}),$$

from which the functions can be written out in a symmetrical form by simple inspection.

Suppose the source and detector to be interchanged in position; then it has been shown on p. 60 that this is equivalent to a transposition of z_2 and z_4 in the original network. The mutual inductances between the detector and each of the other branches in the conjugate network are numerically equal to those between the source and those branches in the original network, and conversely. This is equivalent to a permutation of the subscripts 5 and 6. Making these changes gives for the balance condition of the conjugate network

$$(z_1 + \zeta_1)(z_3 + \zeta_3) = (z_4 + \zeta_4)(z_2 + \zeta_2),$$

which is identical with that for the original network. Hence, the balance condition for a generalized Wheatstone network is unchanged by interchanging the positions of source and detector.

This theory is of extreme generality since it includes all possible bridges of the Wheatstone type in which mutual inductance effects are also present. It also covers a variety of cases in which the bridge is not, at first sight, in the Wheatstone four-branch form, but can be transformed into that form by some slight change. In this connection the following examples are instructive.

Consider first the Campbell frequency bridge shown in Fig. 32(*a*) and described on page 470, the balance condition being there deduced from first principles. It is seen in Fig. 32(*b*) that this network can be re-drawn in such a way as to become a Wheatstone network in which $z_1 = R_2 + j\omega L_2$, $z_2 = \infty$, $z_3 = 0$, $z_4 = -j/\omega C$ are the branch impedance operators, and the mutual inductance operator between z_1 and the alternator is $m_{16} = j\omega M$. Then $a = 0$, $\beta = 0$, $\gamma = -j\omega M$, $\delta = 0$, giving

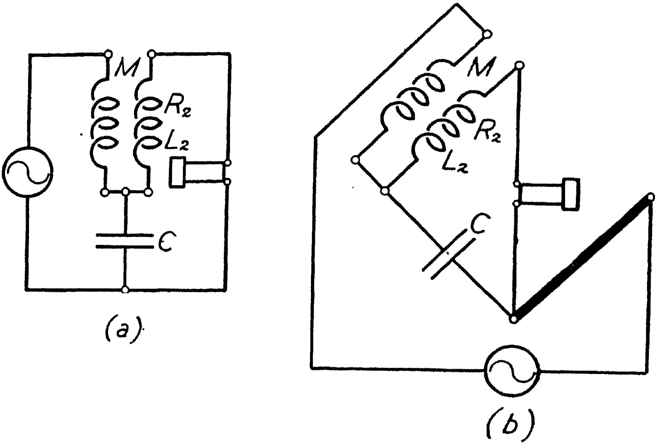

FIG. 32.—WHEATSTONE TRANSFORMATION OF CAMPBELL'S FREQUENCY BRIDGE

$$\infty\left\{\frac{j}{\omega C} + j\omega M\right\} = 0, \text{ i.e. } \omega^2 = -1/MC.$$

Karapetoff* has given an artifice by means of which the theory can be applied to bridges in which the sole connection with the alternator is via mutual inductance. An important case is that of Maxwell's method for the comparison of mutual inductances, p. 418, and Fig. 33(*a*). Suppose a connection to be taken from the terminal of each primary where it joins the alternator, these connections passing to points infinitesimally close to the upper point where the telephone is attached to the network. In the limit, when the points of attachment of these leads and that of the telephone coincide, the extra connections take no current into the bridge, and mutual inductance is the only connection between bridge and alternator. In this case the network becomes a four-branch Wheatstone arrangement of Fig. 33(*b*) in which $z_1 = 0$, $z_2 = 0$, $z_3 = R_2 + j\omega L_2$, $z_4 = R_1 + j\omega L_1$.

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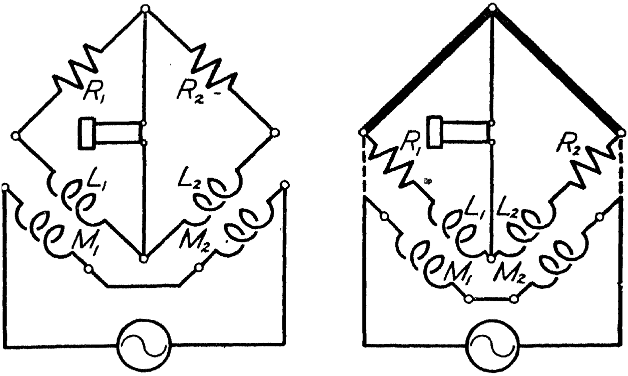
* V. Karapetoff, "General equations of a balanced alternating current bridge," *Phil. Mag.*, 6th series, Vol. 44, pp. 1024–1032 (1922).

Mutual inductances $m_{63} = j\omega M_2$ and $m_{64} = j\omega M_1$ exist between the alternator and z_3, z_4, so that $\alpha = j\omega M_2$, $\beta = 0$, $\gamma = j\omega M_1$, $\delta = 0$. Then from above, $j\omega M_2(R_1 + j\omega L_1) - j\omega M_1(R_2 + j\omega L_2) = 0$ for balance, or $M_1/M_2 = L_1/L_2 = R_1/R_2$.

In most of the Wheatstone networks used in practice, mutual inductance, if present, exists only between one pair of branches.



(a) (b)

Fig. 33.—Wheatstone Transformation of Maxwell's Mutual Inductance Bridge

There will be, therefore, 15 possible bridges of this type, namely, six in which the mutual inductance is between a pair of the branches 1, 2, 3, 4; four with interaction between 1, 2, 3 or 4 and the detector 5; four with interaction between 1, 2, 3 or 4 and the source 6; and one with the source 6 influencing the detector 5. The general operational balance conditions in these 15 cases are as follows—

$z_1z_3 - z_2z_4 + m_{12}(z_3 - z_4) = 0,$ with m_{12} only;
$z_1z_3 - z_2z_4 - m_{23}(z_4 - z_1) = 0,$ with m_{23} only;
$z_1z_3 - z_2z_4 + m_{34}(z_1 - z_2) = 0,$ with m_{34} only;
$z_1z_3 - z_2z_4 - m_{14}(z_2 - z_3) = 0,$ with m_{14} only;
$z_1z_3 - z_2z_4 - m_{13}(z_2 + z_4) - m_{13}^2 = 0,$ with m_{13} only;
$z_1z_3 - z_2z_4 + m_{24}(z_1 + z_3) + m_{24}^2 = 0,$ with m_{24} only;
$z_1z_3 - z_2z_4 + m_{15}(z_3 + z_4) = 0,$ with m_{15} only;

$$\begin{aligned}
&z_1z_3 - z_2z_4 - m_{25}(z_3 + z_4) = 0, &&\text{with } m_{25} \text{ only;}\\
&z_1z_3 - z_2z_4 + m_{35}(z_1 + z_2) = 0, &&\text{with } m_{35} \text{ only;}\\
&z_1z_3 - z_2z_4 - m_{45}(z_1 + z_2) = 0, &&\text{with } m_{45} \text{ only;}\\
&z_1z_3 - z_2z_4 + m_{16}(z_2 + z_3) = 0, &&\text{with } m_{16} \text{ only;}\\
&z_1z_3 - z_2z_4 - m_{26}(z_1 + z_4) = 0, &&\text{with } m_{26} \text{ only;}\\
&z_1z_3 - z_2z_4 + m_{36}(z_1 + z_4) = 0, &&\text{with } m_{36} \text{ only;}\\
&z_1z_3 - z_2z_4 - m_{46}(z_2 + z_3) = 0, &&\text{with } m_{46} \text{ only;}\\
&z_1z_3 - z_2z_4 - m_{56}(z_1 + z_2 + z_3 + z_4) = 0, &&\text{with } m_{56} \text{ only;}
\end{aligned}$$

where $m_{AB} = j\omega M_{AB}$. With any given operators z_1, z_2, z_3, z_4 the two balance conditions for every type of Wheatstone network with mutual inductance between one pair of branches are easily found from the appropriate member of these equations; numerous examples will be found in Chapter IV.

22. The Locus Diagram of a Wheatstone Network. A very large number of bridge networks used in practice are simple Wheatstone arrangements of impedances, i.e. four-branch networks. Other networks, not primarily of Wheatstone form, may be transformed into that form by the application of one or other of the theorems discussed in the preceding sections; several typical examples have already been given, e.g. the Anderson network, the double bridge and networks with mutual inductance. It would appear, therefore, that the Wheatstone network represents a basic type and, for this reason, it is important to examine its theory rather more closely, especially for the condition when the bridge is unbalanced. For this purpose the simplest way is to use the theory of loci developed in Section 9.* The results are very useful in studying the process of balancing a bridge and in estimating its sensitivity.

Consider the bridge network of Fig. 34, to which a harmonic voltage of unit amplitude is applied between the terminals A and B; this voltage is represented by a harmonic vector **1**. In the simplest case let the detector joined to C and D be of infinitely high impedance, represented approximately, in practice, by a thermionic voltmeter or an amplifier. Then we may regard CD as an open-circuited branch and the p.d. between C and D is easily seen to be

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* A. C. Seletzky, "Cross potential of a four-arm network," *Elec. Eng.*, Vol. 52, pp. 861–867 (1933). A. C. Seletzky and J. R. Anderson, "Cross current of a five-arm network," *Elec. Elec.* Vol. 53, pp. 1004–1009 (1934), A. C. Seletzky, "Amplification loci of resistance-capacitance coupled amplifiers," *Elec. Eng.*, Vol. 55, pp. 1364–1371 (1936).

$$\mathbf{v} = \frac{z_1z_3 - z_2z_4}{(z_1 + z_2)(z_3 + z_4)} \quad . \qquad (13a)$$

If the points C and D are bridged by a detector of finite impedance operator z_5, the current $\mathbf{i}$ in the detector is easily shown to be (see p. 60)

$$\mathbf{i} = \frac{z_1z_3 - z_2z_4}{z_5(z_1 + z_2)(z_3 + z_4) + z_1z_2(z_3 + z_4) + z_3z_4(z_1 + z_2)} \qquad (13b)$$

Finally, if A and B are joined to an alternator of impedance operator z_6 and e.m.f. $e_1\mathbf{1}$ then as shown on p. 60 the detector current is

$$\frac{z_1z_3 - z_2z_4}{\Delta} \cdot e_1\mathbf{1} \qquad (13c)$$

where

$$\Delta = z_5z_6(z_1 + z_2 + z_3 + z_4) + z_5(z_1 + z_2)(z_3 + z_4) + z_6(z_1 + z_4)(z_2 + z_3) + [z_1z_2(z_3 + z_4) + z_3z_4(z_1 + z_2)]$$

In all three cases it will be noted that balance occurs, i.e. there is no p.d. between C and D, and the detector is without current when $z_1z_3 - z_2z_4 = 0$.

In all bridges one branch consists of the impedance to be measured, say z_1; two branches, such as z_2 and z_3, are usually fixed impedances; the fourth branch, z_4 say, can be regulated in reactance and resistance successively until balance is secured. Simple inspection of Equations (13*a*), (*b*) or (*c*) will show that the operators are then linear, both in numerator and denominator, in the part of z_4 (reactance or resistance) that is varied. They are, therefore, bilinear operators and the loci of $\mathbf{v}$ or $\mathbf{i}$ are circles, as shown on p. 43. Similar results hold when any other set of three branches are fixed and the fourth is varied either in resistance or reactance.

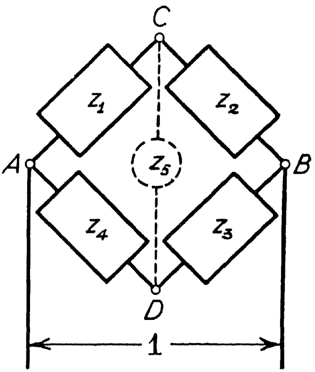

FIG. 34.—ILLUSTRATING THE THEORY OF BRIDGE LOCI

Since the above three equations do not differ in the nature of the loci involved, but only in the degree of arithmetical complexity contained in them, it will suffice to illustrate the properties of the loci by examples of the simplest class, Equation (13*a*); any other case can be similarly analysed.

A considerable number of bridges fall into the resistance ratio-arm class, where $z_2 = R_2$ and $z_3 = R_3$ are fixed resistances. The branch $z_1 = R_1 + jX_1$ is the unknown impedance and $z_4 = R_4 + jX_4$ is an adjustable balancing impedance of similar kind (see also p. 294). Substituting in Equation (13*a*)

$$\mathbf{v} = \frac{R_3(R_1 + jX_1) - R_2(R_4 + jX_4)}{(R_1 + R_2 + jX_1)(R_3 + R_4 + jX_4)}\,\mathbf{1}.$$

Two cases now arise. In the first X_4 *is fixed and* R_4 *varied*, then

$$\mathbf{v} = \frac{[R_3(R_1 + jX_1) - jR_2X_4] - R_2R_4}{[(R_1 + R_2 + jX_1)(R_3 + jX_4)] + (R_1 + R_2 + jX_1)R_4}\,\mathbf{1}$$

which is the form for a circular locus (p. 43), R_4 ranging over positive values from 0 to ∞. The short-circuit and open-circuit points are

$$\mathbf{v}_0 = \frac{R_1R_3 + j(R_3X_1 - R_2X_4)}{(R_1 + R_2 + jX_1)(R_3 + jX_4)}\,\mathbf{1}$$

and

$$\mathbf{v}_\infty = \frac{-R_2}{R_1 + R_2 + jX_1}\,\mathbf{1}$$

The latter is independent of R_3 and X_4 and is a fixed point for all loci when R_1, X_1 and R_2 are fixed. Any third point, e.g. that for which $R_1R_3 = R_2R_4$, will determine the locus.

When the bridge can be balanced by R_4 adjustments, X_4 is such that $R_3X_1 = R_2X_4$; then

$$\mathbf{v} = \frac{R_1R_3 - R_2R_4}{[(R_1 + R_2 + jX_1)(R_3 + jX_4)] + (R_1 + R_2 + jX_1)R_4}\,\mathbf{1}$$

and

$$\mathbf{v}_0 = \frac{R_1R_3}{(R_1 + R_2 + jX_1)(R_3 + jX_4)}\,\mathbf{1}$$

When balanced, $R_1R_3 - R_2R_4 = 0$ so that the locus passes through the origin for this value of R_4. The value of $\mathbf{v}_\infty$ is as before.

In the second case R_4 *is fixed and* X_4 *varied*; then the appropriate forms of Equation (13*a*) are

$$\mathbf{v} = \frac{[(R_1R_3 - R_2R_4) + jR_3X_1] - jR_2X_4}{[(R_1 + R_2 + jX_1)(R_3 + R_4)] + j(R_1 + R_2 + jX_1)X_4}\,\mathbf{1}$$

with

$$\mathbf{v}_0 = \frac{(R_1R_3 - R_2R_4) + jR_3X_1}{(R_1 + R_2 + jX_1)(R_3 + R_4)}\mathbf{1}$$

and

$$\mathbf{v}_\infty = \frac{-R_2}{R_1 + R_2 + jX_1}\mathbf{1} \text{ as before.}$$

Any third point, e.g. that for which $R_3X_1 = R_2X_4$ settles the locus.

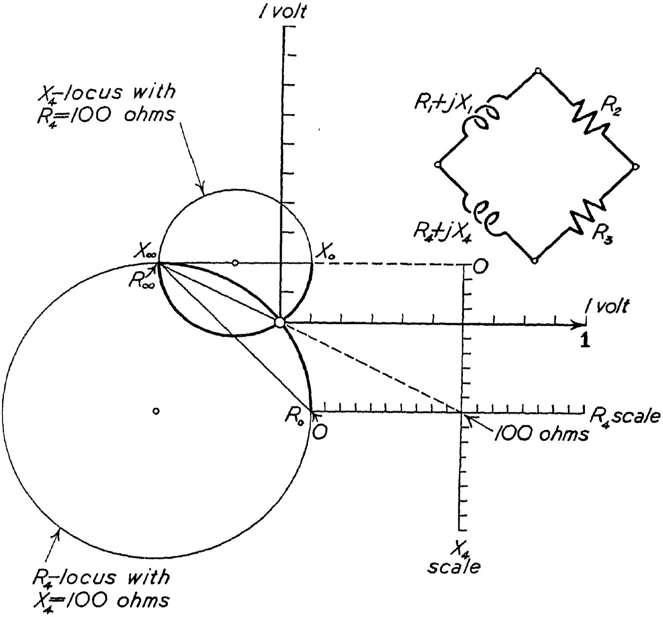

FIG. 35.—DETECTOR VOLTAGE LOCI FOR MAXWELL INDUCTANCE BRIDGE

When the bridge can be balanced R_4 is such that $R_1R_3 = R_2R_4$; then

$$\mathbf{v} = \frac{j(R_3X_1 - R_2X_4)}{[(R_1 + R_2 + jX_1)(R_3 + R_4)] + j(R_1 + R_2 + jX_1)X_4}.$$

and

$$\mathbf{v}_0 = \frac{jR_3X_1}{(R_1 + R_2 + jX_1)(R_3 + R_4)}\mathbf{1}$$

When balance is reached $R_3X_1 - R_2X_4 = 0$ and the locus passes through the origin.

As a simple numerical example consider a Maxwell inductance bridge in which $z_1 = R_1 + jX_1 = 100 + j100$, $z_2 = R_2 = z_3 = R_3 = 100$ and assume the bridge can balance, i.e. draw the two loci for (i) $z_4 = R_4 + j100$ varying R_4 and for (ii) $z_4 = 100 + jX_4$ varying X_4. These are shown in Fig. 35 and are typical of the loci for balanced resistance-ratio bridges. As explained on p. 44 scale lines for the variable R_4 and X_4 are shown and enable the value of **v** appropriate to a desired value of the parameter to be read off by measuring the length of the vector from the origin to the circular locus concerned. Seletzky (*loc. cit.*) gives an example of a balanced bridge of a more complicated kind in which the opposite branches z_2 and z_4, are fixed resistances, z_1 is a fixed inductive resistance and z_3 consists of a fixed resistance and a variable capacitance in series—the Hay bridge of Fig. 133; no further principles are involved. He also works out for the same bridge the detector current locus, using Equation (13*b*).

23. The Balancing Locus of a Wheatstone Network. It has been shown on p. 52 that the process of balancing an a.c. bridge consists in making successive adjustments of resistance and reactance—often in a single branch, though not necessarily so—until the potentials of the branch points C and D are equal at every instant. In some instances certain pairs of balancing adjustments enable balance to be very rapidly attained; in other cases the convergence toward balance may be very slow, while yet other pairs may not permit balance to be secured at all. The rapidity with which balance can be attained and the sensitivity of the network to changes in the balancing elements are matters of considerable practical importance which can be most easily studied by the aid of locus diagrams. In this Section we shall consider an actual example of balancing procedure by the locus method, establishing principles that will be used on p. 299 to study balance convergence. Sensitivity will be examined in the next Section.

Consider the simple Maxwell inductance bridge for which the loci under balance conditions have been worked out in Section 22. Again fixing $z_1 = R_1 + jX_1 = 100 + j100$, $z_2 = R_2 = z_3 = R_3 = 100$, first suppose the bridge seriously out of balance by putting $z_4 = R_4 + j200$, i.e. X_4 has twice the value for which adjustment of R_4 could succeed in establishing balance. Then as R_4 is varied a circular locus is described, and the value of **v** is found to be least when $R_4 = 134$ ohms. Keeping this value of R_4 draw the locus corresponding with a variation of X_4 in $z_4 = 134 + jX_4$; **v** comes to a second minimum when $X_4 = 100$. With this new value of X_4 return to adjustment of R_4, drawing the locus with $z_4 = R_4 + j100$; balance is now secured when $R_4 = 100$. The three loci are drawn in Fig. 36 and the heavy arcs show very strikingly how balance is converged upon from the state of unbalance with $R_4 = \infty$ and $X_4 = 200$, by successive adjustment of R_4 and X_4 to give minimum p.d. across the points C and D, until the p.d. is finally zero. Convergence in this case is particularly rapid. A more detailed study will be made of the whole question on p. 299.

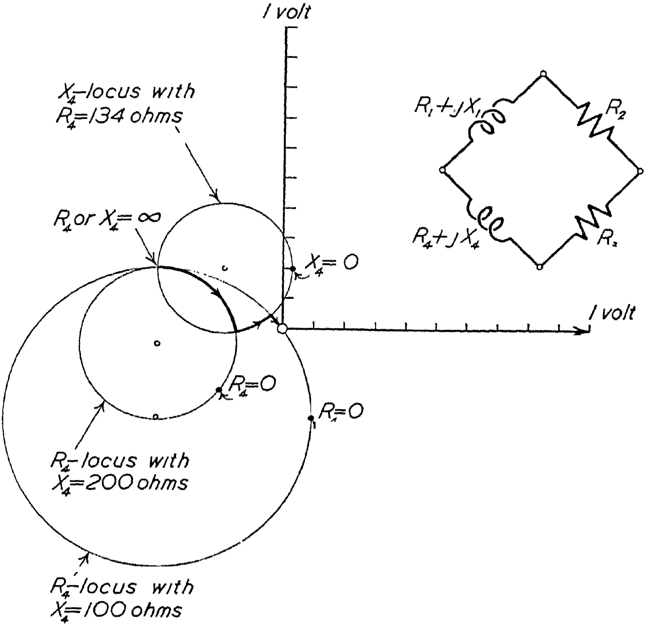
1 volt
X_4-locus with R_4=134 ohms
R_4 or $X_4=\infty$
X_4= 0
1 volt
R_4=0
R_4=0
R_4-locus with X_4=200 ohms
R_4-locus with X_4=100 ohms
R_1+jX_1
R_2
R_4+jX_4
R_3

FIG. 36.—DETECTOR VOLTAGE LOCI TRAVERSED DURING BALANCING OF MAXWELL INDUCTANCE BRIDGE

24. The Sensitivity of Bridge Networks. It is the object of a bridge method to measure a given quantity with the greatest precision, so that the network should be arranged for the greatest sensitiveness. With a given method, that arrangement of the various branches will be most sensitive in which, for a given deviation of adjustment from balance, the current through the detector is greatest.

The theory of sensitiveness of bridges of the Wheatstone type used with direct current is well-known and was worked out by Schwendler* and by Heaviside.† Lord Rayleigh‡ generalized the results by the symbolic vector method to apply to Wheatstone impedance networks. From the equations on page 51 the following results can be deduced.

With a given alternator z_6 and detector z_5 to measure an impedance z_1 the symbolic conditions for sensitiveness are

$$z_3 = \qquad z_2 = \frac{z_5 z_1}{z_5 + z_1}(z_5 + z_6), \; z_4 = \sqrt{\frac{z_6 z_1}{z_6 + z_1}(z_5 + z_1)}.$$

Hence z_3 should be chosen to have the value stated and z_1 measured approximately; then setting z_2 to the sensitivity value, final balance is made by alteration of z_4.

When a certain network has been set up, interchanging the alternator and detector may increase the sensitivity; the rule for their position is the same as for the d.c. bridge: Of the two z_5 and z_6 that which has the larger impedance should connect the junction of the two largest consecutive impedances in the bridge with the junction of the two smallest consecutive impedances, as proved on p. 61.

Lord Rayleigh has pointed out that adjustment of the impedance of the alternator and detector branches may produce further increase in sensitiveness. The best alternator is that which has internal impedance equal to the external impedance across its terminals; the best detector has an impedance equal to the impedance external to its terminals. Assuming these to be adjusted by inclusion of suitable coils or condensers in series with the alternator and detector, or, alternatively, by joining source and detector to the bridge through suitable transformers (p. 232), the greatest sensitiveness occurs when

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* L. Schwendler "On the galvanometer resistance to be employed in testing with Wheatstone's diagram," *Phil. Mag.* 4th series, Vol. 31, pp. 364–368 (1866).

† O. Heaviside, *Electrical Papers*, Vol. 1, pp. 3–8, 8–12 (1892).

‡ Lord Rayleigh, "On the sensitiveness of the bridge method in its application to periodic electric currents," *Proc. Roy. Soc.*, Vol. 49, pp. 203–217 (1891).

$$z_1 = z_2 \qquad z_5$$

i.e. in an equal ratio bridge in which the source and detector have each an impedance equal to that to be measured. Thus, when measuring low impedances, e.g. a small coil, low impedance source and detector are necessary; while with high impedances, e.g. small condensers at low frequencies, they must have high values.

In the modern bridge, certain other conditions must be fulfilled in order that the arrangement may be the most favourable to use with a tuned vibration galvanometer. The reader is referred to the papers of Butterworth* and Jaeger† for further information, and to the various remarks and references in Chapters III (pp. 267–281) and IV.

The detectors, whether vibration galvanometers or telephones, used in modern laboratory practice are usually of very high sensitiveness so that the bridge network can, in most instances, be set up with quite large deviations from the "best" conditions contemplated above. These sensitivity conditions then serve in practice to provide a criterion by which a bridge may be set up *ab initio*; rigorous observance of these requirements is generally less necessary in modern experimental work since the detectors usually employed provide a sufficient margin of sensitiveness for most purposes, especially when amplifiers are used. Special cases are considered in Chapter IV as occasion arises.‡

It may be pointed out here that the theory of the sensitivity of Wheatstone networks applies directly to other more complex networks, such as the Anderson and Kelvin bridges, which can be transformed by the mesh-star theorem into the Wheatstone form.

It is easy to develop expressions for computing the sensitiveness of a network to a change in any balancing adjustment. Using Equation (13*c*) the general expression for the current through the detector is

$$\frac{z_1 z_3 - z_2 z_4}{\Delta} \, {}_1 1$$

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* S. Butterworth, "On the vibration galvanometer and its application to inductance bridges," *Proc. Phys. Soc.*, Vol. 24, pp. 75–94 (1912).

† W. Jaeger, "Günstigste Schaltung der Vibrationsgalvanometer," *Arch. f. Elekt.*, Vol. 4, pp. 262–268 (1916). K. Grufin, "Das Resonanzmaximum beim Vibrationsgalvanometer," *Arch. f. Elekt.*, Vol. 8, pp. 210–213 (1920).

‡ For a number of examples see F. Wenner, *Proc. Phys. Soc.*, Vol. 39, pp. 124–144 (1927).

Assuming z_1, z_2, z_3, z_5 and z_6 to be given and that adjustments are made to z_4,

$$\frac{\partial \mathbf{i}}{\partial z_4} = \frac{-z_2\Delta - (z_1z_3 - z_2z_4)\dfrac{\partial \Delta}{\partial z_4}}{\Delta^2} e_1\mathbf{1},$$

expresses the rate of change of $\mathbf{i}$ with a total vector change in z_4. If $z_4 = R_4 + jX_4$ note that $\partial\mathbf{i}/\partial R_4 \equiv \partial\mathbf{i}/\partial z_4$ and that $\partial\mathbf{i}/\partial X_4 = j\partial\mathbf{i}/\partial z_4$, so that the effect of resistance and reactance changes alone may be easily stated. For sensitivity we are interested in the change of current at the balance point; putting $z_1z_3 - z_2z_4 = 0$ the rate of change at balance is

$$\left(\frac{\partial \mathbf{i}}{\partial z_4}\right)_0 = -\frac{z_2}{\Delta_0} e_1\mathbf{1}$$

where Δ_0 is the value of Δ at balance. It is easy to show that

$$\Delta_0 = \frac{1}{z_2}\{(z_2 + z_3)z_6 + (z_1 + z_2)z_3\}\{(z_1 + z_2)z_5 + z_1(z_2 + z_3)\}$$

and so

$$\left(\frac{\partial \mathbf{i}}{\partial z_4}\right)_0 = \frac{}{(z_1 + z_2)(z_2 + z_3)\left\{z_6 + \dfrac{(z_1 + z_2)}{(z_2 + z_3)}z_3\right\}\left\{z_5 + \dfrac{(z_2 + z_3)}{(z_1 + z_2)}z_1\right\}} e_1\mathbf{1}$$

Hence the alternator acts as though it were in a circuit of operator $z_6 + [z_3(z_1 + z_2)/(z_2 + z_3)]$ and the detector similarly in a circuit of operator $z_5 + [z_1(z_2 + z_3)/(z_1 + z_2)]$. By adding reactance of the appropriate sign in series with z_6 and z_5, or by the use of transformers, the total operators for alternator and detector in the above denominator may be made entirely real, and the current made the greatest possible for these adjustments, as Rayleigh and Butterworth have shown. Further increase depends on the choice of $z_2^2/(z_1 + z_2)(z_2 + z_3)$.

Proceeding now to finite increments near balance, put $z_2 = z_1z_3/z_4$ in the numerator, then

$$\delta\mathbf{i} \doteq \frac{-z_1z_2z_3}{(z_1 + z_2)(z_2 + z_3)\left\{z_6 + \dfrac{(z_1 + z_3)}{(z_2 + z_3)}z_3\right\}\left\{z_5 + \dfrac{(z_2 + z_3)}{(z_1 + z_2)}z_1\right\}} \cdot \frac{\delta z_4}{z_4} \cdot e_1\mathbf{1},$$

expressing the increment of current at balance in terms of a fractional increase $\delta z_4/z_4$ in the branch z_4. This expression can be put into a more manageable form, due to Schering,* in the following way. Suppose the voltage e_1 to be maintained between *A* and *B*, i.e. let the impedance z_6 be disregarded; then we can write for Fig. 34,

$$\delta\mathbf{i} \doteq \frac{-z_1z_2}{(z_1 + z_2)^2\left\{z_5 + \dfrac{(z_2 + z_3)}{(z_1 + z_2)}z_1\right\}} \cdot \frac{\delta z_4}{z_4} \cdot e_1\mathbf{1},$$

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* H. Schering, "Die Empfindlichkeit einer Wechselstrombrücke," [illegible] *Zeits.*, vol. 52, pp. 1133–1134 (1931). J. Krönert, "Empfindlichkeit von Wheatstone-Gleichstrombrücken in der Nullmethode," *Arch. f. tech. Mess.*, 910–12 (Jan. 1933).

or
$$\delta\mathbf{i} = \frac{-1}{\dfrac{(z_1+z_2)(z_2+z_3)}{z_2}\left\{1 + z_5\dfrac{(z_1+z_2)}{(z_2+z_3)z_1}\right\}} \cdot \frac{\delta z_4}{z_4} \cdot e_1\mathbf{1}$$

Now
$$(z_1+z_2)(z_2+z_3)/z_2 = [z_1z_2 + z_2^2 + z_2z_3 + z_1z_3]/z_2 = z_1 + z_2 + z_3 + z_4$$
when $z_1z_3 = z_2z_4$. Also, if z_{CD} is the operator of $(z_1 + z_4)$ and $(z_2 + z_3)$ in parallel, i.e. the operator for the network joined to the detector terminals, then
$$z_{CD} = \frac{(z_1+z_4)(z_2+z_3)}{z_1+z_2+z_3+z_4};$$
but $\qquad z_1 + z_2 + z_3 + z_4 = (z_1+z_2)(z_1+z_4)/z_1$, when $z_1z_3 = z_2z_4$

so that $\qquad z_{CD} = \dfrac{(z_2+z_3)}{(z_1+z_2)}z_1$ under the same conditions.*

Substituting, at balance, the rate of increase is
$$\frac{\partial\mathbf{i}}{\partial z_4} = -\frac{1}{(z_1+z_2+z_3+z_4)\left[1+\dfrac{z_5}{z_{CD}}\right]} \cdot \frac{1}{z_4}e_1\mathbf{1}$$
and the increment,
$$\delta\mathbf{i} = \frac{}{(z_1+z_2+z_3+z_4)\left[1+\dfrac{z_5}{z_{CD}}\right]} \cdot \frac{\delta z_4}{z_4}e_1\mathbf{1}$$
which is a very useful form, applied by Schering to study the sensitivity of the Schering bridge. For more accurate use at somewhat wider deviation from balance Krönert has provided the necessary second-order terms.

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* Likewise it can be easily shown that the alternator, in the general case, acts on an impedance $z_{AB} = (z_1 + z_2)(z_3 + z_4)/(z_1 + z_2 + z_3 + z_4)$, i.e. the impedance of the network across *A* and *B* when z_5 is open-circuited. At balance $z_{AB} = (z_1 + z_2)z_3/(z_2 + z_3)$. The denominators of the general expressions may therefore be rewritten to contain $(z_6 + z_{AB})(z_5 + z_{CD})$.

CHAPTER III

APPARATUS

1. Introduction. The reader will have gathered from the remarks which have been made in the preceding chapters that the apparatus required for the construction and use of a bridge network will consist of suitable standards with which to construct the branches, of a means of supplying the network with alternating current at a definite frequency, and of a detecting instrument to indicate when balance is attained. More specifically, the apparatus required for the purpose of making alternating current bridge measurements will be classified under the following three headings—

1. Standards of Resistance, Self and Mutual Inductance, and Capacitance.
2. Sources of Alternating Current.
3. Detectors.

STANDARDS OF RESISTANCE

2. General Considerations. A perfect resistance for use with alternating current should be absolutely non-reactive and should offer the same resistance at all frequencies. This ideal is never attained in practice, owing to the effects of inductance and capacitance in the windings, and to the influence of certain secondary phenomena which accompany the use of alternating currents. It is possible, however, to construct a resistance in which these disturbing effects can be reduced to a minimum, so that the imperfections to which they give rise can be made of negligible importance and the ideal very nearly approached.

The various factors which have to be considered in the design of a non-reactive resistance winding are: (i) the inductance and capacitance of the coils; (ii) eddy currents; (iii) permanence of value and temperature influences; (iv) effects arising from the grouping of resistance coils to form "plug" or "dial" boxes. Throughout the following discussion the word "coil" does not necessarily mean that the resistance wire is wound in a spiral upon a cylindrical bobbin, but is intended to signify any convenient way in which the wire is arranged in a compact form.

The standards of resistance which are to be treated in this chapter are those ordinarily used in alternating current bridge work, capable of carrying small or moderate currents and ranging in value up to about 10,000 ohms. Low-value four-terminal resistors for heavy currents, such as are used in current transformer testing, and very high resistors, such as are employed as voltage dividers, fall outside the scope of the present discussion; these special resistors are, however, occasionally used in bridge networks in exceptional cases.*

3. Residual Inductance and Capacitance in Resistors. The inductance of a coil can be reduced to a minimum by arranging the windings so that they have the least possible magnetic effect. This can be attained by winding the coil in such a way that portions of it which lie close to one another carry equal currents in opposite directions; the resultant magnetic field of the coil is thereby made very small.

The self-capacitance of the coil, and the effects due to it, can be minimized by subdivision of the winding in such a way that the neighbouring portions of the coil possess only a very small capacitance and have small differences of potential between them.

There will always be a small residual magnetic effect, no matter what process of winding be used, since the inductance could only be made zero by arranging the "go" and "return" portions of the winding to be in absolute coincidence. In a similar manner, since the portions of the coil must be very close in order to minimize the inductance, there will necessarily be a residue of self-capacitance. It is the object of a satisfactory method of winding resistance coils to reduce the residual inductance and capacitance as far as possible, and it will be clear that such a method will probably involve a compromise between the effects of residual inductance, on the one hand, and the influence of residual capacitance on the other.

The residuals of a resistor can be dealt with in three different ways, viz.: (i) the inductance and the capacitance can be independently reduced to be negligibly small; (ii) the effects of inductance can be balanced against those of capacitance; (iii) the resistor can be made in such a way that the residuals may be calculated from its dimensions. The first two methods are used in the construction of coils for use in resistance boxes; the third method is applied in work of the highest precision where a knowledge of the residuals is necessary.

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* Low- and high-valued resistors are considered in detail in Chapter IX of *Instrument Transformers*, B. Hague, 1936 (Pitman), to which the reader is referred. For a good summary of resistor construction see J. Krönert, "Präzisions Drahtwiderstände für Gleich-und Wechselstrom," *Arch. f. tech. Mess.*, Z111-1 (Nov., 1931).

4. Methods of Constructing Resistors with Small Residuals. In the last section it has been shown that the principle underlying all methods for the reduction of the residual effects in a resistor, is to subdivide the winding in such a way that neighbouring parts of it have small capacitances and only slight differences of potential between them. At the same time, the winding must be so distributed that its total magnetic effect is very small. It is generally easy to secure the latter result, but it is by no means so simple to remove the effects of capacitance.

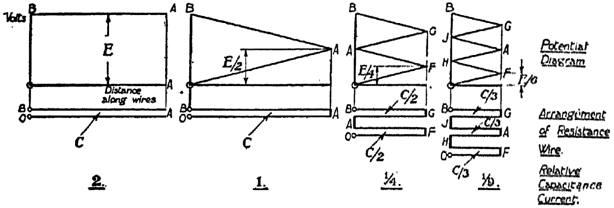

FIG. 37.—ILLUSTRATING THE REDUCTION OF CAPACITANCE EFFECTS IN NON-INDUCTIVE RESISTORS

In order to appreciate the principle upon which all methods of reducing self-capacitance effects are based, consider the case of a "non-inductive" resistance composed of two parallel portions, *OA*, *AB*, of a wire folded back on itself at its mid-point *A*. Let a sinusoidal potential difference of *E* volts be maintained on the terminals *O*, *B*, Fig. 37. Assume first that the wires are separated at *A* and that their distributed capacitance may be represented by *C*; then the potential of *OA* being zero and of *BA* being *E* volts, a definite displacement current will flow in the intervening dielectric. Now let the wires be joined at *A*; a current will flow between *O* and *B* along the wires, producing a uniform fall of potential down them. The average potential difference acting on the distributed capacitance is, therefore, halved; so that the capacitance current is also halved.

Again, let each of the original portions OA, BA be folded back upon themselves, the two loops so formed being connected in series. Then, since the loops are half the length of the original loop, the capacitance of each will be approximately $C/2$. The average potential difference across one loop is now $E/4$, so that the capacitance current is reduced to one-eighth of the value first obtained. By a similar process of reasoning, subdivision of the wire into three equal loops will reduce the capacitance effect to one-eighteenth of the original value, and so on.

In general, subdividing a given length of wire into n non-inductive sections will give a self-capacitance effect approximately $1/n^2$ as great as that obtained if the wire were arranged in one non-inductive winding.

To a first approximation the self-capacitance effect in a coil of resistance R and inductance L can be represented by a condenser of capacitance C connected in parallel with the coil; this condenser is referred to as the self-capacitance of the coil. Then, as stated in Fig. 10, the impedance operator for such an arrangement is

$$z = \frac{R + j\omega\,[L(1 - \omega^2 CL) - CR^2]}{(1 - \omega^2 CL)^2 + \omega^2 C^2 R^2}.$$

Now in a resistor both L and C are small quantities, so that to a first approximation the effective resistance and inductance of the coil become

$$R' \doteqdot R\left\{1 + \omega^2 C(2L - CR^2)\right\}$$

and

$$L' \doteqdot L - CR^2.$$

The phase-displacement between the voltage applied to the coil and the current flowing into it is

$$\phi \doteqdot \arctan \frac{\omega(L - CR^2)}{R} \doteqdot \arctan(\omega L'/R)$$

to the same order of approximation.

The quantity L' is the effective residual inductance of the coil, or, simply, its *residual*. If the coil be such that the effects of inductance preponderate over those of capacitance ($L > CR^2$), L' is positive. If, on the other hand, the capacitive residual is the greater, then CR^2 exceeds L and L' becomes negative. The time-constant of the coil is

$$T = \frac{L}{R} - CR = L'/R$$

which is positive or negative, according as $L \gtrless CR^2$, and is of the order 10^{-7} second or less in a well-designed resistor.

SIMPLE WINDINGS. A simple coil in which wire is wound upon a bobbin has, in general, a small self-capacitance effect, since the potential difference between neighbouring turns is only a small fraction of that applied to the coil. The inductance of such a coil is, however, considerable. To reduce it, a coil can be wound as shown in Fig. 38 (*a*), in which the first few turns wound in one direction are followed by an equal number of turns wound in the opposite direction of rotation, and so on until the end of the coil is reached. Curtis and Grover* have shown that this method is very suitable for high resistance† coils, if the direction of winding be reversed sufficiently frequently to keep the residual inductance small. Curtis and Grover's winding is made in the following way: a cylinder of biscuit porcelain (i.e. baked, but unglazed, so that it can be cut by a steel tool),

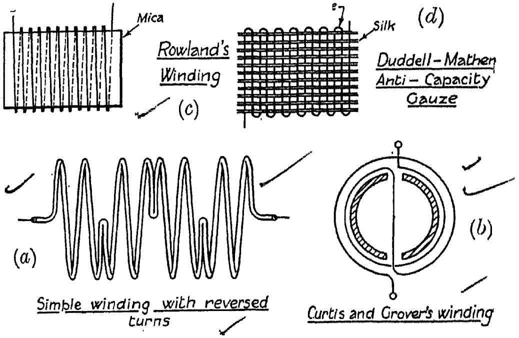
Mica
Rowland's Winding
(c)
Silk
(d)
Duddell-Mather Anti-Capacity Gauze
(a)
Simple winding with reversed turns
(b)
Curtis and Grover's winding

FIG. 38.—SIMPLE WINDINGS WITH LOW TIME-CONSTANTS

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* H. L. Curtis and F. W. Grover, "Resistance coils for alternating current work," *Bull. Bur. Stds.*, Vol. 8, pp. 495–517 (1913).

† It is convenient to refer here to other ways of constructing very high resistances with negligible residuals which have had some application in bridge work. Kundt's resistances consist of a tube of porcelain, upon which a very thin layer of platinum glaze is chemically deposited, the glaze being divided into two spiral strips joined at one end of the tube. Values of about 10^6 ohms can be secured, but the resistances have a large temperature coefficient. P. Wenk and M. Wien, "Eine neue Form von Hochfrequenzwiderständen," *Phys. Zeits.*, Vol. 35, pp. 145–147 (1934), describe similar resistors in which the metal film is applied by kathode sputtering. L. F. Curtis, "Lacquer-coated resistors," *Rev. Sci. Insts.*, Vol. 4, pp. 679–680 (1933), uses graphite lines on short Pyrex glass rods, the whole protected by glyptal lacquer; values up to 10^{12} ohm are described. H. Schering and R. Schmidt, "Die Messung des Phasenwinkels grosser Drahtwiderstände durch Vergleich mit Widerständen aus Mannit-Borsäure-Lösung," *Arch. f. Elekt.*, Vol. 1, pp. 421–432 (1913), have used a tube filled with an aqueous solution of mannitol ($C_6H_8(OH)_6$) and other salts. See also p. 365 Hague's *Instrument Transformers* (Pitman).

2·7 cm. diameter and 15 cm. long, is prepared and slit through diametrically for about two-thirds of its length. Unslotted portions are left at each end for strength, and the cylinder is then re-baked to harden it. Winding is carried out by putting on one turn, passing the wire across the slit and then winding a turn in the reverse direction, and so on, as shown diagrammatically in Fig. 38 (*b*). The winding is thus reversed at every turn and it is found that a 10,000 ohm coil can be made in this manner to have a time-constant of about 10^{-8} second. The coil is a little difficult to wind, but the good result repays the trouble involved.

Another way to reduce inductance of a simple coil is to diminish the cross-sectional area of the former upon which the wire is wound. This is, in effect, the method devised by Rowland and Penniman,* who have described a simple and effective winding, which has been much used for wattmeter voltage circuit resistors and to some extent for bridge work. Fine resistance wire is wound upon a thin sheet of mica or bakelite in the form of an inductive coil; since the cross-section of the sheet is not great, the inductance of the winding is small. The inductance can be further reduced by the artifice of winding a few turns successively in opposite directions. The time-constant of such resistance cards lies between 10^{-6} and 10^{-7} second. Care must be exercised in arranging the cards to form resistance boxes, since the sheets have considerable area and the capacitance between adjacent cards may have an influence on the time-constant (*see* Fig. 38 (*c*)).

The same result is attained by the anti-capacity gauze of Duddell and Mather.† A fabric or ribbon is woven in which the warp consists of silk or cotton threads and the weft is the resistance wire. A piece of the material 90 mm. wide and 1 metre long, woven to have a resistance of 3,300 ohms, has an effective inductance of 0·08 millihenry and a time-constant of $2{\cdot}4 \times 10^{-8}$ second. Such material is very suitable for the construction of ratio boxes since the inductance and resistance of the ribbon are each proportional to its length‡ (*see* Fig. 38 (*d*)).

These woven resistors have been improved by the Leeds and Northrup Co. In the usual weave, shown in Fig. 38 (*d*) and to a larger scale in Fig. 39 (*a*), successive wires pass on opposite sides of the warp and are not so close together as they would be if arranged on the same side, as in Fig. 39 (*b*). Using the same size of wire a resistor of 1,000 ohms had a time-constant of $0{\cdot}8 \times 10^{-8}$ sec. when woven as (*a*) and $0{\cdot}2 \times 10^{-8}$ sec. when woven as (*b*).

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* H. A. Rowland and T. D. Penniman, "Electrical Measurements," *Amer. J. Sc.*, 4th series, Vol. 8, pp. 35–57 (1899).

† W. Duddell and T. Mather, "Improvements in non-inductive resistances," *British Patent*, No. 5,171 (1901). *See* also W. E. Ayrton, "An improved method of covering wire for electrical purposes, and in the orderly arrangement of multiple electrical conductors," *British Patent*, No. 785 (1881).

‡ In connection with the importance of this fact, *see* p. 437.

Another important method introduced by Ayrton and Mather* was developed by A. Campbell and has been adopted by several manufacturers. As shown in Fig. 39 (*c*) a number of inductive sections, such as *AA*, are wound in a certain sense, and over them are put an equal number of similar sections, *BB*, wound in the opposite sense. The two windings are joined in parallel and carry equal currents in opposite directions. Since there is no p.d. between any point of an *A* section and the neighbouring point of a *B* section there is no appreciable self-capacitance effect; the residual is, therefore, purely inductive and very small. The method is particularly valuable for resistors of about 50 to 500 ohms. For bridge work the sections may conveniently be wound in notches cut in the edge of mica or bakelite cards. An example of a low resistor wound with strip on mica is shown in Fig. 39 (*d*). Leeds and Northrup Co. use the method in a woven resistor, see Fig. 39 (*e*); 100 ohm resistors wound as (*b*) and as (*e*) had time-constants of $0{\cdot}52 \times 10^{-8}$ and $0{\cdot}43 \times 10^{-8}$ sec. respectively.

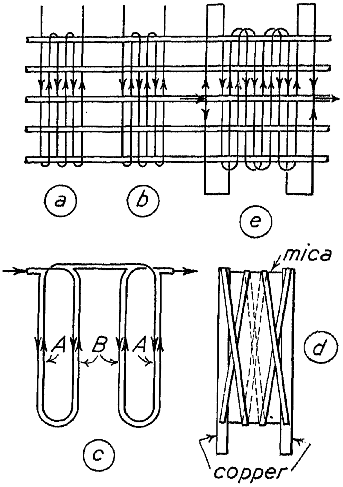

FIG. 39.—IMPROVED WOVEN RESISTORS. PARALLEL-OPPOSITION RESISTORS

BIFILAR WINDINGS. One of the oldest and simplest ways of reducing the inductance of a coil is the method of bifilar winding.

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* W. E. Ayrton and T. Mather, "The construction of non-inductive resistances," *Proc. Phys. Soc.*, Vol. II, pp. 269–275 (1892). For application to slider rheostats see "Induktion und Kapazitätsfreier Widerstand mit Kreuzwicklung," *Elekt. Zeits.*, Vol. 33, p. 721 (1912).

In this a length of wire is taken and folded back upon itself at its middle point. The bifilar conductor so formed is then wound upon a cylindrical bobbin to produce a spiral coil,* as shown in Fig. 40 (*a*). Then, since two wires carrying current in opposite directions lie side by side, the inductance of the coil will be very small, and can be still further reduced by twisting the bifilar leads together before winding the coil.

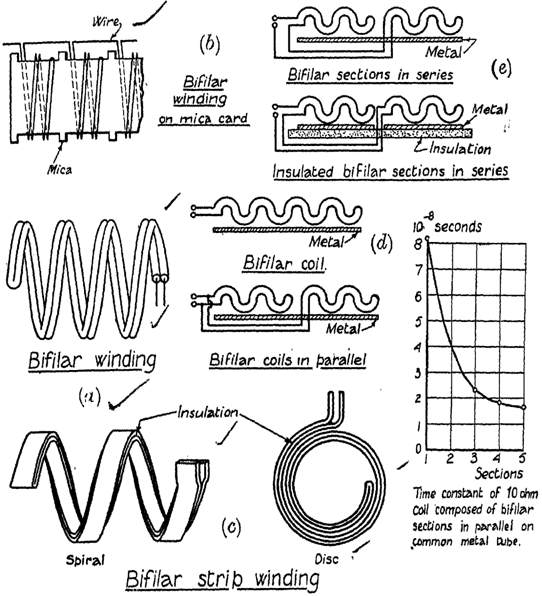
Wire
Mica
(b)
Bifilar winding on mica card
Metal
Bifilar sections in series
(e)
Metal
Insulation
Insulated bifilar sections in series
Bifilar winding
(a)
Metal
(d)
Bifilar coil
Metal
Bifilar coils in parallel
10^{-8} seconds
8
7
6
5
4
3
2
1
0
1
2
3
4
5
Sections
Time constant of 10 ohm coil composed of bifilar sections in parallel on common metal tube.
Insulation
Spiral
(c)
Disc
Bifilar strip winding

FIG. 40.—BIFILAR RESISTORS

A simple bifilar winding may, however, possess considerable capacitance, since the "go" and "return" wires lie close together and have the full potential difference between their terminals. In a resistor less than 100 ohms, in which there is a small quantity of wire, the capacitance effect will be small compared with that of the residual inductance. In a high resistance coil, above 100 ohms, the capacitance will preponderate over the inductance, since such a coil will contain a great length of fine wire. Indeed, so important does the capacitance effect become in high resistance bifilar coils that it was noticed at a very early date by Kohlrausch* in the course of his work on the resistance of electrolytes. In his later work he avoided the use of coils having a resistance greater than 2,000 ohms. More recently, Taylor and Williams† have endeavoured to reduce the capacitance by spacing out the turns of a 1,000 ohm coil wound with twisted bifilar strand.‡ The 100 ohm bifilar coils are usually nearly non-reactive, but may vary somewhat with the size of wire and the bulk of the coil.

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* The individual turns of this and other coils shown in Figs. 38, 39, 40 and 41 are shown opened out so that the manner of winding may be displayed.

A type of bifilar winding very frequently employed is shown in Fig. 40 (*b*). A length of wire is taken and a number of bifilar loops formed from it. The loops are then wound upon a sheet of mica, the parts of the wire forming each loop being frequently twisted together before winding. The proximity of currents flowing in opposite directions in the loops, and the small section of the mica sheet, greatly reduces the inductance. Division of the wire into a number of loops in series diminishes the capacitance.

It has been pointed out above that in a low resistance coil, where the amount of wire is small, the capacitance effect in a bifilar winding is negligible in comparison with the inductance effect. The bifilar winding is, therefore, nearly always used in the construction of resistances of 10 ohms or less. In order to reduce the residual inductance as far as possible, the coils are wound with manganin strip so that the current-carrying conductors lie as close as is feasible. A suitable length of the material is prepared and folded upon itself at the middle of its length with a layer of silk tape or mica between the halves. The composite conductor is then wound upon a bobbin to form a spiral or a disc coil, Fig. 40 (*c*). The capacitance introduced by putting the halves of the strip close together helps to compensate the inductance and to reduce the time-constant.

Low resistance coils may also be made by connecting in parallel a number of higher resistance, bifilar wound sections. Fig. 40 (*d*)* shows the result of the application of this principle to a 10 ohm resistance. In this diagram the effect of the number of sections connected in parallel upon the value of the time-constant is clearly shown. The greater the number of sections in parallel, the smaller will the total inductance become ; the self-capacitances of the sections will be added, so that for the whole coil the capacitance is increased. Hence, in the time-constant $T = \frac{L}{R} - CR$, L is reduced and C increased, so that T will diminish and may ultimately become negative. For example, the diagram shows that the time-constant of a 10 ohm coil wound in one bifilar section is reduced to one-fifth by making the coil of 5 sections of 50 ohms in parallel.

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* F. Kohlrausch, *Ann. der Phys.* Vol. 138, pp. 280–298, 370–390 (1869), and "Ueber das Leitungsvermögen einiger Electrolyte in äusserst verdünnter wässeriger Lösung," *Ann. der Phys.*, Vol. 26, pp. 161–226 (1885).

† A. H. Taylor and E. H. Williams, "Distributed capacity in resistance boxes," *Phys. Rev.*, Vol. 26, pp. 417–423 (1908).

‡ See also S. L. Brown, "Distributed capacity in resistance boxes," *Phys. Rev.*, Vol. 27, pp. 511–514 (1908).

Coils of higher resistance, say up to 1,000 ohms, can be made by connecting bifilar sections in series (*see* Fig. 40 (*e*)). The capacitance of the whole is thereby reduced, since the self capacitances of the sections are in series. It is frequently of importance to wind each section upon a separate metal tube, so that capacitance between the windings and the bobbin is made smaller than would be the case were the sections wound on a common tube. By subdividing the coil into a sufficient number of sections, Nukiyama and Shoji† have been able to design a 100,000 ohm resistance with a time-constant of only $2{\cdot}78 \times 10^{-6}$ second ; each section is designed as a transmission line with distributed inductance and capacitance, the dimensions being chosen to keep the residual down to the desired value.

CHAPERON WINDINGS. Perhaps the most important method of overcoming the defects of a bifilar coil is that introduced by Chaperon.‡ A coil is made by winding with single wire in an even number of layers. The first layer is wound to the end of the bobbin, the second layer being wound back to the starting point in the reversed direction, and so on. Thus, two wires carrying current in opposite directions lie over one another, thereby reducing the inductance while, at the same time, the capacitance effect is small, since the potential difference between neighbouring wires is slight. The winding is shown diagrammatically in Fig. 41.

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* From results given by K. W. Wagner and A. Wertheimer, "Über Präzisionswiderstände für hochfrequenten Wechselstrome," *Elekt. Zeits.*, Vol. 34, pp. 613–616, 649–652 (1913). For further information on bifilar coils, *see* W. Hüter, "Kapazitätsmessungen an Spulen," *Ann. der Phys.*, Vol. 39, pp. 1350–1380 (1912).

† H. Nukiyama and Y. Shoji, "On a design for a bifilar type of non-reactive resistance coil," *Tech. Rep. Tohoku Univ.*, Vol. 4, pp. 1–20 (1924).

‡ G. Chaperon, "Sur l'enroulement des bobines de résistance destinées aux mesures par les courants alternatifs," *Comptes Rendus*, Vol. 108, pp. 799–801 (1889) ; also "Mesure des résistances polarisables par les courants alternatifs et le telephone," *J. de Phys.*, Vol. 9, 2me. série, pp. 481–484 (1890). Also *see* J. Cauro, "Sur la capacité électrostatique des bobines, et son influence dans la mésure des coefficients d'induction par le pont de Wheatstone," *Comptes Rendus*, Vol. 120, pp. 308–311 (1895), for a slight improvement on the Chaperon winding ; also *see* Fig. 41.

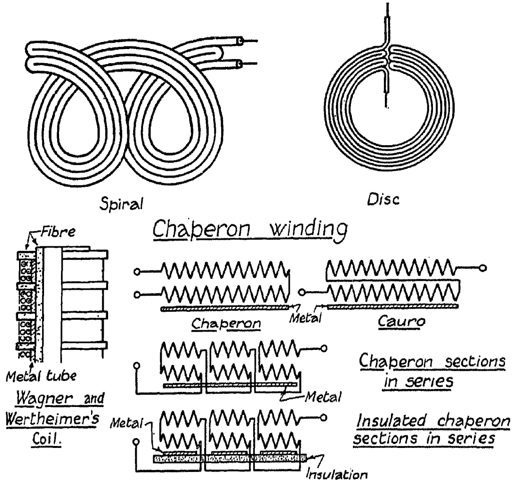
Spiral
Disc
Chaperon winding
Fibre
Metal tube
Wagner and Wertheimer's Coil.
Chaperon
Metal
Cauro
Metal
Chaperon sections in series
Metal
Insulated chaperon sections in series
Insulation

FIG. 41.—THE CHAPERON WINDING AND ITS MODIFICATIONS

Wagner and Wertheimer* have made an important investigation of Chaperon's winding, and have introduced certain improvements in the design of resistors. They have shown that considerable control can be exercised over the time-constant of a coil by arranging the winding in the form of a number of Chaperon sections connected in series. It is usual to wind these sections upon a metal tube, in order that the heat liberated from the coil when it is carrying current may be conducted away. These experimenters have shown that the additional capacitance effect between the windings and the common metal tube may have a considerable effect upon the time-constant, and have proposed a winding (Fig. 41) in which

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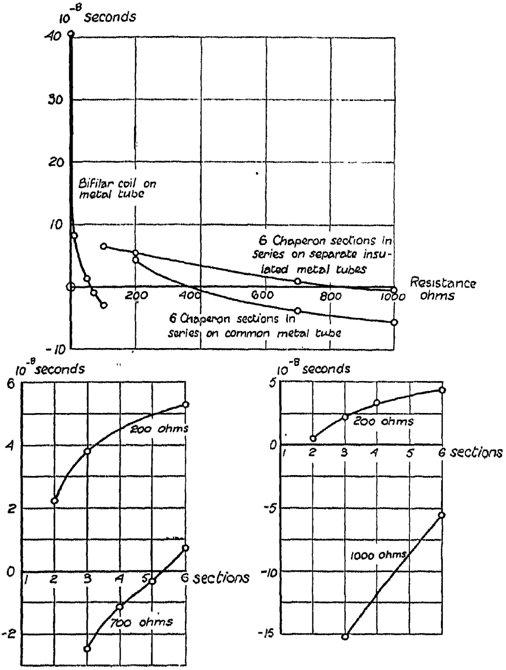
* See *loc. cit.*; also W. Hüter, "Der Phasenfaktor von Rheostatenwiderständen mittlerer Grösse," *Ann. der Phys.*, Vol. 40, pp. 381–386 (1913), for tests on Chaperon coils of 20 to 500 ohms.



Chaperon sections in series on separate insulated tubes

Chaperon sections in series on common metal tube.

FIG. 42.—COMPARISON OF THE TIME-CONSTANTS OF RESISTORS WOUND IN VARIOUS WAYS

the sections are wound upon separate insulated metal tubes. The effects of the subdivision of a coil into a number of Chaperon-wound sections, both on common and on separate tubes, is clearly shown in Fig. 42. The influence of the method of winding coils of various resistances is also illustrated in Fig. 42, Chaperon's, Wagner and Wertheimer's, and the bifilar method being compared.

5. **Methods of Constructing Resistors with Balanced Residuals.** In the methods just described the principle involved is the reduction of the residual inductance and capacitance so that both are very small quantities, the time-constant being thereby made very low. It is possible, however, to construct a coil of approximately zero time-constant by arranging the winding in such a way that the inductance effect and the capacitance effect balance one another. Thus, Mather and Sumpner* have described coils wound with doubled strip between which a layer of insulating material of high dielectric constant is placed. By adjusting the thickness of the insulating layer, or by opening out the strips at the terminals, the necessary compensation can be effected. In many of the methods described in the preceding section, e.g. resistance gauze, compensation may also be arranged.

The theory underlying compensated resistors has been given by G. A. Campbell† and others. A resistance coil can be represented as a resistance R in series with inductance L, the capacitance being equivalent to a condenser C shunting the whole. Referring to page 89, it will be seen that it is not possible to make a resistor which shall be invariable with frequency and at the same time have zero phase-angle unless L *and* C be simultaneously zero. Assuming that ϕ is to be made zero, then

$$L = CR^2$$

which makes $$R' \doteqdot R[1 + \omega^2 C^2 R^2].$$

Since C is to be made small, $R' \doteqdot R$, and the change of resistance with frequency will be negligible.

In the case of a bifilar coil, particularly when of high resistance, the inductance and capacitance must be treated as distributed along the wires. Let R, L, and C_1 be the total distributed resistance, inductance, and capacitance; then the effective resistance and inductance of a bifilar resistor are

$$R' \doteqdot R\left[1 + \omega^2 C_1\left(\frac{1}{3}L - \frac{2}{15}C_1R^2\right)\right], \text{ and } L' \doteqdot L - \frac{1}{3}C_1R^2,$$

to a high degree of approximation (*see* p. 47). From the second of these equations it is seen that the equivalent capacitance acting at the terminals of a bifilar resistor is $\frac{1}{3}$ of the total distributed capacitance, i.e. $C = \frac{1}{3}C_1$. The phase angle and time-constant will be zero if $L = \frac{1}{3}C_1R^2$ and the change of resistance with frequency will then be very small. See also p. 121 for influence of earth-capacitance.

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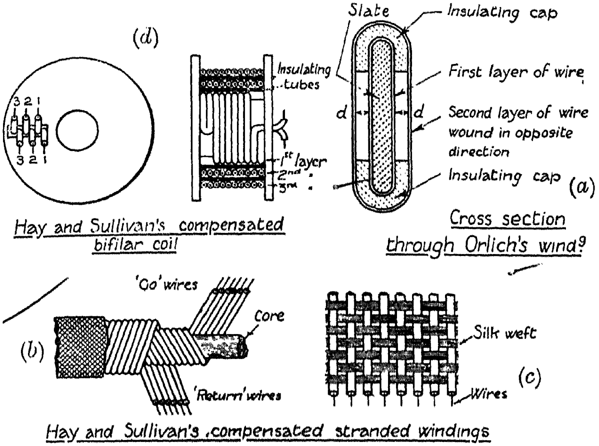
[*] T. Mather and W. E. Sumpner, "Improvements in constructing or winding coils for electrical apparatus," *British Patent*, No. 13,154 (1889).

[†] G. A. Campbell, "Resistance boxes for use in precise alternating current measurement," *Elec. World*, Vol. 44, pp. 728–729 (1904). *See* also Curtis and Grover, *loc. cit.*, pp. 499–500.

One of the first attempts to make use of the compensation principle is that of Brown.* His resistor consisted of a pair of German silver wires stretched out parallel to one another upon a board at a distance apart such that the inductance and capacitance effects are balanced.

Fig. 43 (*a*) shows a winding devised by Orlich† for large resistors up to 25,000 ohms. A piece of slate 5 cm. broad, 12 cm. long, and 3 to 4 mm. thick has its edges well rounded, and is then wound with half the required resistance wire in the form of an inductive spiral. Insulating caps are then put over the wound layer, and the remaining half of the wire is then wound over the caps with a direction opposite to that of the first layer. The caps are proportioned so that the two halves of the winding are separated by a distance *d* cm. By choosing *d* to have a suitable value, the capacitance between



Hay and Sullivan's compensated bifilar coil

Cross section through Orlich's windg

Hay and Sullivan's compensated stranded windings

FIG. 43.—METHODS OF CONSTRUCTING RESISTORS WITH BALANCED RESIDUALS

the two layers can be balanced against the inductance of the winding.

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* S. L. Brown, *loc. cit.*; also "The residual of inductance and capacity in resistance coils. A standard resistance with balanced inductance and capacity," *Phys. Rev.*, Vol. 29, pp. 369–391 (1909).

† E. Orlich, "Über eine Kompensation der Kapazität in grossen Widerstanden," *Verh. d. Deutsch. Phys. Ges.*, Vol. 12, pp. 949–954 (1910).

Hay and Sullivan* have described a variety of methods specially intended for the construction of resistors to be used at very high frequencies. A multiple conductor is used, consisting of a number of insulated strands connected in parallel. Two such conductors being taken, one is wound in a uniform spiral upon a core of insulating material. The second is then wound over the first in a spiral of the opposite hand, the distance between the two layers being proportioned in such a way that the desired compensation of inductance by capacitance is effected. The conductors are joined at one end, so that the core is covered by two concentric tubular conductors of which one forms the "go" and the other the "return" circuit (Fig. 43 (*b*)). An alternative to this construction is to fold a stranded conductor at the middle of its length, the distance apart of the two halves being regulated by means of an intervening insulating layer, so that when the composite conductor is wound into a disc coil (as in Fig. 40 (*c*)) compensation is attained. The multiple conductor in this case may very conveniently be made in the form of a ribbon in which silk or cotton weft is woven across a warp which is composed of the wire strands connected in parallel (Fig. 43 (*c*)).

The same inventors† have also designed a compensated bifilar winding, also shown in Fig. 43 (*d*). A multilayer coil is used, the distance apart of the layers being such that the capacitance between the layers is small compared with that between the wires of each layer. A bifilar layer is wound so that it has a positive time-constant, i.e. residual inductance preponderates. Over this a second bifilar coil is wound which, having a greater length of wire, will have a smaller positive time-constant, and so on with additional layers until one is arrived at which has $T = 0$. Subsequent layers will then have a negative time-constant. Such a number of layers is wound that, when they are all connected in series, the time-constant of the coil is zero.

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* C. E. Hay, "Alternate current measurements, with special reference to cables, loading coils, and the construction of non-reactive resistances," *Journal P.O.E.E.*, Vol. 5, pp. 451–454 (1913); also *Professional Papers*, No. 53, pp. 18–19. C. E. Hay and H. W. Sullivan, "Improvements in Electrical Resistances," *British Patent*, No. 22,375 (1912). Described and illustrated in this chapter by permission of H. W. Sullivan, Esq.

† C. E. Hay and H. W. Sullivan, "Improvements in Electrical Resistances," *British Patent*, No. 21,109, (1913).

It is interesting here to compare the methods used by Messrs. H. Tinsley and Co.* In low resistance coils the compensation is effected by using a very finely stranded conductor of silk-covered manganin wires wound evenly on a bobbin, one set of ends of the wires being connected so that the going and returning currents are closely intermingled. The self-inductance may then be cancelled by the mutual inductance between the strands. In coils above 10 ohms the effects of self-capacitance become large enough to be a useful means of compensating the self-inductance so that at 100 ohms a non-reactive coil may be made by this means alone.

For 100 and 1,000 ohm coils this firm adopts the expedient of using a silk-covered manganin conductor of suitable diameter and winding the bulk of this wire so that a slightly capacitive coil is produced. The remainder of the wire is then wound to add the requisite amount of inductance to effect compensation, which can be done after the coil is impregnated and stoved. The adjustment can sometimes be operated mechanically by moving the two parts of the coil relatively to one another by means of a screw, so that the residual may be made zero after the coil is sealed in its case. If R_c be the resistance of the larger part of the coil and C its self-capacitance, while r, l are the resistance and residual inductance of the added portion, it is easy to show if C be very small that $R \fallingdotseq R_c + r$ and $L \fallingdotseq l - CR_c^2$. Hence L may be made zero and R will be sensibly independent of frequency provided that C is small enough.

6. Secondary Effects in Resistance Coils. In the preparation of resistance standards there are a few matters of a secondary nature to which attention must be directed.

In order to reduce as far as possible any changes in the value of resistance coils with temperature, the material used for the windings is usually an alloy of very low resistance/temperature coefficient. For this purpose, manganin† is much used, this material being an alloy of copper (85 per cent), manganese (12 per cent), and nickel (3 per cent). The resistivity is about 42×10^{-6} ohms per centimetre cube at 0° C., the temperature coefficient being between zero and

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* D. C. Gall, "Non-inductive coils and resistance boxes," *Journal Sci. Insts.*, Vol. 5, pp. 222–225 (1928).

† Discovered by E. Weston in 1889, and investigated in detail at the Physikalische Technische Reichsanstalt. Other high resistivity alloys, such as eureka, constantan, nickel-chrome, etc., are also used.

0·0017 per cent per °C. Except in work of the highest precision, it is usually unnecessary to take account of temperature corrections. The manganin is drawn into the form of wire or rolled into flat strip or ribbon; it is then insulated with double silk covering.

Resistance coils are frequently wound upon brass tubes insulated with silk or cotton fabric "ironed on" with varnish. Heat may easily pass out of the coils when they are carrying current, and for the same reason it is preferable to wind them in a single layer. When wound, the coil is dipped in shellac to protect the wire from oxidation, and baked for a number of hours at a temperature of about 140° C. so that the coil is freed of moisture and the shellac solvent. The coil may then be adjusted and mounted for use. The heating also serves to anneal the manganin after the strain of winding, and a coil of greater permanence is produced.

Gall* has described in some detail the technique of winding and adjusting bifilar coils. With brass tubes $\frac{5}{8}$ in. diameter and 4 in. long about 7 watt can safely be dissipated with good air circulation. A sample of the wire to be used should always be tested, as the resistivity often varies considerably from the tabulated value; in computing the required length 2 to 3 per cent extra is allowed for adjustment. Suitable sizes and lengths are given in the following table—

Resistance, ohms	0·1	1	10	100	1,000	10,000
S.W.G. . .	18	20	24	33	40	44
Length, metres .	1·5	1·5	6	11	26	120

Bifilar conductor is most easily prepared by providing a drum with exactly 0·5 metre circumference. Then for 6 metres of wire 6 turns are taken upon the drum, the wire then folded on itself, and tied upon the brass tube. The bifilar conductor is then uniformly wound on the tube, drawing wire from the stock reel and from the drum until the latter is empty. After securely tying the wire to the tube, it is cut off and the resistance measured. Brushing or dipping gives a thin coating of shellac which is dried by baking at 140° C. for several hours; when cold the coil may be adjusted. The insulation at the ends should be bared with glass-paper to avoid damage to the wire, and small portions of wire are then successively removed until the resistance is within 0·2 per cent of the desired value, preferably slightly below it. The wire is then hard-soldered to copper end wires or lugs, using borax flux and 500° C. melting-point silver solder filings melted in a fine flame; the borax should be scraped off after cooling. The coil is then given several thin coats of shellac, dried off at 100° C. and put aside to age, after which it may be assembled in the instrument by soft-soldering the copper ends to the terminal rods. Final adjustment is accomplished *in situ* by scraping the ends; for this purpose fine wires should be first hard-soldered to thicker manganin end pieces. It is most important not to subject the wire to mechanical strain, by kinking or spiralling the free ends or by binding them with silk.

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* D. C. Gall, "Winding and adjusting resistance coils," *Journal Sci. Insts.*, Vol. 11, pp. 137–144 (1934).

It is found, however, that in the highest class of work such coils have small, erratic variations of resistance depending upon the humidity of the air. The moisture absorbed by the insulation causes a swelling of the latter, the resulting pressure exerted upon the wire producing the small variations of resistance. Rosa* has shown that the humidity trouble may be removed by dipping the coils into paraffin wax after the baking is complete, or by hermetically sealing the coils in brass cases containing oil. Such coils do not vary initially more than 1 part in 100,000 in fourteen months.

A possible cause of error in resistors is the so-called "skin-effect" in the wire. It is easy to show that at a frequency of 3,000 cycles per second a manganin wire must be as large as 2 mm. diameter before the skin-effect produces a change in the value of resistance so large as 1 in 10^5. Now bridge coils are not usually intended to carry much current, and are, therefore, wound with much finer wire or with strip. Hence, the skin-effect is negligible up to the higher telephonic frequencies.

Other secondary effects act upon the residual capacitance of the coil. The dielectric surrounding the wires is usually very poor and its dielectric constant will alter quite appreciably with temperature, changes of 1 per cent per °C. being observed. For such changes to have little effect on the effective value of resistance, the residual capacitance should be very small. Dissipation of energy in the poor dielectric will also be reduced by diminution of the residual capacitance. The absorption of moisture by the dielectric will change the capacitance and can be avoided by coating the coil with paraffin wax or by hermetically sealing it away from the action of the air.

7. Application of Methods in Practice. In the preceding sections the student has been shown a number of ways in which resistors may be constructed to be as nearly non-reactive as possible. Some of the methods are particularly suited to the production of high resistances, some to the production of low resistances, and others to the construction of coils of middle values. It will now be instructive to examine two typical sets of resistance coils, the first designed in America, and the second in Germany, in order to see the methods adopted and the degree of perfection attained. A comparison will also be made of coils commercially obtainable, produced under works conditions.

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* E. B. Rosa and H. D. Babcock, "The variation of resistances with atmospheric humidity," *Bull. Bur. Stds.*, Vol. 4, pp. 121–140 (1908). E. B. Rosa, "A new form of standard resistance," *Bull. Bur. Stds.*, Vol. 5, pp. 413–434 (1909). E. H. Rayner, "The effect of design on the stability of manganin resistances," *Journal Sci. Insts.*, Vol. 12, pp. 294–297 (1935) discusses still further the influence of humidity and of mechanical constraint.

The hygroscopic nature of shellac was noticed at an earlier date during the insulation of the coils for the standard current weigher at the N.P.L., *see Phil. Trans. Roy. Soc. A.*, Vol. 207, p. 497 (1908).

Resistance of Coil.	Material of Spool.	Shape of Resistance Material.	Size of Resistance Material.			Method of Winding.	Effective Inductance of Sample Coil.	Time Constant.
			Width.	Thickness	Diameter			
Ohms.			mm.	mm.	mm.		Micro-henrys.	Seconds.
1	Brass or Porcelain	Strip	3 to 5	0·1		Bifilar	+ 0·05	5×10^{-8}
10	,,	,,	1	0·1		,,	+ 0·2	2×10^{-8}
10	,,	Wire			0·24	Three 30 ohm bifilar sections in parallel on same spool	+ 0·3	3×10^{-8}
100	,,	,,			0·24	Bifilar	− 1·6	$-1{\cdot}6 \times 10^{-8}$
1,000	Porcelain	,,			0·10	Five 200 ohm bifilar sections in series on same spool	− 16	$-1{\cdot}6 \times 10^{-8}$
5,000	,,	,,			0·05	Inductive for 20 turns, then reversed for 20 turns, and so on	+ 210	$+4{\cdot}2 \times 10^{-8}$
5,000	,,	,,			0·05	Inductive, reversed every turn	+ 30	$+0{\cdot}6 \times 10^{-8}$
10,000	,,	,,			0·05	Inductive, reversed every turn	+ 100	$+1 \times 10^{-8}$

Curtis and Grover's Coils. By much experiment, Curtis and Grover* have devised a set of coils which have very small residuals. Single layer coils wound with silk-covered manganin upon spools 2·5 cm. diameter are used. The finished coils are varnished with shellac, baked at 120° C. for 10 hours, and then protected from the air by a coating of paraffin wax or by hermetical sealing.

Tests were carried out chiefly at frequencies between 1,200–1,500 cycles/second; results are given in the above table.

It will be seen that the time-constants of the coils are in all cases very small, in no case exceeding 5×10^{-8} second. At 3,000 cycles per second, this would correspond to a phase displacement between the voltage applied to such a coil and the current through it of only 3·5′. With coils below 100 ohms, efforts are directed to the reduction of residual inductance. A 100 ohm bifilar coil has a small time-constant and requires no further treatment. With coils of 1,000 ohms and higher values, attention is chiefly directed to the reduction of capacitance effects. It becomes important to wind coils of such values upon insulating spools.

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* H. L. Curtis and F. W. Grover, "Resistance coils for alternating current work," *Bull. Bur. Stds.*, Vol. 8, pp. 495–517 (1913). Similar methods have been independently developed by A. Campbell at the N.P.L.

Wagner and Wertheimer's Coils. These investigators have also made a very thorough examination of the methods by which non-reactive coils are constructed, measurements on a large number of coils made in a great variety of ways being recorded. It will be seen from the above table that Curtis and Grover, for coils below 5,000 ohms, favour the use of bifilar wound coils. Wagner and Wertheimer,* on the other hand, employ almost entirely the Chaperon winding in one or other of the forms illustrated in Fig. 41. Measurements were made at a frequency of 1,590 cycles per second for a variety of bridge currents. Photographs are given of typical coils of each class. A comparison of these figures with those in the preceding table will prove very instructive.

tance of Coil.	Material of Spool.	Arrangement of Winding.	Time Constant. Seconds.
0·1	Metal Tube	Bifilar wound manganin strip with mica between	$+9 \times 10^{-8}$
1		,, ,, ,, ,, ,, ,, ,,	$+4{\cdot}5 \times 10^{-8}$
10		4 Bifilar sections of 40 ohms each, in parallel	$+1{\cdot}25 \times 10^{-8}$
20		4 ,, ,, 80 ,, ,, ,,	$+0{\cdot}5 \times 10^{-8}$
30		2 ,, ,, 60 ,, ,, ,,	$+0{\cdot}7 \times 10^{-8}$
50	,, ,,	Bifilar coil	$+0{\cdot}63 \times 10^{-8}$
70	Fibre core with separate metal tube for each section	2 Bifilar sections of 35 ohms in series	$-0{\cdot}55 \times 10^{-}$
100	Metal Tube	,, 50 ,,	Very small
200		2 Chaperon 100	$+0{\cdot}33 \times 10^{-}$
300	,, ,,	3 ,, 100	Very small
500	Fibre core with separate metal tube for each section	166⅔	$+0{\cdot}5 \times 10^{-8}$
700		140	$-0{\cdot}29 \times 10^{-8}$
1,000	,, ,,	166⅔	$-0{\cdot}50 \times 10^{-8}$
1,000	Porcelain Tube	166⅔	$+0{\cdot}23 \times 10^{-8}$
1,000		200	$-0{\cdot}21 \times 10^{-8}$
3,000		500	$-1{\cdot}20 \times 10^{-8}$

Commercial Coils. Resistors manufactured under works conditions of mass production show a fairly wide range of values of time-constant, as would be expected, depending upon the method of winding adopted by each firm and the quality of the resistor. Several manufacturers use a bifilar winding for 0·1 ohm coils, Ayrton-Mather parallel winding for 1, 10, and 100 ohm coils, and simple inductive winding on mica cards for 1,000 and 10,000 ohm coils. Other firms use special methods, some of which have already been described.

An examination of figures published in the lists of eight leading British and American firms gives the following conclusions. For a 0·1 ohm unit the time-constant ranges from 13 to 100 $\times 10^{-8}$ second; for a 1 ohm unit from 5 to 40 $\times 10^{-8}$, with an average of about 12 $\times 10^{-8}$; for a 10 ohm unit from 2 to 8 × 10^{-8}, with an average about 2 × 10^{-8}; for a 100 ohm unit from 0·2 to 1 × 10^{-8} with an average about 0·4 × 10^{-8}; for a 1,000 ohm unit between $\pm 5 \times 10^{-8}$; and for a 10,000 ohm. unit between $\pm 18 \times 10^{-8}$. The precise value in any case is dependent upon the arrangement of the units in the resistance box and also on whether shielding is provided or not; values can be obtained from the makers in any particular instance.

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* K. W. Wagner and A. Wertheimer, "Über Prazisionswiderstände für hochfrequenten Wechselstrom," *Elekt. Zeits.*, Vol. 34, pp. 613–616, 649–652 (1913). K. W. Wagner, *Elekt. Zeits.*, Vol. 36, pp. 606–609, 621–624 (1915).

8. Resistance Boxes. When a suitable set of resistance coils has been prepared, it is necessary to mount them in some way so that they can be conveniently put into circuit in the bridge network. In work of the very highest class, the coils are usually mounted separately, each being sealed up in its own metal case. Massive terminals are provided and the coil is often immersed in oil, so that its temperature may be accurately determined and maintained constant. In ordinary laboratory practice, it is more convenient to group sets of coils within a common case, suitable terminals or switches being provided so that any desired combination of coils may be connected in series. Such a piece of apparatus is called a "resistance box."

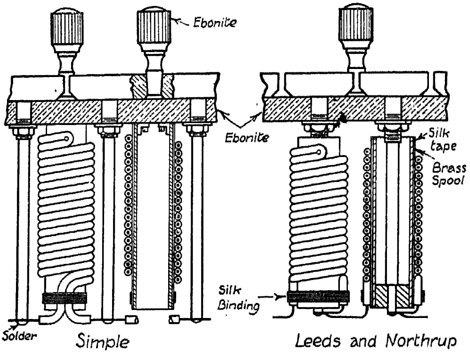


Fig. 44.—Methods of Mounting Resistance Coils

There are a few points connected with the arrangement of coils in resistance boxes which it is proposed now to mention quite briefly. These are (i) mounting of coils, (ii) methods of connecting the coils in circuit, (iii) effects of grouping of coils on residuals.

Mounting of Coils. Most coils for a.c. work are wound upon spools, usually of metal, and are mounted in the resistance box with the axis of the spool vertical. It is common practice to fix the spools directly to the ebonite lid of the box, and to carry down from the contact blocks on top of the lid suitable rods to which the coils may be connected (Fig. 44).

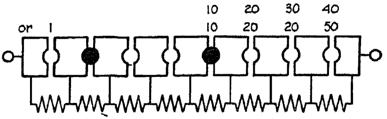
In the simplest arrangement, as shown in the diagram, one rod is connected to each block and forms the common lead for two neighbouring coils. In resistance boxes intended for precision working each coil is frequently provided with its own pair of rods, two rods being attached to each contact block.

The Leeds and Northrup Company have improved on this construction by attaching the metal spool of the coil to the contact blocks ; the radiating surface of these then serves for the dissipation of heat generated in the winding and increases the permissible number of watts which may safely be allowed in each coil.

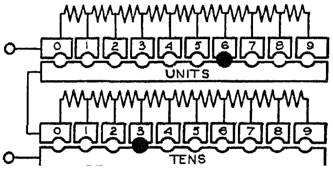
Methods of Connecting the Coils in Circuit. The range of resistance commonly required in boxes designed for bridge work is from 10,000 ohms down to 1 ohm. It is desired to obtain any value of resistance between these limits in steps of 1 ohm. (For special work, boxes are frequently used in which the values are one-tenth of these figures.) For this purpose a certain number of resistance units are mounted in the box and some arrangement of plug contacts or dial switches provided, so that the coils may be combined in series to any value within the desired limits and with the specified step.

The oldest way is to provide a number of coils connected as shown in Fig. 45 between contact blocks of brass mounted on the lid of the box. The insertion of a plug into the conical hole between two blocks short-circuits the coil connected across them and cuts it out of circuit. Any coil is put in circuit by the *withdrawal* of a plug, and it will be clear that the number of plugs required will be equal to the number of coils. The values of the coils may be 1, 2, 2, 5, with a similar grouping for the tens, hundreds, and thousands, two denominations only being shown in the diagram. Or the values may be 1, 2, 3, 4, etc. With either arrangement the desired range can be covered in steps of 1 ohm by removing the requisite plugs.

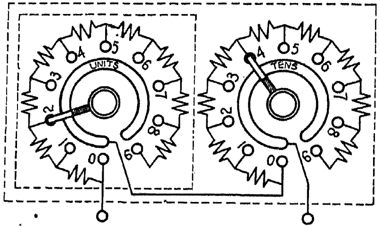
The disadvantages of this arrangement are numerous. Obviously, a large number of plugs, equal to the number of coils, will be required, and these must fit interchangeably in the holes, if errors due to plug contact resistances are to be made small. Again, the largest number of plugs is inserted, and the consequent contact resistance is greatest, when low resistances are required in circuit. Moreover, when a setting



Simple arrangement of coils



Straight decade arrangement of coils



Dial decade arrangement of coils

FIG. 45.—METHODS OF GROUPING COILS IN RESISTANCE BOXES

has been made, the value of resistance must be found by adding the numbers corresponding to the plugs withdrawn.

To overcome these objections, the "decade" principle shown in Fig. 45 has been introduced, and is almost exclusively used in resistance boxes for precise work. In this method there are ten 1-ohm coils for the units, ten 10-ohm coils for the tens, and so on, each group being called a "decade." The 10 coils in series in any decade can then be compared with the first coil of the next higher group, thus providing a check among the various coils. If this advantage be not required, nine coils per group are sufficient for successful operation of the principle. Only one plug per decade is required, so that contact errors are very much smaller than in the old form of resistance box. This plug is *inserted* to put a desired number of coils in series. The value of resistance in circuit is at once read off from the positions of the plugs in the decades, no summation being required.

The greatest advantage of the decade principle is that it readily allows of the use of rotary dial switches, these being much more convenient to manipulate in practice than a number of loose plugs. All high-grade resistance boxes are constructed on this plan, the success of which depends entirely on the certainty of contact and lowness of contact resistance of the dial switches. A typical arrangement is shown in Fig. 45.

The design of decade switches is chiefly directed to the production of a low and constant contact resistance, the essential conditions being constant pressure of the brush on the contact surface and cleanliness. With a suitable pressure the contact develops a high polish which is essential to good contact; if the pressure is too high, the brush may score and tear the contact surface. This polish should be produced by working the brush on the surface and without the use of any abrasive material. To preserve the good contact it is essential to exclude dust and grit; for this reason the switches in the best class of resistance boxes are always enclosed within the box. Also the brushes are arranged to be slightly oblique to the circular path of travel; this makes them self-cleaning, since particles of dirt will be pushed off the surface instead of becoming embedded in the brush and scoring peripheral grooves in the surface. The insulating panels upon which the contacts are mounted nearly always suffer some distortion with lapse of time; it is essential, therefore, for the switches to be either self-aligning or arranged in such a way that they are independent of this deformation. It is a great practical convenience for the contact positions of the brush to be located definitely by some form of ball or cam click. With all reasonable precautions, laminated phosphor-bronze brushes on brass contacts have a resistance of 2,000 to 3,000 microhms, and have a limit of variation of about 100 to 300 microhms.

Fig. 46 illustrates several types of decade switch that have proved satisfactory in practice. Fig. 46 (*a*) shows a double brush arrangement with surface-mounted contacts made by Leeds & Northrup. Fig. 46 (*b*) gives a dual-contact switch* made by H. Tinsley & Co. that fulfils all the conditions with great perfection. Two laminated phosphor-bronze brushes are clamped together by a hollow bolt and make contact by pressure due to the elasticity of the brush assembly alone. The brush is guided between two flat vertical cheeks which exert the required tangential force and is prevented from moving radially by a pin passing through the hollow bolt. There is sufficient clearance between the

* *British Patent*, No. 329,445 (1927).

cheeks, the pin, and the brush to ensure that the latter may be self-aligning. In the example illustrated, the contacts are on a sub-panel below the cover of the resistance box, but the arrangement is easily adapted to surface mounting or for other circumstances.

Many manufacturers make up resistance boxes from units consisting of a decade switch complete with resistors, a principle devised by Paul* many years ago. Fig. 46 (*c*) shows an arrangement adopted, with modification of detail, by the Cambridge Instrument Co. Fig. 46 (*d*) is an individual decade unit by Muirhead & Co., shown complete with its shield on the left and the interior displayed on the right; several such units are easily arranged in a common case to form a decade box. Somewhat similar units are made by the General Radio Co. In all these decades the insulation upon which the contacts are mounted is small in amount and firmly reinforced by metal rings, thus avoiding distortion with its attendant deterioration of the contact resistance.

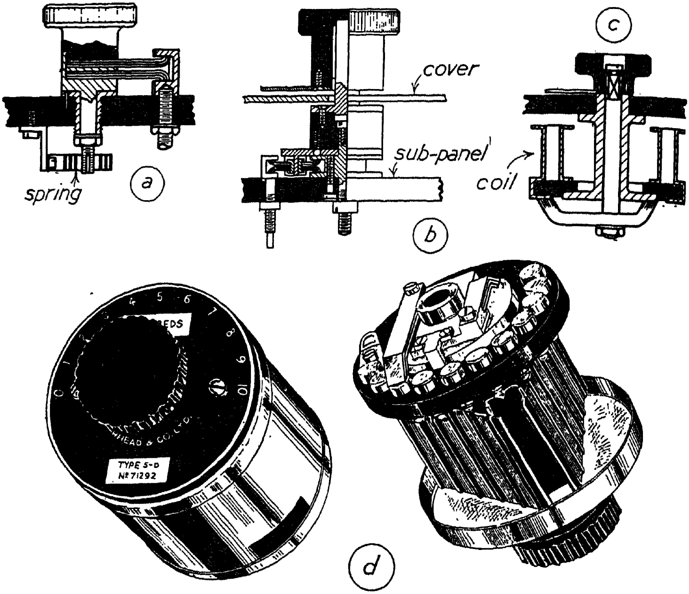


FIG. 46.—DECADE SWITCHES

For permanence and good appearance, resistance boxes are usually made of hard wood, such as teak, mahogany, or walnut. In unshielded boxes the cover is usually an ebonite, keramot, or bakelite panel; if

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* *British Patent*, No. 4,388 (1903).

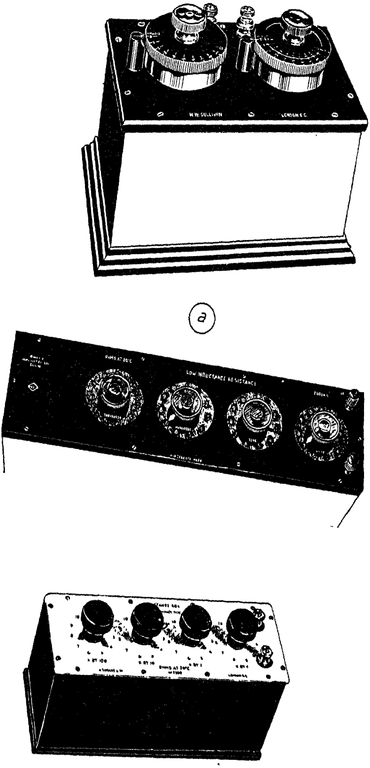

FIG. 47.—TYPICAL RESISTANCE BOXES

surface contacts are used, these are fixed to the cover, but, if the contacts are within the box, they are attached to an internal sub-panel and the cover merely serves as a means of excluding dust. In shielded boxes the shield consists of a copper or aluminium lining; the contacts are always on an insulating sub-panel and the cover is of metal, in intimate contact with the lining. The shield is always provided with a terminal by means of which its potential can be located with respect to the terminals of the resistor. Fig. 47 illustrates typical boxes of various types. A three-decade box by Sullivan, Fig. 47 (*a*), is unique in that the two lower decade switches are coaxial. Fig. 47 (*b*) shows a four-decade box by the Cambridge Instrument Co. An example by Tinsley of a shielded box with metal cover is shown at (*c*).

Referring to Fig. 45, it will be observed that the decade principle requires more coils than the older simple arrangement; it is possible, however, to design decades with as few as four coils of suitably chosen values.* An example using 6 coils per decade has been introduced by Leeds & Northrup Co.† for a.c. purposes and is of considerable interest; see Fig. 48 (*a*). Each of the six coils has a value equal to twice the decade step, e.g. in the units decade the coils are 2 ohms each. Five of the coils are mounted upon a rotor and are connected to contact segments thereon, as shown; the sixth coil and the brushes are stationary. It will be seen that for even settings of the decade both brushes bear on the same segment, short circuiting the stationary coil; for odd settings the brushes touch neighbouring segments, putting the fixed coil in parallel with one of the moving set. Thus even values are given by an appropriate number of the rotor coils in series, while odd values add to these a rotor coil and the fixed coil in parallel. Measurements on a 1,000 ohm decade at 50,000 cycles per second gives for 1 × 1,000 ohms a time-constant of -1×10^{-8} second and for 10 × 1,000 ohms one of -7×10^{-8} second.

Fig. 48 (*b*) shows the principle of another decade arrangement which is much used in resistors intended for use in substitution methods, particularly at very high frequencies. As the resistance coils are cut out, an inductive copper coil is inserted sufficient to keep the total inductance constant.‡ Since $L' = L - CR^2$ it follows that this type of compensation can only be usefully employed so long as L' increases approximately in proportion to the value of R; i.e. C must be small.

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* See *Dictionary of Applied Physics*, Vol. 2, p. 693.

† L. Behr and R. E. Tarpley, "Design of resistors for precise high-frequency measurements," *Proc. Inst. Rad. Eng.*, Vol. 20, pp. 1101–1113 (1932).

‡ R. F. Field, "Constant inductance resistors," *G. R. Exp.*, Vol. 8, No. 10, pp. 6–9 (1934).

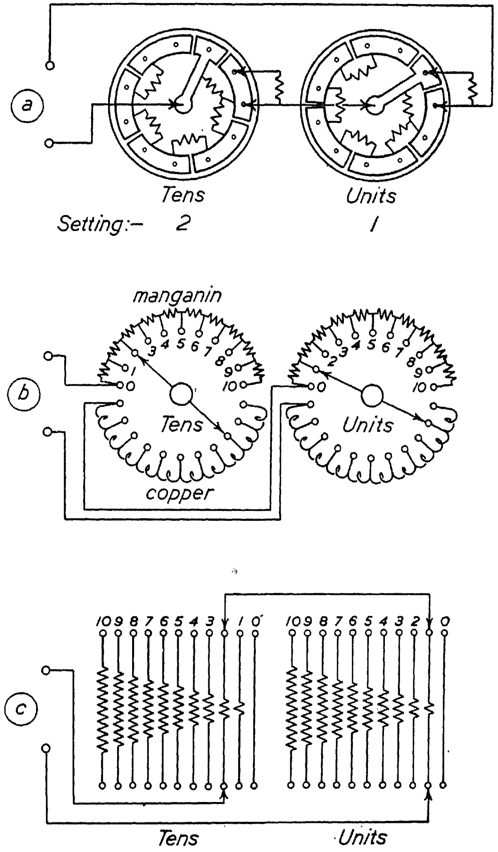

FIG. 48.—OTHER DECADE GROUPINGS

It is applied, therefore, only to low resistance decades. The General Radio Co. make constant inductance boxes with decades of 10 × 10, 10 × 1 and 10 × 0·1 ohms. Messrs. Tinsley & Co. have a box suitable for use at frequencies up to 1,500 kilocycles per second. The arrangement is fully shielded, the terminals being replaced by a shielded concentric socket and plug. Four decades, from 10 × 100 down to 10 × 0·1 ohms, are provided, fitted with special constant inductance dual-contact switches; the mean change of inductance at any setting does not exceed $0{\cdot}01\mu$H. In all these constant inductance boxes the initial resistance at the zero setting is larger than the contact resistance of the switches by the amount of the copper coils. This and the constant inductance are easily accounted for by a preliminary balance of the bridge, after which the box provides pure resistance increments.

In most bridge methods two of the branches of the network are very frequently non-reactive resistances which are required to stand in a definite ratio to one another, usually a simple integral multiplier. These resistances may be advantageously combined in a special type of resistance box, called a "ratio box," of which a variety of forms are in use; plugs may be used, as shown in Fig. 49, but many makers prefer rotary switches.

Fig. 49 shows a simple type having two sets of coils of 10, 100, and 1,000 ohms, so that, by withdrawal of the appropriate plugs, all decimal multiples and submultiples of 10 lying between 100 and 1/100 may be obtained.

When inductances are to be measured by the Campbell method, ratios of 9, 99, etc., are required (*see* p. 435). For this purpose a ratio box is constructed, in which the two coil-groups are of 10, 90, 900 each, thus giving the decimal ratios of 10/10, 100/100, 1,000/1,000; 1,000/100, 1,000/10, 100/10 and the reciprocals of these; and, in addition, the required values of 90/10, 900/100, 990/10. Boxes on this principle are made by the Cambridge Instrument Co. and by Messrs. H. Tinsley & Co.; they were originally made by Paul.

The ratio boxes so far described suffer from the defect that there are as many plugs as there are coils, with their consequent contact errors. It is, however, easy to group the coils so that only one plug in each ratio branch is necessary. A simple box on this principle is drawn in Fig. 49, arranged to give decimal ratios; this advantage is easily obtained with dial switches.

An excellent arrangement for a ratio box is that designed by Schöne,* Fig. 49. Here the bars *C* and *D* form two of the branch points—say the pair to which the detector is attached. The coils are connected between the contact blocks and the common bar *B*. By insertion of one plug between *C* and the blocks, and a second plug between *D* and the blocks, any desired pair of coils may be used to produce any required decimal ratio. Moreover, when using a ratio of unity, reversal of the ratios in the bridge is easily carried out by transference of the inserted plugs to the other end of the contact blocks

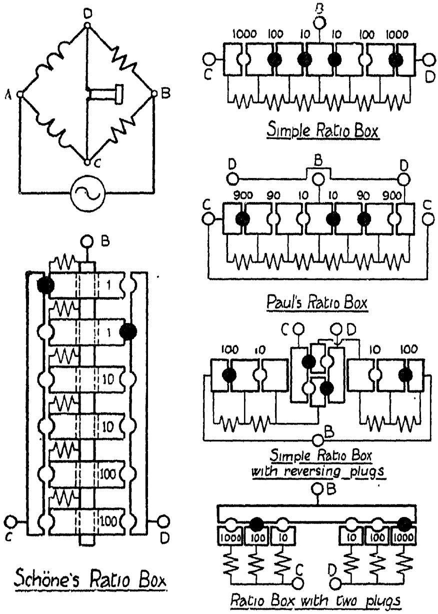

FIG. 49.—RATIO BOXES

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* O. Schöne, "Ueber eine Stopselanordnung für Brückenzweigwiderstände der Firma Siemens and Halske A.G.," *Zeits. f. Inst.*, Vol. 18, pp. 133–135 (1898). The device is used in the high precision ratios of the Leeds & Northrup Company.

which are in use, so that equality of the ratio coils may be readily checked (*see* p. 524).

Effects of Grouping of Coils on Residuals. The coils which are to be put into a resistance box are wound in such a way that their time-constants are as small as possible. Hence care should be taken when grouping them together that "grouping residuals" are kept down to the minimum. Such additional residuals may be introduced by the connections inside the resistance box, by the contact blocks upon the cover, and by the proximity of the coils to one another and to neighbouring objects.

With low resistance coils, less than 100 ohms, the connections from the contact blocks to the coils should have a very low inductance. The normal methods of construction, shown in Fig. 44, in which a rod is fixed to the block or in which the coil spool itself is used as a connector, usually suffice. The small inductance introduced by the rods is practically a constant residual since, whatever value of resistance be plugged in circuit, there are never more than two rods in series.

With high resistance coils, it is necessary to keep all capacitance effects in the connecting leads down to a negligible value. With the normal methods of construction, the capacitance betwen the leads and the contact blocks to which the coil is connected act as a permanent capacitive shunt on the coil, whether it be in circuit alone or in combination with others. To reduce this effect the contact blocks should be small, they and the connecting rods being as far apart as is convenient.

Hartshorn* has pointed out that the "box residual" is often much greater than the actual residuals of the coils up to 10 ohms or even 100 ohms resistance. In a certain four-decade box the residual with all the decades at zero was 0·62 μH. Each of the ten 0·1 ohm resistances had a residual of 0·02 μH, these residuals being additive like the resistances. Similarly the ten 1 ohm coils had residuals of 0·1 μH each, and the ten 10 ohm coils residuals of 0·3 μH, in both cases being simply additive. The effective residual of each 100 ohm coil was slightly negative, but, due to the effects of inter- and earth capacitances, the residuals were not additive; the total residual of this decade became rapidly more and more negative as the top of the dial was reached. This effect will be discussed in detail later.

From these figures it will be seen that the residual due to the connections inside a resistance box may be much larger than that of the individual low resistance coils. Hence, if single low resistances, up to 10 or 100 ohms, be required, it is better to use separate coils rather than take the desired value from a resistance box.

Earth and intercapacitance of the various coils may thus have a pronounced effect upon the time-constant as measured at the resistance desired number of tubes can be joined in series by shielded copper connectors, as shown in Fig. 51; the path of the current is from the insulated end of the wire and back by the outer tube, like a concentric cable short-circuited at the distant end. The following table gives the calculated time-constants for the decade, the values being independent of frequency up to 100,000 cycles per second.

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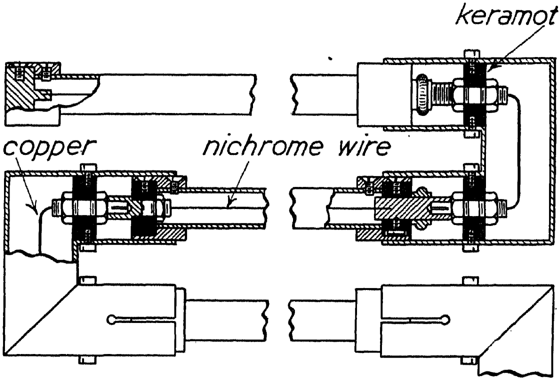
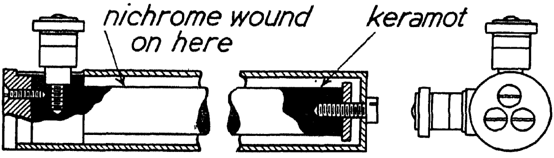
* L. Hartshorn, "Standards of phase angle," *World Power*, Vol. 8, pp. 171–180, 234–240 (1927).

Resistance in ohms .	100	200	300	400	500	600	700	800	900	1,000
Time-constant, seconds; $10^{-8} \times$	0·4	0·3	0·0	–0·4	–0·9	–1·5	–2·2	–3·0	–4·0	–5·1



Assembly of 100 ohm units.



Assembly of 1000 ohm unit.

FIG. 51.—ASTBURY'S RESISTORS WITH CALCULABLE RESIDUALS

The decade 1,000 to 10,000 ohms consists of a series of single-layer solenoids wound with double-silk insulated nichrome wire 2·8 mils diameter, upon rods of keramot 1 cm. diameter with a shoulder carrying a terminal at one end and a brass plate on each end face. The winding is attached to the terminal, and to the more remote brass plate; the whole is assembled in the brass shield tube, which serves as the return conductor, as shown in Fig. 51. Ten units, having resistances of 1, 2, 3 . . . 10 × 1,000 ohms, are provided; the 1,000 ohm standard is about 1·9 cm. long. The calculated time-constants are given in the table below for two frequencies; the effect of eddy currents in the shield and of stray capacitances limits the applicability of these standards to about 3,000 cycles per second.

Resistance in ohms .	1,000	2,000	3,000	4,000	5,000	
Time-constant, seconds; 10^{-8} ×	7·5 7·4	7·5 7·4	7·0 6·9	5·6 5·5	4·4 4·2	at $f = 0$ at $f = 1{,}000$
Resistance in ohms .	6,000	7,000	8,000	9,000	10,000	
Time-constant, seconds; 10^{-8} ×	3·2 3·0	1·6 1·4	− 0·4 − 0·6	− 2·2 − 2·4	− 4·4 − 4·6	at $f = 0$ at $f = 1{,}000$

VARIABLE LOW RESISTORS. In making a bridge measurement, adjustments of the balancing resistances to the nearest ohm can be made in dial or plug resistance boxes. A fine adjustment rheostat, which can be varied continuously between zero and 1 ohm, is essential for accurate settings with a sensitive bridge method. A simple arrangement is a pair of parallel wires upon which a conducting slider can be moved. The small inductance of such a rheostat can be calculated for any position of the slider and, if necessary, allowed for.

A more convenient arrangement of a low inductance 1 ohm resistor is illustrated in Fig. 50. In this a loop of No. 22 S.W.G. Eureka wire is fixed upon the edge of an ebonite disc 16·7 cm. diameter. The sides of the loop are 0·51 cm. apart. The disc is capable of rotation about a central axis, contact being made on the wires by means of two spring brushes. The maximum resistance is about 1·2 ohms, the inductance then being about 0·6 microhenry. Such a resistor, used in conjunction with a dial box reading to 1 ohm, proves very satisfactory in obtaining accurate settings. A scale marked on the disc, read against a pointer carried by one of the terminals, gives the value of the resistance in any position (these details are omitted from the drawing).

In many bridge methods it is very convenient to use a variable low resistor in which the inductance remains constant whatever value the resistance may have. A simple and excellent device which very nearly fulfils this condition is the mercury tube rheostat of Dellinger and Wenner.* In this instrument advantage is taken of the fact that the specific resistance of mercury is about 60 times that of copper. Referring to Fig. 50, a U tube is constructed of ebonite, porcelain, or thin glass, the cross-sectional area of one limb being 10 times that of the other. The limbs are filled with mercury, and at their open ends are fitted with terminal caps, or, if the limbs be of glass, cups are blown upon them in order to contain any overflow of mercury, the terminals dipping therein. Each limb is provided with an amalgamated copper rod which fits the bore of the tube as closely as possible. The resistance is varied by sliding the rods up or down inside the limbs of the U, the copper short-circuiting more or less of the mercury column because of its superior conductivity. The displaced mercury overflows into the terminal caps or the glass cups; spring clips retain the copper rods in any desired position. Settings of resistance can be made with very great precision, and with perfectly uniform variation, since there are no mechanical contacts across which the current must pass. The device can also be constructed in a single tube form with terminal caps at the upper and lower ends. A U tube with limbs 12 cm. long, one being 3 mm. and the other 1 mm. diameter, has an adjustment range for the large limb up to 0·01 ohm, and for the smaller up to about 0·1 ohm. The resistor has the disadvantage of a large temperature coefficient.

Another much-used device is the Campbell constant inductance rheostat made by the Cambridge Instrument Co., illustrated in Fig. 50. In this, a resistance wire of manganin and a parallel copper wire are wound in grooves on the edge of a disc of slate, together with a compensating winding of copper placed between them. A contact carried on a moving arm touches the resistance wire and the copper one in such a way that the resistance is varied by rotation of the arm ; a pointer attached thereto indicates the value of the resistance on a fixed scale. It will be seen from the diagram that no matter to what value of resistance the rheostat may be set, the inductance remains constant and is very small. A range of resistance from 0·10 to 1·1 ohms is obtainable and can be read to 0·01 ohm.

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* J. H. Dellinger, "Note on a variable low resistance," *Phys. Rev.*, Vol. 33, pp. 215–216 (1911).

F. Wenner, "The four-terminal conductor and the Thomson bridge," *Bull. Bur. Stds.*, Vol. 8, pp. 584–585 (1913).

F. W. Grover and H. L. Curtis, *loc. cit.*, pp. 465–466 (1913).

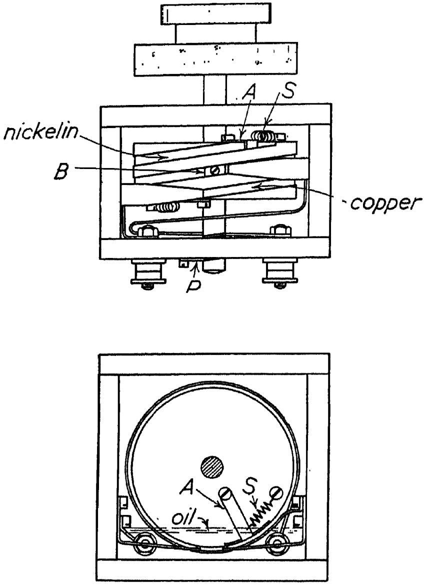

FIG. 52.—WILMOTTE'S CONSTANT-INDUCTANCE RHEOSTAT

Another pattern has a range one-tenth of these values. Similar rheostats are manufactured by several other makers; in some cases a slow motion drive for the brush arm is provided, which is a great convenience in precise settings.

Another type of constant inductance rheostat has been designed by Wilmotte* and is made by Messrs. H. Tinsley & Co.; it has a range of 1·1 ohm, an inductance of 0·2 μH. and may be satisfactorily used up to a frequency of 10^6 cycles per second. As Fig. 52 shows, a drum of bakelite has a shallow helical groove of 1 cm. pitch cut in its periphery, rather over two turns being provided. Resistance variation is obtained by arranging two narrow strips 0·05 mm. thick, one of nickelin and one of copper. One end of each is attached to the block *B* at the middle of the groove, the copper strip occupying the lower turn and the nickelin strip the upper turn. The two remaining ends are attached to pivoted arms, *A*, the wires being kept taut in the grooves by tension springs *S*. Fixed phosphor-bronze brushes with silver contacts touch the strips, one on the copper section near the middle block, and the other on the nickelin near the end; the contacts must be clear of the sides of the groove. A screw-thread of 1 cm. pitch on the spindle engages with a pin *P* and imparts axial travel to the drum as it is rotated, thus varying the relative amounts of copper and nickelin in use without changing the geometric shape of the circuit. The inductance of the drum is approximately cancelled by a reverse turn of copper wire from one of the terminals to the central brush. The resistance is read on a spiral scale marked on an index drum; the zero resistance is of the order of 0·1 ohm. The whole is enclosed in an oil-tight bakelite box and the contacts operate under pure medicinal paraffin oil.

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* R. M. Wilmotte, "A quick and sensitive method of measuring condenser losses at radio frequencies," *Journal Sci. Insts.*, Vol. 5, pp. 369–375 (1928).

In cases where it is desired to keep the inductance of a circuit strictly constant and to vary its resistance by a definite step, say 0·1 ohm, the following artifice* may be adopted. Let links of two metals having different resistivities, e.g. copper and manganin, be prepared, the links having the same section, length, and shape. The dimensions of the links must be chosen so that the latter differ in resistance by the desired amount. Suitable mercury cups are provided into which one or other of the links may be placed in a perfectly definite position. Substitution of the copper for the manganin link, or *vice versa*, will then produce a pre-determined change of resistance without any change of inductance.

STANDARDS OF SELF AND MUTUAL INDUCTANCE

The standards of self and mutual inductance used in bridge measurements are of two kinds. The first are those which have a single definite value of inductance, and are known as fixed standards. In the second class the inductance may be varied continuously between certain limiting values, without altering the resistance of the inductive coils. Such are known variously as variable standards, inductors, inductometers, etc., and are much used in a.c. bridges.*

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* Curtis and Grover, *loc. cit.*, pp. 465 (1913). Also *see* C. E. Hay, *P.O. Reprint*, No. 53, pp. 21–22 (1912).

10. Fixed Standards. Fixed value standards of self and mutual inductance are of two classes, absolute and secondary. In absolute standards the coils are arranged in such a way that the inductance may be calculated from their accurately measured dimensions. Secondary standards, on the other hand, are wound in any convenient manner, and are then calibrated by comparison with absolute standards in a bridge or by other means. They serve as general laboratory standards of reference, whereas the absolute standards are only used in measurements of the highest accuracy, such as would be undertaken at the National Physical Laboratory and institutions of a similar nature.

SELF-INDUCTANCE. A reference standard of self-inductance, whether absolute or secondary, should be constructed so that it fulfils the following important conditions—

(i) It should be permanent in form and constant in value.

(ii) The resistance should be low in comparison with the inductance.

(iii) The winding should be carried out in such a way that the inductance is the largest possible for a given amount of wire, the coil being of a convenient size.

(iv) The value of the inductance should be independent of the current strength.

(v) The effective inductance and resistance of the standard should be little affected by frequency.

(vi) It is often desirable that the inductor should be astatic with respect to external fields.

In the following paragraphs it will be shown how these various conditions are complied with in the design of absolute and secondary inductances.

In absolute standards of self-inductance, everything is done to construct a coil which is unalterable in value, and of which the dimensions can be measured to the highest attainable degree of accuracy. A brief description of an actual coil will serve to show how this is carried out in practice. The Bureau of Standards* at Washington possesses a standard of self-inductance, having a value of about 0·2 henry, which has been constructed in the following way. A hollow cylinder of white marble, 54 cm. external diameter and 11 cm. in radial thickness, is accurately ground so that it is as perfectly cylindrical and circular in section as modern machine tools can make it. The diameter of the cylinder is accurately measured at a large number of places, so that its true average diameter is known, and also its variations from perfect roundness and cylindrical shape. Upon the surface of the cylinder a single-layer coil is uniformly wound†, the total length of the coil being about 46 cm., there being about 15 turns to the centimetre. The coil is wound in sections so that the inductances of portions of the winding may be inter-compared, thereby providing a check on the calculations. From the measured dimensions of the cylinder, and the number of turns in the winding, it is possible to calculate the inductance of the standard to a very high degree of accuracy.

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* For a good summary *see* G. Zickner, "Absolute Induktivitäts-Normale," *Arch. f. tech. Mess.*, Z121–1 (Feb., 1935); "Induktivitäts-Normale (regelbare Modelle)," *ibid.*, Z123–1 (March, 1935); "Induktivitäts-Normale (unveränderbare Modelle)," *ibid.*, Z121–2 (May, 1935). Also *see* A. Campbell and E. C. Childs, *The Measurement of Inductance, Capacitance and Frequency* (Macmillan, 1935).

A coil constructed in the manner just described is built on sound mechanical lines, and will, therefore, be permanent in form and unalterable in value. No attempt is made to distribute the wire of the winding in the most economical way, attention being directed more to the precise localization of every turn. By the use of marble, the inductance is made independent of the strength of current in the wire, since marble is practically non-magnetic. In order that the standard may be reasonably free from the effects of frequency, three things are done, (i) the skin-effect in the winding is reduced by the use of small wire; (ii) self capacitance of the winding is kept small by using a single-layer coil with the turns reasonably widely spaced; (iii) the insulation resistance of the winding is maintained at as high a value as possible.‡ These effects are discussed in detail below.

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* J. G. Coffin, "Construction and calculation of absolute standards of inductance," *Bull. Bur. Std.*, Vol. 2, pp. 87–143 (1906). Somewhat similar coils are found at the National Physical Laboratory and at the Physikalische-Technische Reichsanstalt. For a description of the 0·01 and 0·05H. standards at the latter institution, *see* E. Grüneisen and E. Giebe, "Eine neue Bestimmung der absoluten elektrischen Widerstandseinheit," *Ann. der Phys.*, Vol. 63, pp. 179–200 (1920).

† In most absolute standards the position of every turn is made quite definite by winding accurately drawn, bare copper wire in an accurate screw thread cut in the surface of the marble cylinder.

‡ For a method of winding absolute inductances so that the insulation resistance is high and can be readily tested, *see* W. E. Ayrton, T. Mather, and F. E. Smith, *Phil. Trans. Roy. Soc., A.*, Vol. 207, pp. 469–470 (1908).

In general laboratory practice calibrated secondary standards are quite sufficient. Wien*, in 1896, introduced the type of coil which, with slight constructional modifications, is in use at the present day. Maxwell† has proved that if wire be wound in a channel of square section cut in the rim of a circular

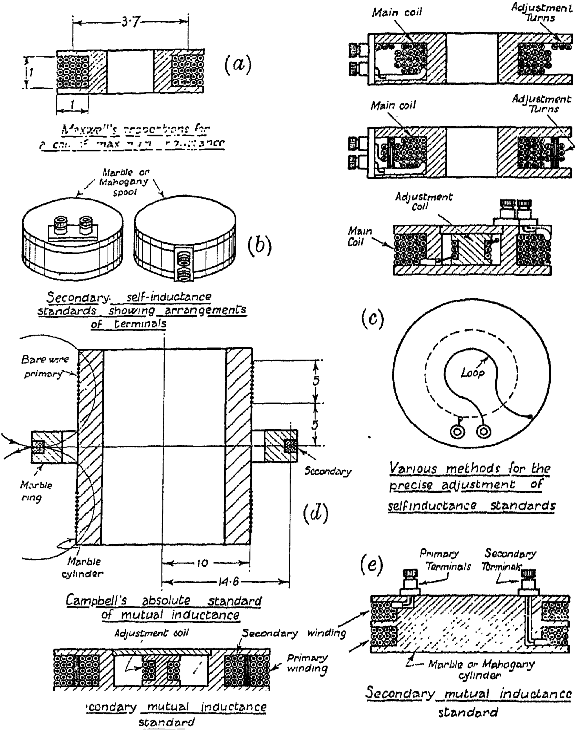
3·7
1
1
(a)
Maxwell's proportions for a coil of maximum inductance
Marble or Mahogany spool
(b)
Secondary self-inductance standards showing arrangements of terminals
Main coil
Adjustment Turns
Main coil
Adjustment Turns
Adjustment Coil
Main Coil
(c)
Loop
Various methods for the precise adjustment of selfinductance standards
Bare wire primary
5
5
Marble ring
Secondary
(d)
Marble cylinder
10
14·6
Campbell's absolute standard of mutual inductance
Adjustment coil
Secondary winding
Primary winding
condary mutual inductance standard
(e)
Primary Terminals
Secondary Terminals
Marble or Mahogany cylinder
Secondary mutual inductance standard

FIG. 53.—FIXED VALUE STANDARDS OF SELF AND MUTUAL INDUCTANCE

bobbin, the inductance of the resulting coil will be a maximum for a given length and thickness of wire when the mean diameter of the coil is 3·7 times the side of the square section of the channel (Fig. 53 (*a*)). Wien's coils were wound to fulfil this condition, the wire being thus distributed in the most advantageous way, i.e. so that a coil is produced which has the largest time constant.

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* Max Wien, "Einheitsrollen der Selbstinduction," *Ann. der Phys.* Vol. 58, pp. 553–563 (1896). Coils of 0·001, 0·01, and 0·1 henry are described.

† *Treatise*, Vol. 2, Sec. 706, pp. 345–346; *see* also J. E. Ives, "On the dimensions of large inductance coils," *Phys. Rev.*, Vol. 16, pp. 112–114 (1903).

The design of fixed value self-inductance standards for laboratory purposes is a matter of considerable importance and has been fully investigated by H. B. Brooks* at the Bureau of Standards. The principal factor determining the design of such coils is the time-constant, which should be as large as possible; in other words, a given length and thickness of wire should be wound to form a coil with the largest possible ratio of inductance to resistance. For ease of winding the circular form of coil is to be preferred and the cross-section of the winding space should be square; there is an infinite number of possible forms of coil that may be made with a given length of wire. At one extreme is a winding with one very large circular turn, resulting in a low value for the inductance; at the other extreme is a coil in which the mean radius is equal to half the side of the square cross-section, i.e. there is no hole through the centre of the coil, the inductance being less than can be obtained by an intermediate form. Between these extremes is a coil of such proportions that the maximum inductance for the given length of wire is obtained. Maxwell derived the optimum form from the approximate inductance formula

$$L = 4\pi a n^2 \left(\log \frac{8a}{R} - 2\right) \text{cm.}$$

where a is the mean radius, n the number of turns and $R = 0{\cdot}44705b$ is the geometric mean distance of the section, where b is the side of the square, arriving at the result that the mean diameter $2a$ should be equal to 3·7 times the side b. Using these values in the above equation gives as a simple working expression for a coil of maximum inductance for a given length of wire,

$$L = 6\pi a n^2 \text{ cm.} = 18{\cdot}85 a n^2 \text{ cm.}$$

Using a more accurate expression for L and a graphical method of computation, Shawcross and Wells† show that the relation $2a = 3b$ gives a value of time-constant about 0·5 per cent greater than Maxwell's coil, a result verified theoretically by Hak* from Stefan's more accurate formula,

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* H. B. Brooks, "Design of standards of inductance, and the proposed use of model reactors in the design of air-core and iron-core reactors," *Bur. Stds. Journal of Res.*, Vol. 7, pp. 289–330 (1931).

† R. E. Shawcross and R. I. Wells, "On the form of coils to give maximum self-inductance for a given length and thickness of wire," *Elecn.*, Vol. 75, p. 64 (1915). They demonstrate that the square channel is the best shape and that it is better to use too large rather than too small a mean diameter, so that Maxwell's figure errs on the right side.

$$L = 4\pi a n^2 \left[\left(1 + \frac{\alpha^2}{6} \right) \log \frac{\sqrt{8}}{\alpha} - 0{\cdot}8483 + 0{\cdot}204\alpha^2 \right] \text{ cm.}$$

where $\alpha = b/2a$. Hak shows that the true relation is $2a = 2{\cdot}95b$, but the maximum is so flat that the more convenient value of 3 may be substituted, as the following table shows—

$2a/b$	2·90	2·94	2·95	2·96	3·00	3·10
L/L_{max}	0·999971	0·999994	1·000000	0·999994	0·999953	0·999594

With a in cm. and $2a = 3b$ a convenient formula for L is

$$L = 16{\cdot}83 a n^2 \text{ cm.},$$

replacing the above formula.

Assuming small currents, so that thermal limitations do not enter into consideration, the factors determining the design of a fixed value inductance are the size, weight, and cost of the coil and a sufficiently high time-constant. Small coils usually have a time-constant of about 1 millisecond, larger coils of 2 to 10 milliseconds; the greater the time-constant, the larger, heavier, and more costly the coils become. To facilitate design Brooks gives a number of charts from which preliminary values of the side of square section, the wire size, and weight of copper can be found for a given time-constant; corrections may be applied to take account of the insulation space-factor of standard insulated wires. The use of the charts is clearly illustrated by the solution of several numerical examples, and the process is worthy of study by those interested in the subject. Tables of suitable figures are given from which large scale copies of the charts can be constructed for use in practice.

The question of suitable materials for spools upon which to wind the coils is also a matter of importance. It is essential that the material should be a good insulator, permanent in form, non-magnetic, and have a coefficient of linear expansion about equal to that of copper, i.e. about 17 parts in 10^6 per degree C., so that the turns are not stressed by changes of temperature. It is worth noting that aluminium has a coefficient of expansion about equal to that of bakelite, namely, 23 parts in 10^6 per degree C.; aluminium coils on bakelite formers would offer the same constructional advantages.

The bobbins on which Wien's coils were wound were made of serpentine,* a material which has been much used for the purpose owing to the ease with which it can be procured and machined. Rosa and Grover† have shown that the inductance of a coil wound on a serpentine bobbin depends on the strength of the current in the windings, showing that serpentine is quite appreciably magnetic. For example, if a coil of 1 henry be wound on a wooden bobbin—which is certainly non-magnetic—the inductance can be altered by 2 or 3 millihenrys if a lump of serpentine be laid upon the coil. It is desirable, therefore, to use some material for the bobbins which is more nearly non-magnetic.

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* J. Hak, "Die eisenlose Drosselspule vom kleinsten Kupfergewicht," *E.u.M.*, Vol. 46, pp. 249–250 (1928); "Résolution graphique des formules pour le calcul de l'inductance des bobines sans noyeau," *Rev. Gén. de l'Él.*, Vol. 23, pp. 583–586 (1928). Also see E. B. Rosa and F. W. Grover, *Bull. Bur. Stds.*, Vol. 8, pp. 1–237 (1912) for inductance formulae.

The best material for the construction of inductance standards is white marble.‡ This material is practically free from magnetic properties, it possesses a high insulation resistance, and can be easily machined and tooled. Coils wound on it are very permanent in value owing to the rigidity and permanence of form of the marble bobbins. Marble fulfils all the conditions admirably, provided that the temperature does not exceed 60° C. If heated above 60° C. and then allowed to cool to room temperature marble spools show an undesirable property of irreversible, permanent growth; § they are suitable, therefore, only for coils in which the temperature rise is very small. Phenol condensation products, such as bakelite, etc., have expansibilities of about 24 parts in 10^6, but shrink considerably when carried through temperature cycles in excess of 60° C. Ebonite has an expansibility of about 70 parts in 10^6, and also shows this shrinkage effect. The value for loaded ebonite (keramot) is a little lower, and this material withstands wide range temperature changes much better, without the same tendency to distortion. By far the most desirable material seems to be porcelain. Forty samples of porcelain showed linear coefficients of expansion between 1·6 and 19·6 parts in 10^6 per degree C., including the value for copper within the range; the material is entirely free from the phenomenon of growth observed in marble.

For ordinary laboratory work, coils may be wound on spools of well-seasoned Cuba mahogany. If the wood is thoroughly dry, and is impregnated with wax to prevent absorption of moisture, coils of great constancy will result. The impregnation is best done before the bobbins are finally turned. This construction is very cheap and simple, the spool being quite free from magnetic impurities. The coils may have a fairly considerable inductance-temperature coefficient since the expansibility of the wood across the grain is about twice, and with the grain one-fifth, that of copper.

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* *Serpentine.*—A mineral composed of magnesium silicate with some ferrous inclusions. Chemical analysis shows that about 0·6% of iron may be present, chiefly in the form of magnetite. The mineral may be of a dull red, brown, or greenish colour, veined or mottled. It is widely distributed over the world, and is found in England on the Cornish coast.

† E. B. Rosa and F. W. Grover, "The use of serpentine in standards of inductance," *Bull. Bur. Stds.*, Vol. 1, pp. 337–348 (1905).

‡ *Marble.*—A mineral formed of a crystalline and compact mass of carbonate of lime. Its magnetic susceptibility is about $-0{\cdot}8 \times 10^{-6}$, its permeability 0·999988; its density is about 2·74 and coefficient of linear expansion 10 parts in 10^6 per degree C.

§ W. H. Souder and P. Hidnert, "Thermal expansion of insulating materials," *Bur Stds. Sci. Papers*, Vol. 15, pp. 387–417 (1920).

A further item requiring attention is the size and placing of the terminals. These should be as small as possible, made of high resistivity material to reduce the eddy current effects to a minimum, and should be placed in the weakest part of the magnetic field near the edge of the bobbin, or preferably on its rim (*see* Fig. 53). If placed close together this will also ensure very small mutual inductance between the coil and any small loop formed by the leads running from the terminals to the bridge. Fixing screws should be of resistance material, such as German silver or constantan, or still better, of ivory.

Briefly, a secondary self-inductance standard is a coil of Maxwell's proportions wound upon a rigid, non-magnetic spool of marble or mahogany. After winding, the whole is immersed in hot paraffin wax, which completely fills the interstices between the wires, and when cold sets into a solid mass, which retains its shape permanently and thus holds the wires in position. The greatest care should be taken to see that the wax is not overheated, as its insulating properties are thereby greatly impaired.

Frequency Errors. In general, the effective inductance and resistance of a coil will vary with frequency, owing to the following three causes: (i) eddy current effects in the wire; (ii) the self capacitance of the winding*; and (iii) imperfect insulation of the coil.† As these effects are of considerable importance in accurate measurements, especially at high frequencies, each will be considered in turn, together with the method adopted to make the coil as little dependent on frequency as possible.

Eddy current effects may be practically eliminated by winding the coil with twisted stranded conductor, each strand being separately insulated from its neighbours. Dolezalek has shown that the independent strands need not be less than 0·1 mm. diameter, since the effects of eddy currents in smaller wires upon the resistance and inductance of the coil are, at frequencies up to 2,000 cycles per second, less than the effects due to changes of temperature. For higher frequencies, the conductor must be very finely stranded, so that the "skin effect" does not appreciably affect the distribution of current over the section of the wire.‡

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* F. Dolezalek, "Ueber Prazisionsnormale der Selbstinduktion," *Ann. der Phys.*, Vol. 12, pp. 1,142–1,152 (1903). Further information is to be found in E. Orlich, "Über Selbstinduktionsnormale und die Messung von Selbstinduktionen," *Elekt. Zeits.*, Vol. 24, pp. 502–506 (1903); and E. Giebe, "Prazisionsmessungen an Selbstinduktionsnormalen," *Zeits. f. Inst.*, Vol. 31, pp. 6–20, 33–52 (1911).

† A. Campbell and J. L. Eckersley, "On the insulation of inductive coils," *Elecn.*, Vol. 64, pp. 350–352 (1910).

‡ *See*, however, S. Butterworth, "Eddy current losses in cylindrical conductors with special applications to the alternating current resistances of short coils," *Phil. Trans. Roy. Soc.*, *A.*, Vol. 222, pp. 57–100 (1921).

In large inductances a considerable quantity of wire is wound upon the spool, and such coils will, therefore, possess appreciable self capacitance, due to the capacitance between the individual turns and layers of wire. To a first approximation the self capacitance of a coil of inductance L and resistance R may be represented by a condenser of capacitance C joined in parallel with the coil. Referring to Fig. 10, the impedance operator for such an arrangement is

$$z = \frac{R + j\omega[L(1 - \omega^2 CL) - CR^2]}{(1 - \omega^2 CL)^2 + \omega^2 C^2 R^2}.$$

Remembering that C is small, so that its square may be neglected, the effective resistance of the coil is approximately

$$R' \fallingdotseq R(1 + 2\omega^2 CL),$$

and the effective inductance is

$$L' \fallingdotseq L(1 + \omega^2 CL),$$

L and R being the true inductance and resistance at very low frequency.

When L is small, then C is very minute and the effect of self capacitance is negligible, except when ω is very great. In large inductances, on the other hand, C is important and the effect may be far from negligible even at low frequencies. For example, Dolezalek records the case of a coil for which $L = 0{\cdot}9365$ henry and $C = 1{\cdot}6 \times 10^{-10}$ farad. At a frequency of $\omega/2\pi = 10{,}000/2\pi = 1{,}590$ the effective resistance of the coil would be nearly 3 per cent higher than at low frequency, while L' would be about 1·5 % higher than L.

In practice, the effect of self capacitance is allowed for by measuring L' at two frequencies and therefrom determining L and C. The value of L' can then be calculated at any other frequency. The effect of C on R' is twice as important as its effect on L', hence correction for self capacitance is essential in measurements of effective resistance in which large self-inductances—or small inductances at high frequency—are used. In order to allow of these corrections being made, some makers mark not only L and R, but C also upon the spool.

The insulation resistance of an inductive coil may be represented by a resistance R_2 in parallel with it. If L be the true inductance and R the true resistance of the coil, Fig. 10 states that the impedance operator is

$$z = \frac{R_2\{[R(R + R_2) + \omega^2 L^2] + j\omega LR_2\}}{(R + R_2)^2 + \omega^2 L^2}.$$

The effective resistance of the coil at frequency $\omega/2\pi$ is

$$R' = \frac{R(R + R_2) + \omega^2 L^2}{(R + R_2)^2 + \omega^2 L^2} \cdot R_2,$$

and the effective inductance is

$$L' = \frac{R_2{}^2}{(R + R_2)^2 + \omega^2 L^2} L.$$

If the insulation conductance be $G = 1/R_2$ these expressions may be written as

$$R' = R + \frac{R_2(\omega^2 L^2 - R^2) - R(\omega^2 L^2 + R^2)}{(R + R_2)^2 + \omega^2 L^2}$$

and $$L' = L/\{(1 + RG)^2 + \omega^2 L^2 G^2\}\,.$$

From the first expression, provided that the second term be positive, R' will always exceed R, the two values being more nearly equal as R_2 becomes greater. Again, L' is always less than L unless G be vanishingly small. Thus, if the insulation resistance be high, the frequency variation of inductance and effective resistance will be small. It should be clearly understood that G is the alternating current conductance of the coil insulation and will itself depend on frequency; it bears no simple relation to the ordinary d.c. insulation conductance.

Summarizing, a secondary standard of self-inductance can be corrected for frequency effects: (i) by ensuring uniform distribution of current in the wire by the use of stranded conductors; (ii) by measuring and allowing for the effects of self capacitance in small inductances at high frequencies or in large inductances at all frequencies; (iii) by ensuring high insulation resistance, especially in large inductances, by dipping the coil in wax after winding, and by fixing the terminals on an ebonite block (*see* Fig. 53 (*b*)).

Temperature Coefficient. The inductance of a standard coil will vary with any change of dimensions caused by alteration of temperature. The wire is wound on under tension and remains in contact with the former for ordinary temperature changes; the temperature coefficient of inductance, therefore, is of the same order as the coefficient of linear expansion of the former, but its exact value depends on several auxiliary factors such as the kind of conductor used, the method of winding and the impregnating medium, if any. Giebe (*loc. cit.*) gives temperature coefficients of inductance between − 23 and + 13 parts in 10^6 per degree C. for stranded, wax-impregnated coils. As mentioned on p. 133, most insulating materials used for formers do not return to their original size after temperature cycles, resulting in permanent changes in the value of inductance; the coefficient of expansion usually exceeds that of copper and the turns tend to be slackened. It is most desirable that the former should have a temperature coefficient equal to that of copper, so that the wire is unstressed by temperature change, and that the material should be cyclic.

Griffiths* has described an ingenious method for compensating the change of inductance; although originally devised for single-layer coils used in high-precision wavemeters, the same principle can be applied in the design of multi-layer and second-grade coils suitable for bridge work. Fig. 54 illustrates the essential features of the construction. The coil former is a grooved cylinder of keramot fixed to end cheeks of bakelite. By slotting the cylinder as shown, the mean diameter of the coil is determined by the expansion of the cheeks, and these are arranged to have a coefficient of expansion of the same order as that of copper; rise of temperature increases the diameter and the inductance. To compensate this effect the keramot cylinder is given a coefficient of expansion sufficiently greater than that of the bakelite, so that the fall of inductance due to the axial lengthening of the cylinder with rise of temperature counteracts the rise due to the increased diameter. Coils up to 3,000μH. have inductance-temperature coefficients less than 5 parts in 10^6 per degree C. and up to 1H. less than 10 in 10^6; they are made by H. W. Sullivan, Ltd.

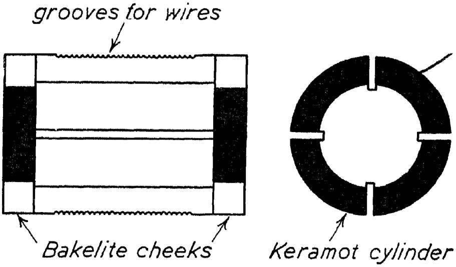

FIG. 54.—GRIFFITHS'S TEMPERATURE COMPENSATED SELF-INDUCTANCE

Adjustment. Such standards are usually wound to some exact value of inductance, e.g. 0·1 henry, and are precisely adjusted within, say, 0·01 per cent of the marked value. With large inductances, say 1 henry, the desired precision can be attained by winding the coil to the nearest turn. For example, a 1-henry coil of 2,350 turns, with a mean diameter of 15 cm., changes by about 0·04 per cent for single turn adjustment, remembering that the inductance of a coil varies as the square of the number of turns. With smaller inductances, in which there are fewer turns, the addition or removal of a turn makes a much larger alteration of inductance. For such coils, single turn adjustment is too rough a method, other devices being necessary.* For example, the coil can be wound so that it has an excess of turns ; if, then, one or more turns of the last layer are wound at a diameter greater than that of the latter, the self-inductance of the whole coil will be less than if the last layer were uniform in diameter. These auxiliary turns may be put on the coil cheek or on a slip-over tube. Again, if the coil be wound one turn too small, the inductance can be very accurately adjusted by means of a small auxiliary coil sunk in a pit in the main bobbin and connected in series with the main coil. In a variant of this, the auxiliary coil consists of a loop of wire sunk in a groove on the outside of the coil former. In both cases the adjusting turns must be firmly fixed relatively to the main coil (*see* Fig. 53 (*c*)).

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* W. H. F. Griffiths, "Notes on standard inductances for wavemeters and other radio frequency purposes," *Exp. W. and W. Eng.*, Vol. 6, pp. 543–549 (1929); "Inductances of high permanence," *Journal Sci. Insts.*, Vol. 6, pp. 354–357 (1929); "Inductances for radio frequencies," *W. Eng. and Exp. W.*, Vol. 11, pp. 305–307 (1934). *See also* E. B. Moullin, "The temperature coefficient of inductance with special reference to the valve generator," *Proc. Inst. Rad. Eng.*, Vol. 23, pp. 65–84 (1935). H. A. Thomas, "Coils and condensers of negligible temperature coefficient," *Journal Sci. Insts.*, Vol. 14, pp. 221–227 (1937). J. Groszkowski, *Proc. Inst. Rad. Eng.*, Vol. 25, pp. 448–464 (1937).

Astatic Arrangements. In some cases it is essential to have fixed value self-inductance standards that are nearly astatic with respect to local magnetic fields; it is not possible then to retain the form giving optimum time-constant, a lower value being tolerated in order to permit of the desired astatic construction being made. The most perfect astaticism is secured by toroidal ring winding,† a somewhat tedious process, but worth undertaking if perfection is desired. A lesser degree of astaticism is obtainable by the use of two coils in series, held in a fixed relation to one another, the coils having opposite instantaneous polarities when viewed from the same side. Thus, the coils may be fixed coaxially one above the other, or side by side in the same plane; other variants will occur to the reader. Small relative movements of the coils may be utilized to effect fine adjustment of the inductance of the combination to the desired value. These and other systems are considered by Campbell and Childs in their book (*see* Bibliography), to which the reader is referred for numerical details.

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* G. Moore, "The final adjustment of precision inductances," *Elec. Rev.*, Vol. 88, pp. 504–505 (1921).

† I. Fröhlich, "Ringförmiges Inductionsnormale," *Ann. der Phys.*, Vol. 63, pp. 142–153 (1897) describes a primary standard wound toroidally on a marble ring. Some firms, e.g. Leeds and Northrup Co., Dubilier & Co., construct toroidal inductors. J. C. Vogel, "A direct-reading inductance standard," *Bell Lab. Rec.*, Vol. 4, pp. 399–401 (1927) describes a toroidal variable inductor with permalloy core; see also p. 320.

MUTUAL INDUCTANCE. A fixed value reference standard of mutual inductance should satisfy the following conditions—

(i) It should be permanent in value.

(ii) The resistances of the two windings should be small and the mutual inductance high.

(iii) Eddy currents in the windings should be negligible.

(iv) Capacitance effects between the primary and secondary windings should be reduced to a minimum.

Absolute standards of mutual inductance are to be preferred to absolute self-inductances for a variety of reasons: (i) the value of mutual inductance can be calculated with greater certainty from the dimensions of the coils than is the case with self-inductance, since formulae for mutual inductance are of much higher theoretical accuracy than those for self-inductance; (ii) the effective self-inductance of a coil will vary with frequency owing to eddy currents in the wire and to self capacitance of the winding. Both these effects are much smaller in mutual inductances if the primary and secondary windings are reasonably far apart. On account of these considerations, the National Physical Laboratory use an absolute standard of mutual inductance as the ultimate reference standard in bridge measurements.

This standard is constructed on the principles laid down by A. Campbell.* The primary winding is an accurately measurable single-layer coil of bare copper wire, wound in a screw thread cut on the surface of an accurate marble cylinder. The primary consists of two equal sections in series of 75 turns each, the marble cylinder being 30 cm. diameter. The secondary is a coil of many layers wound in a groove in a marble ring which surrounds the primary cylinder, the ring being adjustable axially and radially with respect to the latter.

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* A. Campbell, "On a standard of mutual inductance," *Proc. Roy. Soc., A.*, Vol. 79, pp. 428–435 (1907). For a description of a similar mutual inductor made by Mr. Paul for the Japanese Government, *Elecn.*, Vol. 88, p. 159 (1922); also D. W. Dye, "Calculation of a primary standard of mutual inductance of the Campbell type and comparison of it with the N.P.L. standard," *Proc. Roy. Soc., A.*, Vol. 101, pp. 315–332 (1922). Further details of the N.P.L. standard are given by A. Campbell, *Proc. Roy. Soc., A.*, Vol. 87, pp. 391–414 (1912). *See also* R. M. Wilmotte, "On the field of force near the neutral point produced by two equal coaxial coils, with special reference to the Campbell standard of mutual inductance," *Proc. Roy. Soc. A.* Vol. 107, pp. 716–724 (1925); the book by Campbell and Childs; and p. 518.

The secondary has 488 turns in a channel of 1 sq. cm. area and 43·8 cm. mean diameter, and is set midway between the two primary helices. The calculated value of the mutual inductance between the primary and secondary coils is $10{\cdot}0177_8$ millihenrys at 15° C.

When the coils have the relative proportions shown in Fig. 53 (*d*), Campbell has shown that their mutual inductance is a maximum. The circumference of the secondary coil then lies in zero magnetic field, so that the change in the value of mutual inductance is a minimum for small axial displacements of the secondary and for small changes in its radius. The entire accuracy of construction is thereby thrown on the single-layer primary which can be very precisely measured; it thus becomes possible to use a multilayer secondary in order to get relatively large mutual inductance values. Mechanical details of construction are very similar to those already mentioned in connection with absolute self-inductances.

Secondary mutual inductance standards of fixed value are made for ordinary laboratory work by winding primary and secondary coils in grooves cut in circular bobbins of marble* or of wood† (*see* Fig. 53 (*e*)). The wire is preferably stranded to reduce eddy current effects. Such standards can be readily calibrated, and some correction for the effects of self capacitance will be necessary at high frequencies. The coils can be wound in several sections so that a variety of values of mutual inductance can be obtained.

If such a secondary standard is to be accurately adjusted to some definite value of mutual inductance, Campbell's‡ second method of construction is convenient (*see* Fig. 53 (*f*)). In this the primary consists of a multilayer coil; the secondary consists of two coils concentric with the primary, one being much smaller than the other. The two secondary coils are in series. One turn added to or taken from the small coil produces a very small change in the mutual inductance, which can thus be set with accuracy. For example, consider a coil of radius 10 cm. concentric with which is a coil of a cm. radius; if both coils have 1,000 turns each, the following figures show how M varies with a.

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* A. Campbell, "On the measurement of mutual inductance by the aid of a vibration galvanometer," *Proc. Phys. Soc.*, Vol. 20, pp. 626–638 (1907). The standard is nominally 0·05 henry.

† E. B. Rosa, "Mutual inductances for laboratory use," *Phys. Rev.*, Vol. 24, p. 241 (1907). A mutual inductance of 0·01 henry wound on a mahogany spool was constructed by Prof. T. Mather in 1892 and has been in use for many years in the laboratories of the City and Guilds College.

‡ A. Campbell, "Inductance measurements," *Elecn.*, Vol. 60, pp. 626–627 (1908); also *N.P.L. Researches*, Vol. 4, p. 245 and Fig. 3 (1908).

a cm.	1	2	3	4	5	6	7	8	9
M millihenrys	2	8	18	34	55	85	126	181	279

If the primary were of 10 cm. radius and the main secondary of 9 cm. each turn on a central 2 cm. coil would only have one-thirtieth of the effect of one turn on the main secondary.

The particular sources of error in the use of mutual inductance standards will be discussed in Section 12.

11. Variable Standards. In adjusting the balance of a bridge, a step-by-step change of self or mutual inductance, such as is provided by a set of fixed value standards, is not sufficiently fine. What is required is a standard by which the inductance may be steadily and continuously varied between certain limits without altering the resistance of the part of the apparatus which lies in one of the balancing branches of the network. A piece of apparatus in which this may be performed is called by different writers a variable inductance, an inductor, a variometer, an inductometer, etc.

The desirable features to be possessed by an inductometer are as follows: (i) In self inductometers the time-constant should be large, and the space occupied by the inductometer small. In mutual inductometers the resistances of the windings should be low and the mutual inductance relatively high. (ii) The range between the maximum and minimum values of inductance should be large. (iii) Inductometers may conveniently possess a linear scale for easy interpolation, but a logarithmic scale, giving equal percentage accuracy of reading at all parts of the range is often more desirable. (iv) Astatic arrangement of the coils is desirable. (v) Inductometers should be free from frequency errors due to capacitance, eddy-current, and insulation effects. Temperature effects should be small.

The simplest type of inductometer is that usually attributed to Ayrton and Perry,* illustrated in Fig. 55 (a). In this instrument two circular coils are arranged one within the other, the inner coil being capable of rotation about a diameter within the outer coil, which is fixed to the base of the apparatus in a vertical position. The axis of rotation of the moving coil is vertical, and the angle between the planes of the two coils may be read on a scale of degrees. By using one coil as primary and the other as secondary, the instrument becomes a convenient mutual inductometer; as the inner coil is rotated the mutual inductance varies from a maximum value when the two coils are in the same plane to zero when they are perpendicular, and thence to a negative maximum when they again become coincident. By joining the two coils in series, a variable self-inductance is obtained having a value $L = L_1 +$

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* "The measurement of inductances, capacities, and polarizing resistances," *Elecn.*, Vol. 34, pp. 546–547 (1895). The type of apparatus was used by other investigators at an earlier date, notably by Brillouin and by Hughes. *See* also O. Heaviside, *Electrical Papers*, p. 37 (1892), for use as a variable self-inductance; and Lord Rayleigh, *Phil. Mag.*, 5th series, Vol. 22, pp.

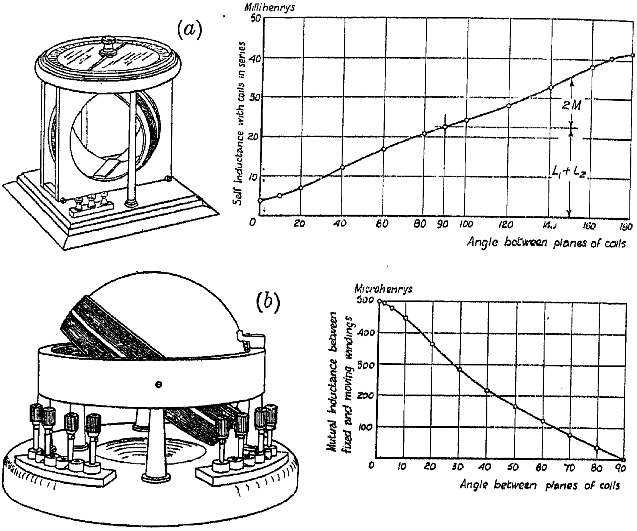

FIG. 55.—AYRTON-PERRY PATTERN INDUCTOMETERS

473–477, 498–500 (1886), for use as mutual inductometer. An even simpler type consisting of a fixed coil and a moving coil sliding on a wooden rod perpendicular to the plane of the first has occasionally been used; *see* Kollert, "Kontinuierlich abstufbare Selbstinduktionsnormale," *Elekt. Zeits.*, Vol. 30, pp 560–561 (1909).

$L_2 \pm 2M$ in any given position of the coils when M is the mutual inductance, L_1 and L_2 being the self-inductances of the fixed and moving coils respectively. L is shown on a separate scale.

In a typical instrument of this type, the variation of self-inductance is shown by the curve in Fig. 55 (*a*). The self-inductance with the coils in series varies from 3·64 to 42·22 millihenrys, and the mutual inductance between ± 9·645 millihenrys. The time-constant is 3·95 milliseconds at the maximum self-inductance reading, the resistances of the fixed and moving coils being 5·44 and 5·26 ohms respectively.

In order to avoid eddy current errors at high frequencies, Wien* and others have constructed inductometers in which the amount of metal is reduced to a minimum and the coils are wound with stranded wire. In addition, each coil is wound in sections, so that several overlapping ranges may be obtained from the same instrument. In an inductometer designed by Dr. H. F. Haworth, ebonite is used throughout. The coils are arranged horizontally, one being wound on the inside of a horizontal ebonite ring, and the other upon a pivoted concentric former. Each coil is of stranded wire and has three sections of 5, 10, and 25 turns respectively. The calibration curve and illustration of the instrument are given in Fig. 55 (*b*). The maximum range of self-inductance is up to 2,044 microhenrys, and of mutual inductance 496 microhenrys. The time-constant for the maximum self-inductance is 0·351 milliseconds.

In the Ayrton-Perry and similar inductometers the two coils are made as nearly equal in radius as is possible, the wire being usually wound upon portions of spherical surfaces. The primary and secondary coils being close together, a large mutual inductance for a given bulk is obtained. This arrangement has the disadvantage that the variation of mutual (or self) inductance is far from linear. Lord Rayleigh (*loc. cit.*) has shown that a practically uniformly divided scale can be obtained if the radius of the inner coil is 0·548 of that of the outer coil.

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* Max Wien, "Ueber ein Apparat zum varieren der Selbstinduction," *Ann. der Phys.* Vol. 57, pp. 249–257 (1896). In this instrument, constructed of wood, the fixed coil has four windings and the moving coil two windings. The windings may be grouped in series as desired and a range of self-inductance from 0·4 to 120 millihenrys covered almost continuously. Full dimensions and calibration curves are given. H. Hausrath, "Induktions Variometer und Widerstands Kombinationen," *Zeits. f. Inst.*, Vol. 27, pp. 302–312 (1907). Describes an instrument made of Stabilite, the fixed coil having four and the moving coil two windings. Self-inductance varies in eight ranges from 0·385 to 144·2 millihenrys. Dimensions are given.

The space occupied by the apparatus for a given maximum M is, however, much greater than when the coils are more nearly equal. It is with the object of producing an inductometer with a high time-constant, a linear scale, and a small bulk that the instruments now to be described were designed. In these instruments two discs of ebonite are arranged over one another, the upper disc being pivoted at the centre of the lower. Each disc carries coils, the mutual inductance between the fixed and moving sets of which can be varied by rotation of the upper disc over the lower.*

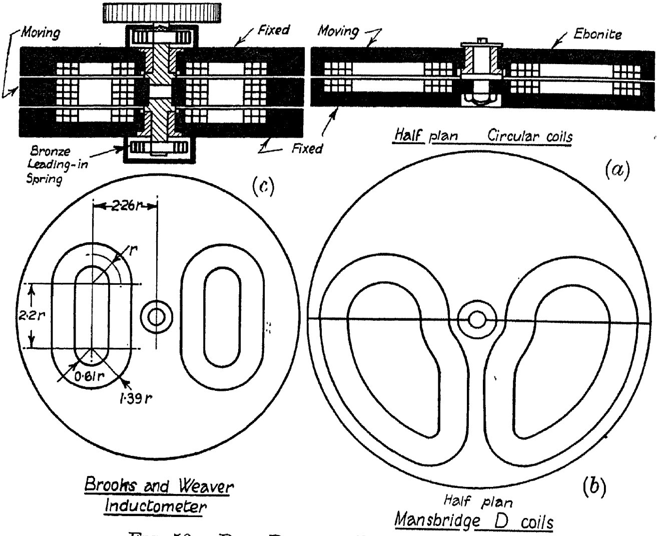

FIG. 56.—DISC PATTERN INDUCTOMETERS

In order that the apparatus should have the largest time-constant for a given quantity of wire, the fixed and moving coils, when over one another and joined in series, should form a coil of Maxwell's proportions wound on a circular former (Fig. 56 (*a*)). It is found that an inductometer made in this way has a scale which is far from linear, although from the point of view of space occupied for a given inductance, the apparatus is of the best possible proportions.

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* A. Larsen, "Der komplexe Kompensator, ein Apparat zur messung von Wechselstromen durch Kompensation," *Elekt. Zeits.*, Vol. 31, pp. 1039–1041 (1910). C. H. Sharp and W. W. Crawford, "Some recent developments in exact alternating current measurements," *Trans. Amer. I.E.E.*, Vol. 39, Part 2, pp. 1525–1529 (1911). For methods of computing coil systems for various scale characteristics see the book by Campbell and Childs.

To obtain a more uniform scale, G. F. Mansbridge* has designed the inductometer shown in half plan in Fig. 56 (*b*), the coils being of *D* or semicircular shape. There are four coils, two embedded in the upper and two in the lower ebonite disc; each coil is wound in sections so that various ranges are obtained. For the higher values, all four coils are in series, the self-inductance varying between 9 and 105 millihenrys. For the lower values, with only a small section of each coil in use, the range is between 0·7 and 12 millihenrys. The time-constant at the maximum reading is about 1·6 milliseconds; the division of the scale is practically uniform over the working range.

The most perfect inductometer of this type is due to Brooks and Weaver.† In this, a much greater time-constant is obtained by the use of link-shaped coils, by the proper proportioning of which a uniform scale can be obtained. Three pairs of coils are used; two being fixed to ebonite or condensite discs, the remaining pair rotating between them. The best results are obtained when the coils have the relative dimensions shown. The coils are connected astatically and are wound with stranded wire. By the use of two sets of fixed coils, the effect of a small axial displacement, due to wear of the bearings, on the calibration is negligible (*see* Fig. 56 (*c*)).

If R_1, R_2, L_1, L_2 are the resistances and inductances of the two systems of coils and M is the mutual inductance between them, the impedance operator for the whole when the fixed and moving systems are in additive series is

$$z_s = (R_1 + R_2) + j\omega(L_1 + L_2 + 2M)$$

If the two systems are put in additive parallel, it is easy to show that the impedance operator becomes

$$z_p = \frac{R_1R_2 - \omega^2(L_1L_2 - M^2) + j\omega(L_1R_2 + L_2R_1)}{(R_1 + R_2) + j\omega(L_1 + L_2 - 2M)}$$

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* G. F. Mansbridge, "Improvements in and relating to iron-free variable inductances," *British Patent*, No. 22,206 (1905). The apparatus is made by the Cambridge Instrument Co.

† H. B. Brooks and F. C. Weaver, "A variable self and mutual inductor," *Bull. Bur. Stds.*, Vol. 13, pp. 569–580 (1917). The instrument is made by the Leeds and Northrup Company in sizes up to 500 millihenrys. Messrs. Tinsley & Co. also make the inductometer, but with the plane of the coils mounted vertically. A series parallel commutator is provided; a range of self-inductance 5 to 140 millihenrys and of mutual inductance − 30 to + 30 millihenrys is obtained. Also see reference on p. 495.

Now let the fixed and moving coils be designed to have equal resistances and inductances, i.e. $R_1 = R_2 = R$ and $L_1 = L_2 = L$; then

$$z_s = 2[R + j\omega(L + M)]$$

and

$$z_p = \tfrac{1}{2}[R + j\omega(L + M)]$$

That is, the effective resistance and inductance of the combination when in parallel are one-quarter of the values when in series, a property pointed out by Heaviside (*Phil. Mag.*, Vol. 23, p. 186 (1887)) and a very useful method of obtaining an additional range of self-inductance from a given inductometer. The device is adopted by several manufacturers, both in fixed and variable standards.*

Variable Self-inductances Without Moving Coils. Variable standards of self-inductance may also be constructed by assembling together a number of inductance coils in a box and providing suitable switch or plug arrangements to enable their inductances to be added in series. In order that this may be accurately secured it is essential that there should be no mutual inductance between the coils. This is easily attained with three coils assembled with their planes mutually perpendicular. A fourth coil can be added, having no mutual inductance with respect to two of the coils; its mutual inductance with the third coil can be made very small when it is remembered that the mutual inductance between two coils falls off very rapidly as their distance apart increases. Brooks (*loc. cit.* on p. 131) has described a set of eight coils, arranged in this manner in two rows of four each, with values of 1, 2, 3, 4 millihenrys and 10, 20, 30, 40 millihenrys respectively, the rows being sufficiently far apart to make mutual inductance between them negligibly small; means are provided to replace any coil cut out of circuit by a non-inductive winding of equal resistance, thus keeping the total resistance constant at all settings. A number of makers list variable self-inductors consisting of a single bobbin wound for the maximum value and tapped to give ten equidistant steps which are controlled either by plugs or by a dial switch.

A. Campbell† has designed a step-variable self-inductance which is a great improvement on the arrangements just mentioned, and possesses the additional feature of continuous variation down to zero self-inductance with negligible variation of resistance over the entire range; the construction is represented diagrammatically by Fig. 57. In this diagram a is a coil from which tappings are brought out to give 10 equal values of inductance; b is a second coil having a range one-tenth of the value of a. Stud switches enable selection of any desired number of sections from each decade to be added in series, suitable non-inductive compensating resistances keeping the total resistance constant for all positions of the switches. The two coils a and b are arranged with their planes at right angles so that there is no mutual inductance between them and their self-inductances are simply additive; other arrangements to attain this astaticism may be used, such as toroidal winding. Fine adjustment is provided by a fixed inductance λ in series with a slide wire, a variable portion r of which is shunted by a condenser k.

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* C. T. Burke, "Variable inductors for bridge measurements," *G.R. Exp.*, Vol. 7, pp. 7–8 (June, 1932). Also p. 495, footnote.

† A. Campbell, *British Patent* No. 317,642 (22nd Aug., 1929).

The effective inductance of the combination is very nearly $\lambda - kr^2$, and by suitable choice of λ, k and r can give a range from a maximum value equal to a little more than a single step of the lower decade down to zero and thence to a negative value. This arrangement is of considerable usefulness in many branches of bridge measurement, and is the first attempt to produce a continuously variable self-inductance without moving coils; it is easily rendered strictly astatic by the use of toroidal coils.

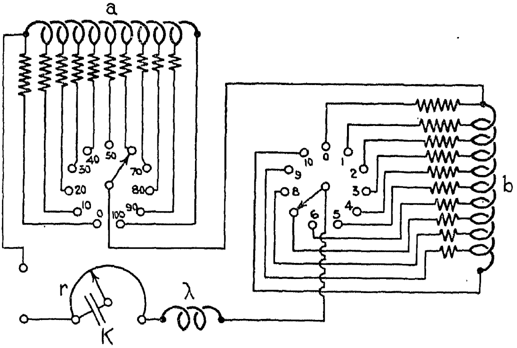

FIG. 57.—CAMPBELL'S VARIABLE SELF-INDUCTANCE

Messrs. Muirhead make an interesting self-inductor for low values (0 to 25μH.) consisting of a helix of copper tube. A sliding contact slides on the inside of the helix, and is actuated by a coaxial screw of equal pitch. A scale reads the angular position of the contact, and a window indicates the turn upon which contact is made.

CAMPBELL'S MUTUAL INDUCTOMETER. Mr. A. Campbell, late of the National Physical Laboratory, has shown that standards of mutual inductance in alternating current bridges have marked advantages over self-inductances, principally for the reason that mutual inductometers can be varied down to zero and then to negative values, while self inductometers have not these properties. This advantage of a mutual inductometer is of great value in the measurement of small inductances, and in the allowance for the inductance of leads, etc., in other measurements. Mr. Campbell has developed a large number of excellent bridge methods containing mutual inductance, and has devised a special mutual inductometer which is very widely used in practice. The arrangement and connections of the inductometer* are shown in Fig. 58, from which it is seen that the instrument is really a transformer of variable ratio.

The primary winding consists of two equal, coaxial, fixed coils, P_1, P_2, wound in the same direction and connected in

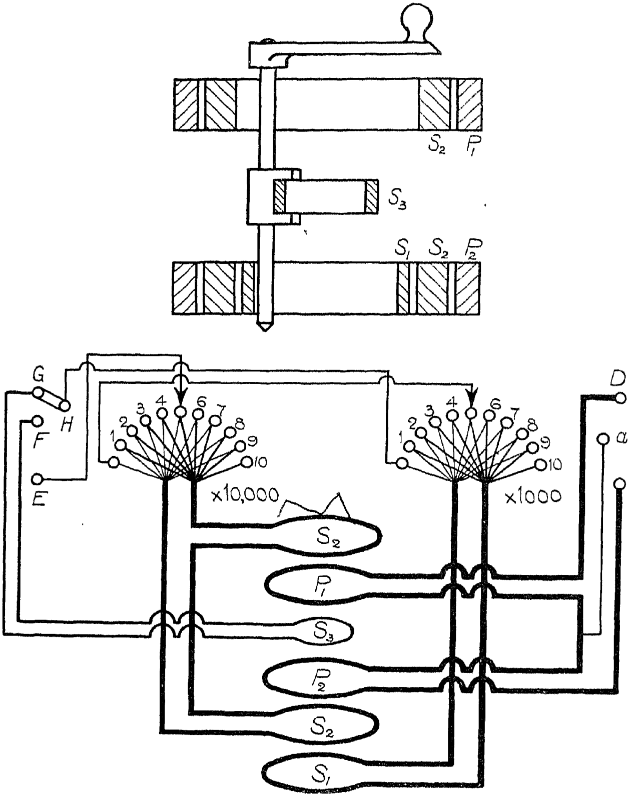

Fig. 58.—Campbell's Mutual Inductometer

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* A. Campbell, " On the use of variable mutual inductances," *Phil. Mag.*, 6th series, pp. 155–171 (1908); *Proc. Phys. Soc.*, Vol. 21, pp. 69–87 (1910). The instruments are made by the Cambridge Instrument Co.

series. Two terminals make connection with the primary, the point of junction of the two coils being brought to a third* terminal *a*. In a typical inductometer the total inductance of the primary was 0·10057 henry, and its resistance 40·15 ohms at 20° C.

The secondary winding is composed of three portions which can be put in series, S_1, S_2, S_3. The coil S_1 is made up of ten sections, each of which has exactly the same mutual inductance with respect to the primary, namely, 1,000 microhenrys in the above-mentioned example. The precise equality of the sections is ensured by the artifice of winding the coil with a stranded† rope of wires in the following way. Ten insulated wires of a suitable length are twisted together to form a stranded rope, which is then wound upon a bobbin to form a coil having a predetermined number of turns. It is then found that the mutual inductance between each strand and the primary winding is almost exactly the same.‡ By the use of a dial switch, any number of strands may be joined in series and their mutual inductances added.

The coil S_2 is wound in the same way as S_1, but partly on the lower and partly on the upper bobbin, and has such a number of turns that each of its ten strands has ten times the mutual inductance of a strand in S_1 with respect to the primary, namely, 10,000 microhenrys. The total range of S_2 is thus 100,000 and of S_1 10,000 microhenrys.

The third section of the secondary is the coil S_3 which is mounted on an axis parallel to that of the primary coils. Moving this coil by means of the handle varies its mutual inductance with respect to the primary from a maximum down to zero and then to negative values. With the choice of suitable proportions§ for the coil, a semicircular scale may be obtained covering an arc about 30 cm. long, the divisions at the upper and lower values being quite open, so that precise readings are possible. In the inductometer mentioned, the range of S_3 is from -40 to 1,060 microhenrys. By means of the link shown, connecting F and H puts the coil S_3 in opposition to the coils S_1 and S_2, thus enabling 1,000 on the scale to be checked against the first section of S_1.

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* For the use of this mid-point in practice, *see* p. 438.

† Stranded inductometers were used by M. Brillouin, *Assoc. Francaise p. l'avancement d. Sc.*, pp. 333–336 (1881); *Comptes Rendus*, Vol. 93, pp. 1010–1014 (1881); *Ann. de l'École Normale*, Vol. 11, pp. 339–424 (1882); and by H. Rowland, *Amer. J. Soc.*, 4th series, Vol. 4, pp. 429–448 (1897); *Phil. Mag.*, 5th series, Vol. 45, pp. 66–85 (1898).

‡ For the method of final precise adjustment to equality, *see* G. Moore, *loc. cit.* (1921).

§ A. Campbell, *loc. cit.* Also *see* S. Butterworth, "On the coefficients of mutual induction of eccentric coils," *Phil. Mag.*, 6th series, Vol. 31, pp. 443–454 (1916). N. F. Astbury, "A simple method of selecting an inductometer scale," *Journal Sci. Insts.*, Vol. II, pp. 319–320 (1934).

In normal working, the link connects G and H, the entire secondary winding being in series and connected to the terminals E, F. The range of the inductometer is thus from $-0{\cdot}00004$ to $0{\cdot}111060$ henry. A smaller instrument of precisely the same pattern has a range of $-3{\cdot}5 \times 10^{-6}$ to $0{\cdot}011104$ henry, and by the provision of terminals tapping one-tenth of the primary winding, a particularly low range inductometer, reading up to 0·0011104 henry, is obtained.

The conductors with which the coils are wound are themselves highly stranded, so that eddy currents therein are reduced to a minimum. No unnecessary metal is used in the construction of the inductometer for the same reason.

The principal defect of Campbell's mutual inductometer arises from the considerable capacitance of the stranded windings. It will be shown in Section 12 that the effect of this is to introduce imperfection in the behaviour of the instrument as a quadrature transformer and to produce errors which may become important at telephonic frequencies, especially in inductometers of the larger sizes. For use at low frequencies, say up to 500 cycles per second, the Campbell inductometer is unsurpassed for accuracy and ease of manipulation.

Mr. Campbell has developed and patented* an improved design in which the impurity (*see* p. 153) is very much reduced, the angular length of the fine adjustment scale increased, and several other valuable features are incorporated. In the older stranded pattern the impurity at 1,000 cycles per second and a reading of 10,000μH. lies between $+0{\cdot}016$ and $-0{\cdot}028$ ohm according as M is in the sense to oppose or to assist self-inductance; the frequency coefficient of mutual inductance at this frequency is about 1 part in 1,000, and varies with the square of the frequency; the angular range of the scale for the moving coil is about 160°. In the newer pattern the impurity has been reduced to 0·006 ohm at 10,000μH. and 1,000 c.p.s. by improved methods of winding, stranding being abandoned, each section being separately adjusted; the change of M with frequency is negligible up to at least 2,000 c.p.s. By using the arrangement shown in Fig. 59 the angular range of the moving coil has been increased to 260°, and by means of a scale range-switch the scale may be either direct reading or a $\frac{1}{5}$ factor can be intro-

* A. Campbell, *British Patent*, No. 244,596.

duced. As this diagram shows, *D* is the primary winding and *E* the moving secondary mounted on a vertical axis *X* which carries a pointer moving over a horizontal scale. An auxiliary coil is now fitted perpendicular to *E* and serves to open out the higher scale divisions. The divisions open out toward the zero and the scale has approximately equal percentage accuracy of reading at all parts. The primary windings are tapped at the middle for use in equal ratio bridges, and also at inductance ratios of $\frac{1}{9}$ and $\frac{1}{99}$, thus dispensing with the use of balancing coils otherwise required for the unequal ratio method of page 433. The standard instrument has a range of 11,000μH., having one decade in 1,000μH. steps, a second decade in 100μH. steps, and a moving coil giving -5 to 105μH.; a zero adjuster, in the form of a mid-point slide wire rheostat between the sections of the primary, and a small moving coil giving $\pm$ 2μH., is also incorporated. A complete diagram of connections is given in Fig. 60. A second instrument, having a range of 110,000μH., is also available, both being manufactured by the Cambridge Instrument Co.

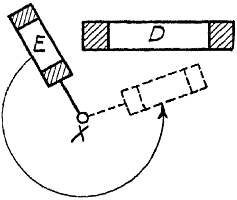

FIG. 59.—CAMPBELL'S LONG SCALE MOVING COIL

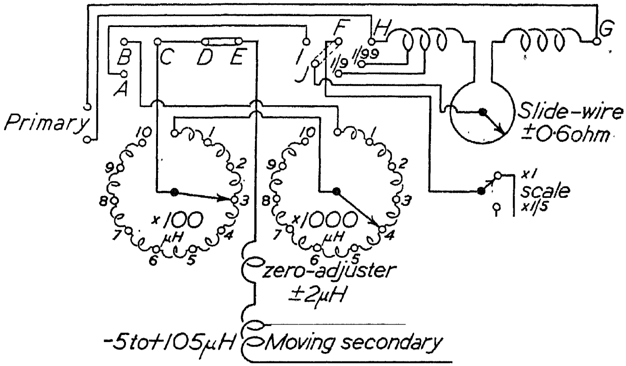

FIG. 60.—CAMPBELL'S IMPROVED MUTUAL INDUCTOMETER

THE BUTTERWORTH-TINSLEY MUTUAL INDUCTOMETER. To overcome the capacitance effects inherent in stranded-coil inductometers of the Campbell type, Mr. Butterworth* has suggested a design of variable mutual inductometer with very small capacitance and consequently much reduced errors at high frequencies; the instrument is made by Messrs. H. Tinsley and Co. The primary consists, as before, of a fixed winding. Each decade of the secondary is composed of three separate coils, having mutual inductances with respect to the primary of 6, 3, and 1 dial units respectively. These three coils may be arranged so that the self and intercapacitances are very small. They must be independently adjusted, but other points on the dial are obtained by addition or subtraction of the mutual inductances of these coils in series; thus 10 is given by 6 + 3 + 1, 9 by 6 + 3, 8 by 6 + 3 - 1, etc. A suitable form of controller-type dial switch has been devised for effecting the necessary grouping, the coils not required in any setting being disconnected. Fig. 61 illustrates the principle of the switching.

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* See *N.P.L. Report* for 1922, pp. 82–83.

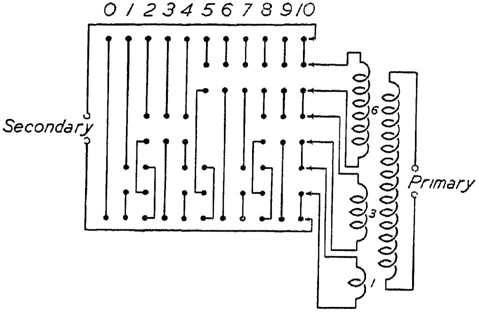

FIG. 61.—TYPICAL DECADE OF BUTTERWORTH-TINSLEY MUTUAL INDUCTOMETER

The standard instrument has two such decades together with a movable coil, as in the Campbell instrument, to provide for continuous variation from the value of one unit of the lower decade down to zero and small negative values. Three sizes of the inductometer are manufactured, having ranges of - 1 to + 1,100 μH, - 10 to + 11,100 μH, and - 100 to + 111,000 μH respectively.

Errors in Inductometers. Self inductometers are subject to the same sources of error as fixed value self-inductances. They should be constructed, therefore, with a view to permanence of value, high time-constant, freedom from magnetic materials, small self capacitance, high insulation resistance, and absence of skin-effect. The methods of construction described above will show how these conditions are met in practice and do not require amplification here. In addition, it should be pointed out that the change of inductance with temperature in inductometers of the Ayrton-Perry type is small, Taylor* having shown it to be less than 1 part in 40,000 per °C.

Mutual inductometers require more discussion. An ideal mutual inductance is one in which the electromotive force induced in the secondary coil is exactly in quadrature with the primary current. Such an inductance is termed by Silsbee† "pure," and is very closely represented by a pair of carefully constructed coils arranged not too close to one another and carrying low frequency current. However, at high frequencies these conditions no longer hold, since the effects of the self and intercapacitances of the coils, of imperfect insulation, and of eddy current losses become appreciable. Arising from these effects a component of secondary electromotive force appears in phase with the primary current, so that the induced secondary electromotive force is no longer $-j\omega M\mathbf{i}_1$ but $(\sigma - j\omega M)\mathbf{i}_1$. The mutual inductance is then said to be "impure," σ being called the "impurity."

12. Impurity Effects in Mutual Inductances. Mutual inductances are frequently used in bridge measurements at high frequencies, so that it becomes important to discuss their imperfections under these conditions. The properties of impure mutual inductances have been examined theoretically and experimentally by Butterworth,‡ his experimental work being supplemented by that of Hartshorn.§ These writers show that the factors producing the impurity are (i) self and intercapacitances of the windings; (ii) eddy current losses in the copper and in terminals, etc.; (iii) leakage and dielectric losses in the insulation; (iv) resistance inadvertently included in common with both windings when these are connected at a common point, as is usually the case in practical work.

It will be supposed in most of the following discussion that the primary and secondary windings have a common point. Campbell has shown experimentally in his classic work on mutual inductors that just as a perfect mutual inductance may be balanced by a second equal mutual inductance in a suitable bridge network, an impure mutual may be balanced at a given frequency by an *M-R* pair, i.e. by a pure mutual in series with a resistance. Hence an impure mutual inductance with a common point may be regarded as equivalent to a pure mutual, together with resistance σ common both to the primary and the secondary circuits, as shown in Fig. 62(*a*). Let the primary terminals *AB* and the secondary terminals *CD* be closed through external applied electromotive forces $\mathbf{e}_1$ and $\mathbf{e}_2$; the positive directions of the mesh cyclic currents $\mathbf{i}_1$ and $\mathbf{i}_2$ being counter-clockwise. Let z_1 and z_2 be the impedance operators of the primary and secondary windings of the pure mutual part of the equivalent circuit, then

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* A. H. Taylor, "On the possible variation of inductance standards with temperature," *Phys. Rev.*, Vol. 20, p. 394 (1905).

† F. B. Silsbee, *Bull. Bur. Stds.*, Vol. 13, p. 414 (1916).

‡ S. Butterworth, "Capacity and eddy current effects in inductometers," *Proc. Phys. Soc.*, Vol. 33, pp. 312–354 (1921).

§ L. Hartshorn, "The properties of mutual inductance standards at telephonic frequencies," *Proc. Phys. Soc.*, Vol. 38, pp. 302–320 (1926). "Standards of phase angle," *World Power*, Vol. 8, pp. 171–180, 234–240 (1927).

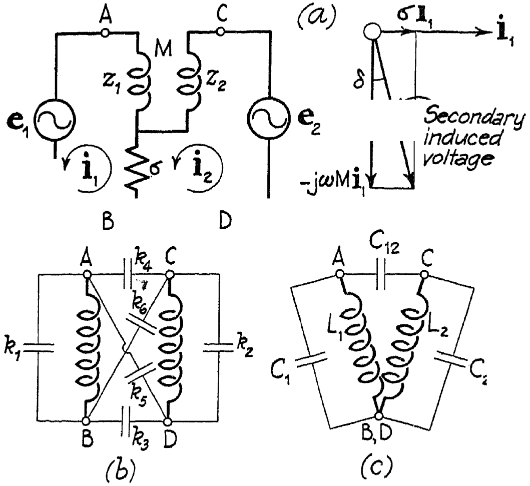

Fig. 62.—Illustrating the Properties of Impure Mutual Inductances

$$z_1\mathbf{i}_1 + \sigma(\mathbf{i}_1 - \mathbf{i}_2) = \mathbf{e}_1 - j\omega M\mathbf{i}_2$$

$$z_2\mathbf{i}_2 + \sigma(\mathbf{i}_2 - \mathbf{i}_1) = \mathbf{e}_2 - j\omega M\mathbf{i}_1$$

where the first term on the right-hand side is the externally applied e.m.f. and the second term is the internally induced e.m.f. in each case. Re-writing the equations in the following way

$$(z_1 + \sigma)\mathbf{i}_1 = \mathbf{e}_1 - (j\omega M - \sigma)\mathbf{i}_2$$

and

$$(z_2 + \sigma)\mathbf{i}_2 = \mathbf{e}_2 - (j\omega M - \sigma)\mathbf{i}_1$$

Hence the secondary induced e.m.f. in an impure mutual inductance may be written as $(\sigma - j\omega M)\mathbf{i}_1$, replacing the e.m.f. $-j\omega M\mathbf{i}_1$, which would be induced if the mutual were free from effects of impurity.* The quantity σ has the dimensions of a resistance; both σ and M in the equivalent circuit are functions of frequency. The phase-defect of an impure mutual, i.e. the angle by which the vectors of primary current and secondary induced voltage differ from exact quadrature, is $\delta = \arctan(\sigma/\omega M)$.

The principal causes of impurity and of frequency errors in mutual inductances are self and intercapacitances and the effects of eddy currents. Butterworth has investigated these causes mathematically and has shown that as regard capacitance effects a mutual inductance is equivalent to the circuit of Fig. 62(*b*), in which k_1, k_2 are the self capacitances of the two windings, while k_3, k_4, k_5, and k_6 represent the effects of distributed intercapacitance by condensers located between the terminals. If a primary current $\mathbf{i}_1$ enters A and leaves B, only a part flows through the coil; the remainder finds its way to B by paths such as $k_3 + k_5$ in series, $k_4 + k_6$ in series, etc. Thus the current through the coil is not $\mathbf{i}_1$ but $\mathbf{i}_1'$ differing slightly in magnitude and phase from $\mathbf{i}_1$. The e.m.f. induced in the secondary is in quadrature with $\mathbf{i}_1'$; this e.m.f. will cause currents to circulate through the various capacitance paths and through the secondary coil, thus producing a small potential drop which must be subtracted from the induced voltage. Hence the net secondary voltage will be different from what would be found were the capacitances absent, and will not be in quadature with $\mathbf{i}_1$; in other words, an impurity σ_c due to capacitance effects has been introduced. The influence of eddy currents will be discussed later.

If the mutual be used as in Fig. 62(*b*) the relative potentials of the two coils may be largely determined by the stray capacitances, so that the mutual capacitance effect, and with it the value of M and σ, may be liable to uncertainty. Hence, in a.c. bridge networks, it is necessary to have one point common to both windings, as shown in Fig. 62(*c*). The circuit is then much simpler, since k_3 is inoperative, k_1 and k_5 are joined in parallel to form C_1, k_2 and k_6 to form C_2, and k_4 is written as the sole intercapacitance C_{12}. Butterworth shows that such a system has the following properties. If M_0 be the mutual inductance at a low frequency and M the mutual inductance at high frequency, $M = M_0 + \Delta M$, and $\Delta M/M_0$ is the frequency coefficient. It is shown that the part of $\Delta M/M_0$ due to C_1 and C_2 is always positive, while that due to C_{12} may be positive or negative according to the sign of M_0. When M_0 is positive, i.e. mutual assisting the self-inductances, the mutual capacitance effect is positive. If M_0 is negative, i.e. mutual opposing the self-inductances, as is usually the case in a.c. bridges, while both L_1 and L_2 are $> M_0$, as is also customary, the mutual capacitance effect becomes negative and opposes that of self capacitance. If $L_1 > M_0 > L_2$ the mutual effect is positive but much smaller than when M_o is positive. Hence a negative mutual inductance has a much smaller frequency coefficient than a positive one and is, therefore, preferable. Similar considerations apply to the phase-defect $\sigma_c/\omega M$, where σ_c is the part of σ arising from capacitance effects. In the case of M_0 being negative, the phase-defect due to mutual capacitance opposes that due to selfcapacitance when L_1 and L_2 are greater than M_0, and in all cases the impurity and phase-defect with a negative M_o are less than with a positive value. Butterworth has shown that the effect of eddy currents in the coils is to introduce a further component of impurity, σ_e, which may be positive or negative according to the sign of M_o, but usually the eddy effects are of much less importance than those of capacitance. By artificially adjusting the capacitances and eddy losses by the addition of condensers and resistances in appropriate positions, Butterworth has demonstrated that σ_c and σ_e may be set off one against the other, resulting in a compensated pure mutual when M_o is negative.

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* Butterworth and other writers following his work write for the secondary induced e.m.f. $(\sigma + j\omega M)\mathbf{i}_1$ but it is to be remembered that the difference is due to the adoption of a different convention for the sign of M. In this book M is taken as positive when the mutual flux assists the self flux with counter-clockwise positive directions for the cyclic currents in all meshes of the network. Butterworth's convention is the converse of this, namely, M is positive when the mutual flux and the self flux are in opposition; see also page 67.

Following the convention* adopted on pages 67 and 155, viz. that M is regarded as positive when assisting L, the impurity introduced into a mutual inductance with a common point by self and intercapacitance of the windings is worked out by Butterworth in the following way. Repeating Fig. 62 (*c*) with slight change of notation, a mutual with self and intercapacitances is represented by the equivalent circuit of Fig. 63 (*a*), in which

$$\xi = R_1 + j\omega L_1,\ \eta = R_2 + j\omega L_2,\ m = j\omega M_o,\ \alpha = j\omega C_{12},$$
$$\beta = j\omega C_2,\ \ \gamma = j\omega C_1.$$

Using the transformation on page 70, the system ξ, η, m can be replaced by a star-connected system without mutual inductance, $x = -m$, $y = \xi + m$, $z = \eta + m$, as shown in Fig. 63 (*b*). With the aid of the star to mesh transformation (equation 11*a* on page 58), this system is equivalent to the mesh shown in Fig. 63 (*c*), where $\alpha' = \alpha + \frac{x}{\mu}$, $\beta' = \beta + \frac{y}{\mu}$ and $\gamma' = \gamma + \frac{z}{\mu}$ where $\mu = xy + yz + zx$. This in turn is converted into a star-system by Equation 10*a* on page 57, as shown in Fig. 63 (*d*), this finally being transformed by the inverse of the theorem on page 70 into two mutually influencing impedances $\xi' = X + Y$, $\eta' = Z + X$, with mutual operator $m' = -X$ as in Fig. 63 (*e*). After a lengthy algebraical treatment in which squares and products of the small capacitances C_{12}, C_1 and C_2 are neglected it can be shown that

$$m' = m - \alpha\,(\xi + m)\,(\eta + m) - \beta m\eta - \gamma m\xi,$$
$$\xi' = \xi - \alpha\,(\xi + m)^2 - \beta m^2 - \gamma\xi^2,$$
$$\eta' = \eta - \alpha\,(\eta + m)^2 - \beta\eta^2 - \gamma m^2.$$

The first expression is the most useful since it gives the frequency change of mutual inductance and the impurity arising from the self and intercapacitances; the other expressions show the corresponding changes in effective resistance and inductance of the windings. Writing

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* See also R. M. Wilmotte, "Capacitative and inductive coupling, including a method of measuring mutual inductance at radio frequencies," *Exp. W. and W. Eng.*, Vol. 7, pp. 485–492 (1930).

$m' = j\omega M - \sigma_c$ where M is the effective mutual inductance and σ_c the impurity due to capacitance effects,

$$M = M_o - C_{12}R_1R_2 + \omega^2 \{C_1L_1M_o + C_2L_2M_o + C_{12}(L_1+M_o)(L_2+M_o)\}$$
$$\equiv M_0 + \Delta M,$$
$$\sigma_c = -\omega^2 \{C_1R_1M_o + C_2R_2M_o + C_{12}[R_1(L_2+M_o) + R_2(L_1+M_o)]\}.$$

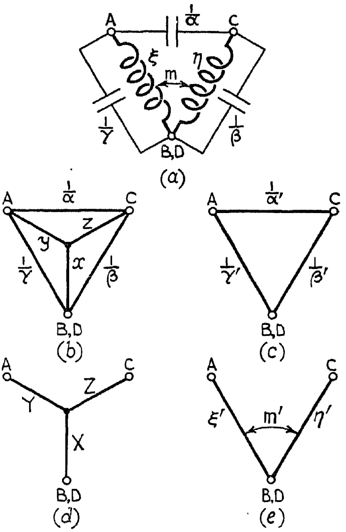

FIG. 63.—TRANSFORMATION OF MUTUAL INDUCTANCE WITH CAPACITIVE IMPURITY EFFECTS

Frequency Coefficient. It is to be noted that the term $C_{12}R_1R_2$ is usually negligible. Then the frequency coefficient of mutual inductance

$$\frac{\Delta M}{M_o} = \omega^2 \left[C_1L_1 + C_2L_2 + C_{12}\frac{(L_1 + M_o)(L_2 + M_o)}{M_o} \right].$$

The terms $\omega^2 C_1L_1$ and $\omega^2 C_2L_2$ may be regarded as the frequency coefficients due to the self capacitances of the primary and secondary windings, and are essentially positive. The third term is the contribution of the intercapacitance and will be positive or negative according to the sign of M_o and the relative magnitudes of M_o, L_1 and L_2. When the mutual inductance assists the self-inductance (M_o *positive*) the third term is positive, and $\Delta M/M_o$ attains its largest value for given values of C_1, C_2 and C_{12}. If the mutual inductance opposes the self-inductance (M_0 *negative*) as is commonly the case in most bridges, then numerically,

$$M \doteqdot M_o + \omega^2 \{C_1L_1M_o + C_2L_2M_o - C_{12}(L_1 - M_o)(L_2 - M_o)\}$$

$$\text{and } \frac{\Delta M}{M_o} = \omega^2 \left[C_1L_1 + C_2L_2 - C_{12}\frac{(L_1 - M_o)(L_2 - M_o)}{M_o} \right].$$

It is most usual to have $L_1 > M_o$ and $L_2 > M_o$, in which case the intercapacitance term is negative and reduces $\Delta M/M_o$ considerably. If, however, $L_1 > M_o$ but $M_o > L_2$, as may in certain cases occur, the third term is positive but much smaller than when M_o has a positive value. Hence, if the capacitances remain unchanged with change of common-point (in which connection see later) a negative mutual inductance (M_o opposing self-inductance) has a smaller frequency coefficient than a positive mutual inductance (M_o assisting self-inductance), and should be used whenever possible.

Impurity and Phase Defect. The phase defect δ_c due to capacitance effects is

$$\delta_c = \arctan\frac{\sigma_c}{\omega M} \doteqdot \frac{\sigma}{\omega M}, \text{ or}$$

$$\delta_c \doteqdot -\left[\omega C_1R_1 + \omega C_2R_2 + C_{12}\left\{ \frac{\omega R_1(L_2 + M_o)}{M_o} + \frac{\omega R_2(L_1 + M_o)}{M_o} \right\} \right],$$

in which the first and second terms are contributed by the self capacitances and do not depend upon M_o, while the terms within curved brackets arise from the intercapacitance and may become positive or negative according to the sign of M_o and the relative magnitudes of M_o, L_1 and L_2. When mutual inductance assists self-inductance (M_o *positive*) σ_c and δ_c are essentially negative and have their largest values, since all terms have the same sign. If mutual inductance opposes self-inductance (M_o *negative*) then numerically,

$$\sigma_c = \omega^2\{C_1R_1M_o + C_2R_2M_o - C_{12}[R_1(L_2 - M_o) + R_2(L_1 - M_o)]\},$$

and

$$= -\left[\omega C_1R_1 + \omega C_2R_2 - C_{12}\left\{ \frac{\omega R_1(L_2 - M_o)}{M_o} + \frac{\omega R_2(L_1 - M_o)}{M_o} \right\} \right].$$

Provided the first two terms together exceed the remainder, σ_c will be positive and δ_c negative. In the common practical case $L_1 > M_o$ and $L_2 > M_o$ intercapacitance effects oppose those of selfcapacitance, and in all cases make σ_c and δ_c for a negative mutual smaller than for positive mutual, all the capacitances being assumed unchanged.

Effect of Changing the Common Point. In the preceding discussion of the impurities due to capacitance effects it has been supposed that the self and intercapacitances do not change with change in the sign of M_o obtained by alteration of the common point; this is not, in general, true since the partial capacitances shown in Fig. 62 (*b*) are not necessarily symmetrically arranged. Consequently, when the sign of M_o is changed by altering the common point $\Delta M/M_o$, σ_c and δ_c are liable to change on account of the altered values of C_1, C_2 and C_{12}. In addition to the frequency coefficient and impurity produced by capacitance there are additional imperfections of a similar character arising from the effects of eddy currents, but these are generally of small importance. Taking all into account, however, it is found in practice that a positive mutual (M assisting L) usually has a larger frequency coefficient and impurity than a negative mutual (M opposing L). This can be illustrated by the following figures for a 10 mH. Campbell stranded inductometer at the top of its scale at 1,000 cycles per second; there are four possible common points, two giving negative and two positive values of M. Also see p. 150.

Sign of M_0	$\Delta M/M_0$	σ ohms	$\sigma/\omega M$ numerically
M opposing L (−)	$9 \cdot 8 \times 10^{-4}$	+ 0·016	0·00026
,, ,, ,, (−)	$8 \cdot 2 \times 10^{-4}$	+ 0·015	0·00024
,, assisting ,, (+)	$15 \cdot 8 \times 10^{-4}$	− 0·028	0·00045
,, ,, ,, (+)	$11 \cdot 8 \times 10^{-4}$	− 0·021	0·00033

In this table ΔM, σ and δ refer to the overall effects of capacitance and eddy currents, the former predominating.

Numerical Values. In general, it is found that ΔM and σ increase nearly proportionally to the square of the frequency, so that they become of greatest importance at high frequencies. It will be valuable to state the magnitude and properties of these frequency corrections for the commoner types of standard inductometers, showing the variation with frequency, scale reading, and change of common point. The following conclusions are drawn from the work of Butterworth and of Hartshorn previously cited.

Campbell Inductometer. The stranded windings of this instrument have considerable self and intercapacitance; for example, if one strand in a decade is in use, the remaining nine strands being free, the decade will act like a 9 to 1 transformer with capacitance load. Consequently, the impurity due to capacitance effects may be large. It is found by experiment that: (i) The mutual inductance increment ΔM is proportional to ω^2. (ii) When M is negative (M opposing L) the fractional increase of mutual inductance with frequency is approximately constant, for a given frequency change, over the whole range of the instrument. (iii) The value of σ increases more rapidly than the theoretical proportionality to ω^2, due to dielectric losses which introduce additional impurity, varying with a power of ω higher than the square. (iv) When M is negative the value of δ is nearly constant for all readings.

Butterworth shows that for a 10 mH. inductometer at the top of its scale the mutual inductance has a positive correction of 9·8 parts in 10,000; the self-inductance of the fixed winding a positive correction of 7·6 parts in 10,000; the impurity is 0·0175 ohm; all the corrections are at 1,000 cycles per second and vary very nearly with the square of the frequency. The instrument is arranged with a common point such that when the coils are in series M opposes the self-inductances.

Hartshorn's experiments show that for a 1 mH. inductometer the corrections are somewhat smaller, since the capacitances are less, while for a 100 mH. instrument the corrections are more serious, as shown in the table for the 100 mH. setting.

Frequency	100	500	1000	1500	2000
ΔM in mH.	0·09	0·24	1·03	2·41	4·79
σ in ohms	—	0·082	0·60	1·99	4·6
$\Delta M/M_0$	$0{\cdot}09 \times 10^{-2}$	$0{\cdot}24 \times 10^{-2}$	$1{\cdot}03 \times 10^{-2}$	$2{\cdot}41 \times 10^{-2}$	$4{\cdot}79 \times 10^{-2}$
$\sigma/\omega M$	—	0·00026	0·00095	0·0021	0·0035

Here again, the mutual inductance is negative (opposing self-inductance) and the common point is *DE* in Fig. 58. The deviation from square law of frequency is greater than in the smaller inductometers. At 1,000 cycles per second the phase defect is about 3 minutes, about equal to that of a condenser with power factor of 0·001.

Butterworth-Tinsley Inductometer. This instrument is specially constructed to have very low self and intercapacitances, resulting in a much purer mutual inductance at high frequencies; the frequency corrections are of the order of one-sixth of the corresponding corrections in the Campbell stranded instrument. Typical results for a 10 mH. standard with a common point and the mutual inductance opposing the self-inductances are given in the table; compare p. 150.

M in μH.	Frequency	ΔM in μH.	σ in Ohms	$\Delta M/M_0$	$\sigma/\omega M$
5,000	1,000	0·7	0·0015	$1{\cdot}3 \times 10^{-4}$	$0{\cdot}5 \times 10^{-4}$
	2,000	2·6	0·007	$5{\cdot}2 \times 10^{-4}$	$1{\cdot}1 \times 10^{-4}$
10,000	1,000	1·3	0·004	$1{\cdot}3 \times 10^{-4}$	$0{\cdot}6 \times 10^{-4}$
	2,000	5·2	0·015	$5{\cdot}2 \times 10^{-4}$	$1{\cdot}2 \times 10^{-4}$

Conditions for Zero Impurity and Frequency Coefficient. Consider a mutual inductance in which the common point has been arranged to make mutual inductance oppose self-inductance (M_0 *negative*). Then the preceding equations show that the frequency coefficient will vanish if

$$C_{12} = \frac{M_0\,(C_1L_1 + C_2L_2)}{(L_1 - M_0)\,(L_2 - M_0)},$$

while the capacitive impurity will be zero if

$$C_{12} = \frac{M_0\,(C_1R_1 + C_2R_2)}{R_1\,(L_2 - M_0) + R_2\,(L_1 - M_0)}.$$

If the first of these conditions be inserted into the expression for σ_c it will be found on simplifying that the result is essentially negative. Hence the capacitive impurity in a mutual inductance which is invariable with frequency is always a negative quantity. The whole impurity, however, contains both capacitance and eddy current terms, and as the latter may be positive or negative it would appear possible to reduce the total impurity to zero. Butterworth suggests that a zero frequency coefficient can be secured by artificial addition to the self or intercapacitances, and a zero impurity by the introduction of artificial eddy losses; he works out appropriate numerical values applicable to a 10 mH. inductometer.

Campbell's M-correcter. A. Campbell* has used a device, related to Butterworth's suggestion, by which the total impurity in a mutual inductor can be reduced to zero and the frequency change of mutual inductance made very small, and has applied it to some important bridge networks. Referring to the diagram in Fig. 64 the mutual inductance M with impurity σ has a common point as shown. A closed loop of resistance ρ and inductance λ is coupled to the primary by mutual inductance m and to the secondary by mutual inductance μ. Writing down equations for the primary and the loop meshes gives

$$(z_1 + \sigma)\mathbf{i}_1 - \sigma\mathbf{i}_2 = \mathbf{e}_1 - j\omega M\mathbf{i}_2 - j\omega m\mathbf{i}_3,$$

$$(\rho + j\omega\lambda)\mathbf{i}_3 + j\omega m\mathbf{i}_1 + j\omega\mu\mathbf{i}_2 = 0,$$

from which is found

$$\left[z_1 + \sigma + \frac{\omega^2 m^2 \mu}{\rho + j\omega\lambda}\right]\mathbf{i}_1 = \mathbf{e}_1 - \left[j\omega M - \sigma + \frac{\omega^2 m\mu}{\rho + j\omega\lambda}\right]\mathbf{i}_2.$$

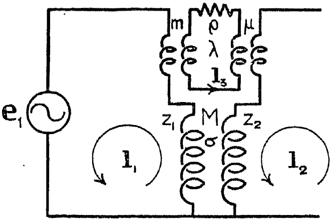

FIG. 64.—CAMPBELL'S M-CORRECTER

Hence the operator for the combination is

$$j\omega M - \sigma + \frac{\omega^2 m\mu}{\rho + j\omega\lambda} = j\omega\left(M - \frac{\omega^2 m\mu\lambda}{\rho^2 + \omega^2\lambda^2}\right) - \left(\sigma - \frac{\omega^2 m\mu\rho}{\rho^2 + \omega^2\lambda^2}\right)$$

Since m and μ may have either sign the impurity term can be made to vanish if $\sigma/\omega^2 m\mu\rho = 1/(\rho^2 + \omega^2\lambda^2)$ at a given frequency; the effective mutual inductance is then $M - \frac{\lambda}{\rho}\sigma$. If M is negative (opposing L) and the impurity σ is positive the effective mutual can be made practically free from error due to the frequency term at a given frequency. If the mutual has a negligible frequency coefficient, the impurity can be made zero at any frequency by the adjustment described, and the mutual will be practically invariable with frequency provided $\omega\lambda$ is small compared with ρ; the impurity adjustment will require alteration to suit each value of frequency.

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[*] A. Campbell, *British Patent*, No. 350,789 (18th June, 1931).

STANDARDS OF CAPACITANCE

The standards of capacitance used in bridge measurements are of two kinds, absolute and secondary. Condensers of the former type are constructed so that their capacitance can be calculated from their accurately measured dimensions. Secondary standards, on the other hand, have their capacitance determined by comparison with an absolute standard or otherwise.

Absolute condensers are not often used in alternating current bridges; their principal application is to determine the ratio of the electromagnetic unit of electricity to the electrostatic unit. For this purpose condensers of quite small capacitance are suitable, and are made of some simple geometrical shape for which the capacitance can be easily calculated. For this reason they have the form of concentric spheres, flat plates or coaxial cylinders, and the dielectric is air.*

All condensers used in alternating current bridges are secondary standards of a capacitance considerably greater than that of the absolute condensers just mentioned. Secondary standards are calibrated and should have the following properties—

(i) They should be true condensers, i.e. the current taken by them when supplied with a sinusoidal p.d. should lead on the p.d. by $\pi/2$ and should be free from harmonics.

(ii) They should be free from losses and absorption effects in the dielectric.

(iii) The capacitance should be definite and permanent, and the standard should be compact for a given value of capacitance so that inaccuracy due to earth capacitances may be very small.

(iv) They should be independent of frequency, wave-form, and temperature.

(v) The insulation resistance should be great and the condensers should be capable of withstanding high voltages.

The condensers used in practice fulfil these conditions in varying degree, according to the way in which they are constructed and to the material used as the dielectric. The dielectrics most frequently employed are air or other gas, oil, mica, treated paper, and glass. In accurate standards the only satisfactory materials are air and mica; paper condensers are sufficiently good for less accurate work or in cases where dielectric imperfections are of little importance. The other materials are only used where capacitance is required without the advantages of permanence, accuracy, and perfection, e.g. tuning condensers, or in filter circuits. Secondary condensers are, therefore, conveniently classified according to the dielectric employed in them, each group having special properties and uses which will now be described.

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* *See*, for example, E. B. Rosa and N. E. Dorsey, *Bull. Bur. Stds.*, Vol. 3, pp. 433–604, 605–622 (1907), for accurate details of all these types and for references. For a cylindrical condenser *see* J. J. Thomson, *Phil. Trans. Roy. Soc.*, Vol. 174, pp. 707–721 (1884); and J. J. Thomson and G. F. C. Searle, *ibid.*, Vol. 181, p. 603 (1891). For a description of various types and the appropriate formulae see G. Zickner, "Absolute Kapazitätsnormale," *Arch. f. tech. Mess.*, Z131–1 (Oct., 1933) and also the book by Campbell and Childs.

13. Air Condensers. Gases, such as air, when used as the dielectric in a condenser subjected to an alternating potential difference are almost entirely free from dielectric losses. A properly constructed air condenser is, therefore, the closest approach to a perfect condenser which can be attained in practice, i.e. one in which the current leads on the p.d. by a quarter period.

Absolute air condensers have usually only a single pair of electrodes, so that their capacitance is very small. To increase the value in secondary air condensers, it is necessary to use several sets of electrodes or plates in parallel, the two sets of plates forming the condenser being held in a suitable framework and separated from one another by supports of solid insulating material. Obviously the amount of solid insulation used to support the plates of an air condenser should be reduced to the minimum, and should stand in a weak electric field. It should have the highest possible insulation resistance; for this purpose ebonite has been much used, but has the disadvantage of yielding in course of time. Amberite,* Pyrex glass and fused quartz are preferable in air condensers of the highest class, as they are more permanent, though these materials are somewhat difficult to work.

Since the dielectric constant of air is low, it follows that a large number of electrodes in parallel must be used, each of large area, to produce a reasonable value of capacitance. An air condenser is, therefore, a very bulky standard, and the bulk is added to on account of two other important facts which impose a limit on the closeness of successive plates. Firstly, the dielectric strength of air is not very high, and, secondly, particles of dust may bridge across between the plates when the condenser is subjected to high voltage. This produces internal leaks and greatly impairs the insulation of the condenser. To reduce this effect it is usual to make the distance between successive plates not less than 2 mm.; if care be taken to dry the air thoroughly, condensers can be made with 1 mm. air space or less.

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* *Amberite.*—A synthetic material composed of amber chips moistened with ether and compressed into a solid mass.

Air condensers are of two classes: (i) in which the capacitance is fixed; (ii) in which the capacitance can be continuously varied. The former serve as fixed value reference standards; the latter as fine adjustments for use in balancing bridges, being frequently used in conjunction with standard mica condensers to carry the capacitance down continuously to the lowest values.

FIXED STANDARDS. Fixed value air condensers are made on the same plan as absolute condensers, but with several sets of plates in parallel to give adequate capacitance. The principal forms are made with cylindrical or with flat electrodes, both types being developed in several different designs.*

An early form of cylindrical condenser is that designed for the Standards Committee of the British Association by Glazebrook and Muirhead.† It consists of 24 concentric brass tubes, 12 forming one electrode and 12 the other. These tubes are mounted upon two stepped brass cones, the lower one standing upon three pillars of ebonite. The upper cone is supported by the case of the condenser. The capacitance is about 0·024 μF.

A later example of a cylindrical condenser is due to Giebe‡ and illustrated in Fig. 65 (*a*). This is composed of seven concentric brass tubes, four forming one electrode and three the other. One set of cylinders is fixed between the plates *AA* upon brass blocks, three projecting from each plate. The other set of cylinders is mounted in a similar way between the plates *BB*. The plates *BB* and the cylinders are slotted out to clear the projections from *AA*, upon which the first set of cylinders is mounted. The plates *AA*, *BB* are insulated from one another by corrugated ebonite cylinders *E*, the whole being clamped solid by the spindle and nut *S*. The lower plate *A* stands on the metal base of the condenser, which, in turn, is supported on ebonite feet. A metal case surrounds

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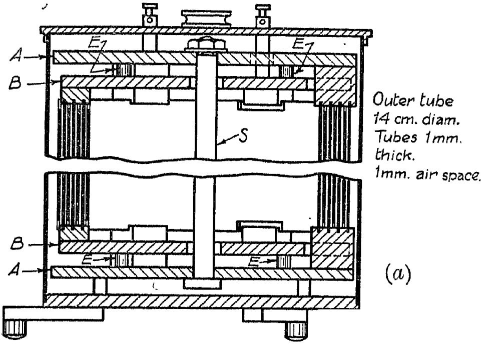
* G. Zickner, "Normal-Luftkondensatoren (unveränderbare Modelle)," *Arch. f. tech. Mess.*, Z131–3 (June, 1934), for a good summary.

† R. T. Glazebrook, "On the air condensers of the British Association," *Elecn.*, Vol. 25, pp. 616–619, 637–640 (1890).

‡ E. Giebe, "Normal Luftkondensatoren und ihre absolute Messung," *Zeits. f. Inst.*, Vol. 29, pp. 269–279, 301–315 (1909).



Giebe's cylindrical air condenser

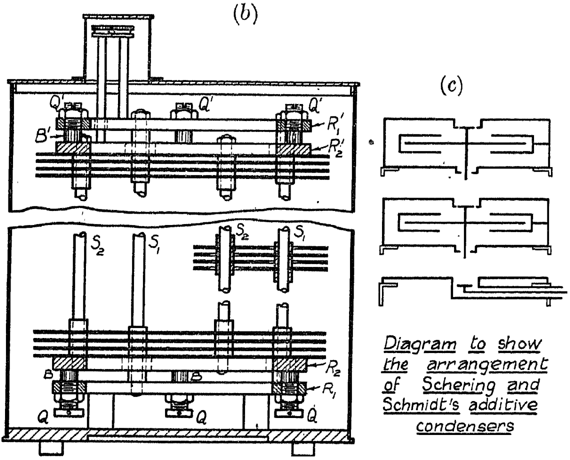


Diagram to show the arrangement of Schering and Schmidt's additive condensers

Giebe's plate air condenser

FIG. 65.—PRECISION AIR CONDENSERS

the cylinders and is connected to A, thus shielding the condenser from external electrostatic influences. With the dimensions given in the diagram, the capacitance is about $10{,}900\mu\mu$F.

Flat plate condensers of various forms are widely used. In a simple type the electrodes consist of square sheets of plate glass, each plate being coated on both sides with tin-foil. The glass sheets are arranged in a pile with small spacing pieces of glass at the corners to insulate successive plates from one another. The glass plates serve merely as convenient supports for the tin-foil coatings, the dielectric being the air between successive plates. The tin-foils on alternate plates are connected together to form the two sets of electrodes.*

In another type, due to Lord Kelvin,† 45 brass plates, each 10·13 cm. square, are prepared; 22 form one electrode and 23 the other. The plates of each set are suitably supported on metal rods, the second set of plates being at 45° to the first. With an air space of 3 mm. between successive plates the capacitance is about $0{\cdot}001\mu$F. A condenser of a similar type constructed by Messrs. H. W. Sullivan is shown in Fig. 66 (*a*). One system of plates is joined to the shield, the other being insulated by quartz; the capacitance is $1{,}000\mu\mu$F. Condensers on this principle have been used for many years at the Bureau of Standards. Fig. 66 (*b*) shows a $500\mu\mu$F. condenser by Messrs. H. Tinsley, with shaped plates; the shield is removed to show the interval arrangement.

Giebe‡ has constructed a precision flat plate condenser in which the plates are circular instead of square. Referring to Fig. 65 (*b*), the construction can be readily made out. The plates are of magnalium,§ 71 in number, each 1 mm. thick and 20 cm. diameter; 35 form one electrode and 36 the other, with a space of 2 mm. between successive plates. A bronze ring R_1 is fixed to the base of the condenser and carries four adjusting screws which support a second bronze ring R_2 upon short cylinders of amber, B. Four equally spaced brass rods, S_1, are screwed into R_1 and pass through clearance holes in R_2. Four other rods, S_2, midway between S_1 are fixed to R_2.

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* For a variety of simple condensers of this type, with and without guard rings, *see* W. E. Ayrton and T. Mather, *Practical Electricity*.

† Lord Kelvin, *Proc. Roy. Soc.*, Vol. 52, pp. 6–10 (1892–3); *see* also *Dictionary of Applied Physics*, Vol. 2, pp. 655–656.

‡ E. Giebe, *loc. cit.*

§ *Magnalium.*—A light, strong alloy of aluminium and magnesium.

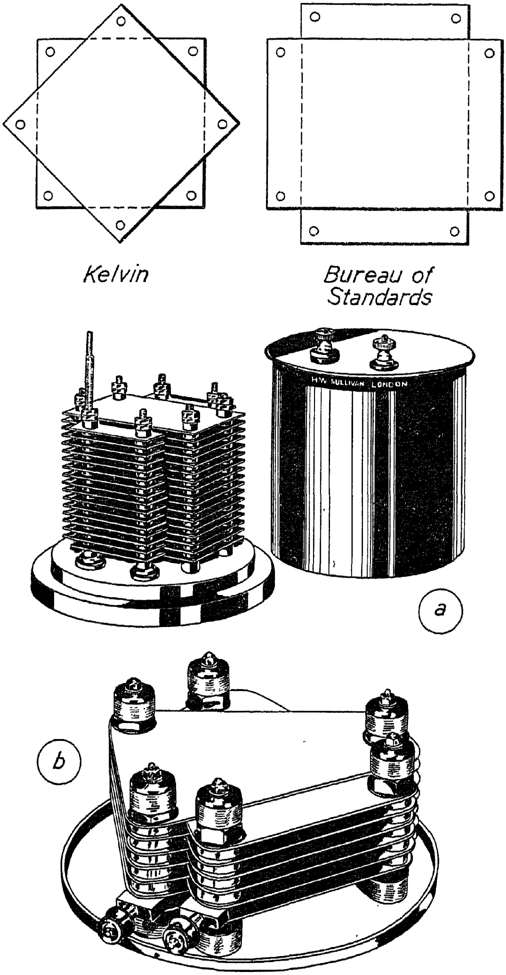

FIG. 66.—EXAMPLES OF CROSSED-PLATE CONDENSERS

The condenser plates are provided with eight holes, four being 5 mm. diameter and four 12 mm. diameter, so spaced that the plates can be dropped over the rods. The condenser is assembled in the following way: a plate is put on the rods to rest on R_2, the rods S_2 passing through the 5 mm. holes. Distance tubes of a length to give 2 mm. air space are put on S_1, and a plate of the second electrode put on the rods with the 5 mm. holes on S_1. Distance tubes 5 mm. long are then slipped on S_2, and a second plate of the first set put on. This procedure is carried out until the whole condenser is assembled. A bronze ring, R_2' is then put on the holes in it fitting S_2 and clearing S_1, nuts on the top of S_2 clamping the set of plates between R_2 and R_2'. Suitable distance tubes are then put on S_1 and a ring R_1' added, the second set of plates being clamped between this and R_1 by nuts on S_1. The air gaps between the two sets of plates are adjusted by moving the ring R_2 with its plates by means of the screws Q; the two sets of plates are clamped together by the screws Q' insulated by amber rods B'. Terminals are attached to R_1' R_2', the whole condenser being surrounded by a case joined to the ring R_1 and its associated plates. The condenser is about 30 cm. high, weighs 19 kilogrammes, and has a capacitance of about 10,000$\mu\mu$F. It will break down at a voltage of 3,000.

In a second type, Giebe uses 107 plates 30 cm. diameter with 1 mm. air gaps. The breakdown voltage in this case is 900 and the capacitance 30,000$\mu\mu$F.

The insulation resistance of condensers constructed on this plan is very high, being of the order of 10^9 to 10^{10} ohms. If care be taken to remove dust in the condenser with 1 mm. gaps, and to dry the air by means of metallic sodium, the insulation resistance can be raised to 10^{15} ohms. In illustration, such condensers, when charged at 120 volts, lose only 5 per cent of their charge in 8 days.*

Giebe's condensers are very permanent in value, and his later researches (*see* p. 187) have shown them to be practically perfect capacitances. They have a small temperature coefficient, of the order of 2 or 3 parts in 100,000 per degree.

In much standardizing work, units of larger capacitance are frequently required, and can be obtained by connecting several air condensers in parallel. As the total capacitance is not very large, the unknown capacitance of the leads used to connect the component condensers in parallel may cause an appreciable error in their combined value. In order that air condensers may be paralleled and have a definite capacitance equal to the sum of their separate values, Schering and Schmidt* have devised the principle illustrated in Fig. 65 (*c*). Their condensers are built on the plan of Giebe's flat plate instruments, but differ in the arrangement of the terminals. A baseplate, forming one terminal of the assembled condenser, carries at its centre an insulated metal socket forming the second terminal. The condenser units contain two sets of plates, one connected to the case and the other insulated therefrom; the insulated set bears a central socket above and a central spring spigot below. Any condenser can be piled on top of any other or on the base, the spigot pressing firmly in the socket; to ensure central registration of each unit, the upper portion of each case is machined to fit interchangeably a recess in the base of the one above it.

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* For an investigation of the insulation of condensers of all kinds, expressed in terms of the time-constant CR, *see* E. Horst, "Über Zeitkonstanten von Kondensatoren," *Arch. f. Elekt.*, Vol. 31, pp. 273–281, 827–831 (1937).

Their condensers had 20 cm. diameter magnalium plates with capacitances of 0·005μF. (37 plates), 0·002 (15 plates), and two of 0·001 (9 plates). The air spaces were 5 mm. in the first two condensers and 5·8 mm. in the second pair. Tests were made by the Maxwell commutator method to measure the capacitances of the separate condensers and their combined values when assembled in various ways, as follows—

Condenser .	*A*	*B*	*C*	*D* .
Measured value	0·0010001	0·0010001	0·0019997	0·0050002
Combination .	$A + C$	$B + C$	$A + B + C$	$A + B + C + D$
Measured value	0·0029994	0·0029994	0·0039990	0·0090010
Calculated value	0·0029998	0·0029998	0·0039999	0·0090001

Plate condensers with a definite parallel-connected capacitance have been made by Mr. Paul on a principle similar to that of Schering and Schmidt.

A further range of precision air condensers has been designed by Giebe† and his associates at the Reichsanstalt, in decades from 1 to $10 \times 10\mu\mu$F. up to 1 to $10 \times 10^4\mu\mu$F.; quartz insulation is used, and some novel methods of construction are adopted. A typical arrangement of a condenser in the $10^2\mu\mu$F. decade is shown in Fig. 67 (*a*), the example being the $8 \times 10^2\mu\mu$F. unit with 13 aluminium plates. Eleven

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* H. Schering and R. Schmidt, "Ein Satz Normal-Luftkondensatoren mit definierter Schaltungskapazität," *Zeits. f. Inst.*, Vol. 32, pp. 253–258 (1912).

† E. Giebe and E. Alberti, "Absolute Messung der Frequenz elektrischer Schwingungen. Ein Normalfrequenz-oder Wellenmesser," *Zeits. f. tech. Phys.*, Vol. 6, pp. 92–103, 135–145 (1925); E. Giebe and G. Zickner, "Über die Kapazitätsnormale der Physikalisch-Technischen Reichsanstalt," *Zeits. f. Inst.*, Vol. 53, pp. 1–12, 49–56, 97–108 (1933).

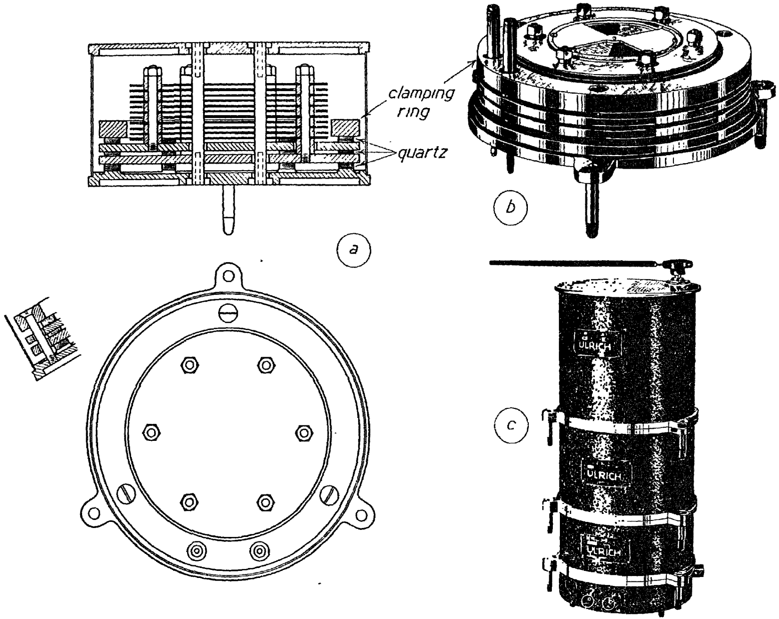

FIG. 67.—REICHSANSTALT ADDITIVE AIR CONDENSERS

of these are 120 mm. diameter and 1 mm. thick, the lowest two being 158 mm. diameter and 5 mm. thick; the air space is 2 mm. The method of construction is self-explanatory in the diagram. For units of 2, 3 . . . $10 \times 10^2 \mu\mu$F. the numbers of plates are 2, 4, 5, 7, 9, 13 and 17 respectively. Accurate adjustment of the capacitance is done by cutting two sectorial areas from the upper plate and rotating a butterfly-shaped plate to an appropriate position over these openings, as shown in the external view, Fig. 67 (*b*). The $2 \times 10^2 \mu\mu$F. unit consists only of the two thick plates and clamping ring; the $1 \times 10^2 \mu\mu$F. unit is similar but the spacing is about 5 mm. In all cases both sets of electrodes are insulated from the casing by quartz rods 5 mm. diameter and 2 mm. thick.

Condensers in the 1 to $10 \times 10 \mu\mu$F. decade are similarly constructed, but, instead of plates, the electrodes are rings 158 mm. outside, 118 mm. inside, spaced 5 mm. apart. From 6 to $10 \mu\mu$F. appropriate numbers of complete rings are used; the $5 \times 10 \mu\mu$F. has a single pair of rings. The smaller units contain segmental rings subtending angles of 270°, 180°, and 90° in the 4, 3, and $2 \times 10 \mu\mu$F. sizes.

Condensers of the 1 to $10 \times 10^3 \mu\mu$F. decade contain plates of cast copper-aluminium alloy 3 mm. thick having a diameter equal to that of the clamping ring, 158 mm. with the assembly bolts passing clear through the latter; the spacing is 1 mm. Otherwise they are similar to the 10^2 decade illustrated in Fig. 67.

All the condensers of the 10, 10^2, and 10^3 series may be stacked to add their capacitances, a suitable terminal base being provided; connection is made between the units by double-ended "banana" connectors. Since both sets of electrodes are insulated, they may be so used; or either may be joined to the case. Similarly-constructed rotary variable condensers, described on p. 175, may be combined with the stack of fixed units to give continuous variation; an example of a complete set is shown in Fig. 67 (*c*),* consisting of base, two fixed units, and a variable unit surmounting the whole.

The largest condensers have values of 1, 3, 5, and $10 \times 10^4 \mu\mu$F. The 1 and 3 units are those described on p. 168 rebuilt with quartz insulation. The others are similarly constructed, the 5 unit with 157 plates 300 mm. diameter at 2 mm. spacing, the 10 unit having 122 plates 350 mm. diameter at 1 mm. spacing. This series will not stack with the others.

VARIABLE STANDARDS. A continuous variation of capacitance is frequently required in bridge work, and is usually provided by a variable air condenser. In its commonest form, it consists of a set of fixed plates between which a set of moving plates can be passed, so that the active area of the condenser is continuously variable. The two sets of plates are generally approximately semicircular, as in Fig. 68, the moving set being rotated about a central axis so as to be more or less interposed within the fixed set. By proper proportioning of the plates, the changes of capacitance can be made over a wide range nearly proportional to the angle turned through by the moving plates, as the calibration curve shows.

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* These condensers are made commercially by several German manufacturers, e.g. W. Meyerling of Charlottenburg, Spindler and Hoyer of Göttingen, O. Selinger of Berlin, and Max Ulrich of Leipzig. Illustrations have been made from material supplied by Dr. Ulrich.

In the commonest construction, semicircular plates of sheet aluminium are used, set on rods with distance pieces between the plates so that they are properly spaced out. The rods supporting the fixed plates are fastened to the ebonite cover of the condenser; the moving plates are assembled on the central spindle which passes through a bearing in the cover and bears an ebonite knob by means of which the plates can be rotated. The entire condenser is enclosed in a metal case or in a glass vessel lined with tin-foil, serving to shield the condenser from external electrostatic influences. In practice, it is best to join the moving plates to the case. (*See* page 184.)

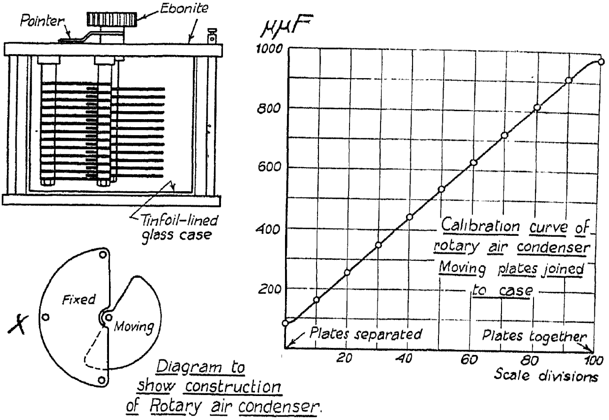

FIG. 68.—VARIABLE AIR CONDENSER AND CALIBRATION CURVE

This construction has many disadvantages. In course of time the upper bearing may wear and the moving plates descend somewhat, thus altering the value of the capacitance. In the best types, therefore, the shaft is supported in bearings at both ends, or, alternatively, in a properly fitted conical bearing at the upper end. The shaft is continued a sufficient distance through the cover, so that the presence of the hand on the operating knob does not appreciably affect the capacitance. Connection is made with the moving plates by a flat spiral spring, one end being soldered to the shaft and the other to a terminal fixed on the ebonite cover. The dielectric losses will be high since far too much solid insulating material, and that not of the best kind, is used in the construction.

In modern condensers the plates are made from aluminium, duralumin or brass sheet, or are machined out of the solid from castings in the manner introduced by Seibt.* A stout construction is thus secured with air spaces of definite dimensions. The whole case, including the cover, is of metal, forming an effective shield. The fixed system is insulated from the case by short pillars or tubes of high grade insulating material, such as quartz, the minimum amount of solid insulation being used and situated in the weakest part of the electric field. The capacitance is indicated by a pointer attached to the central shaft moving over a scale attached to the cover. In the most accurate instruments the pointer carries a vernier, and a slow motion adjustment is provided by gearing the operating knob to the central shaft instead of attaching it directly thereto.† The scale is usually one of degrees and is calibrated. Such rotary condensers are made in various sizes, up to 3,000 $\mu\mu$F., a very usual size being 1,000 $\mu\mu$F.

In standard condensers of the highest precision every precaution is taken to reduce the solid dielectric material, with its accompanying losses, to a minimum, and to ensure permanence of calibration by sound mechanical construction; it will be interesting briefly to examine a few examples in which these and other necessary conditions are satisfied.

The National Physical Laboratory pattern,‡ designed by Tomlinson and made by Messrs. H. Tinsley & Co., is shown diagrammatically in Fig. 69 (*a*). The moving plates are semicircular and are keyed to a horizontal shaft; since the plates are vertical, they are free from distortion due to bending stresses. A large disc, calibrated on its edge in hundreds of $\mu\mu$F., carries lead weights to counterbalance the moving plates. The shaft is mounted in split bronze bearings supported by stiff brackets, one being provided with a spring and thrust screw, by

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* G. Seibt, *Jahrb. d. d. Tel.*, Vol. 5, p. 407 (1911); *Dictionary of Applied Physics*, Vol. 2, pp. 656–657.

† *See* Schering and Schmidt, *loc. cit.*, for a variable condenser of excellent design with a range from 0·0001 to 0·002 μF.

‡ "Standard variable air condenser," *N.P.L. Rep. for* 1924, pp. 84–85 (1925). For a general review see G. Zickner, "Normal-Luftkondensatoren (Regelbare Modelle)," *Arch. f. tech. Mess.*, Z 135–1 (May, 1934).

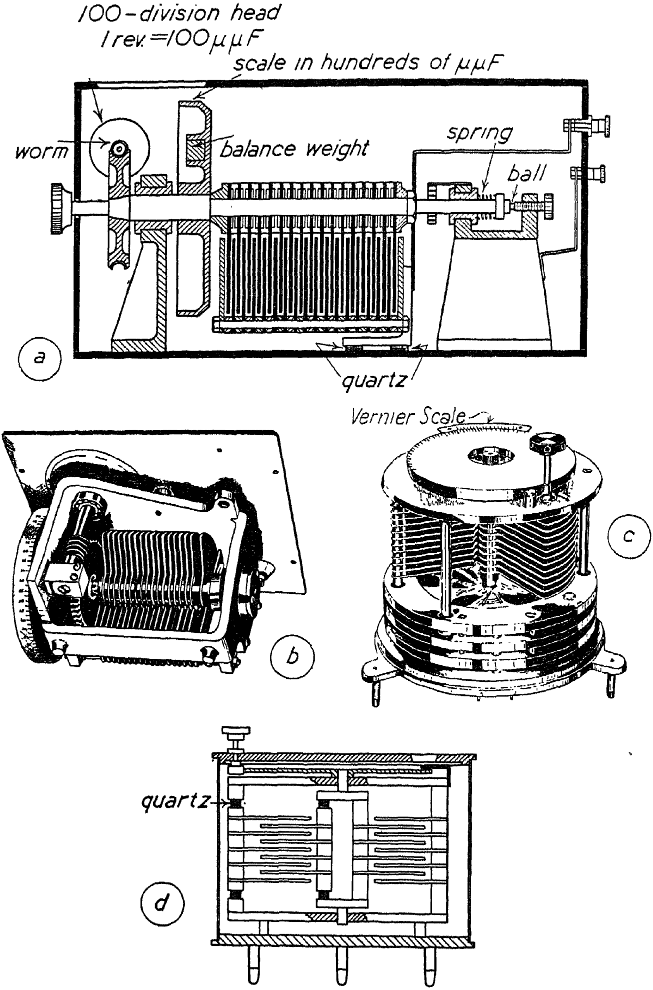

FIG. 69.—PRECISION VARIABLE AIR CONDENSERS

means of which the moving system may be correctly located in an axial direction at the position of minimum capacitance. The fixed plates are also semicircular, clamped between ring castings at their rims and supported by brackets. Each of these is insulated from the base plate by amberite or quartz washers. Rough setting is accomplished by a knob on the shaft; fine setting is obtained by a double-thread worm and wheel, the worm shaft having a head divided into 100 parts. A cam is provided to put the worm out of gear when rough settings are made. The end plate of the moving system is shaped so that one turn of the worm gives a change of $100\mu\mu$F.; hundreds are read on the disc, tens and units on the head. The range of reading is from 200 to $3{,}000\mu\mu$F. The entire condenser is contained in a metal case, an insulated terminal being provided for the fixed system; a second terminal on the case is connected to the moving system through a flat-coil bronze spring.

A somewhat similar principle is employed in the Type 722 precision condenser of the General Radio Co.,* illustrated removed from its metal case in Fig. 69 (*b*). Three sizes are made, with ranges of 25 to 100 $\mu\mu$F., 45 to 500 $\mu\mu$F., and 100 to $1100\mu\mu$F. respectively. Insulation is either isolantite or quartz.

The Reichsanstalt have a series of variable condensers intended for use with the fixed-value condensers described on p. 171. A typical example made by Max Ulrich is shown in Fig. 69 (*c*) and is constructed like the fixed condenser in Fig. 67; the axis of the moving system is vertical. Quartz rods insulate both systems from the frame and case (removed in the drawing). Four sizes are made, with maxima of 130, 210, 350, and $580\mu\mu$F. Two smaller sizes with maxima of 26 and 68 are also constructed. In these the fixed system consists of a single pair of quartz-insulated plates between which a single vane is moved, this being in permanent connection with the frame (see also Fig. 119 on p. 343). All these condensers have air gaps of 2 mm. and a uniform case diameter of 165 mm.; they may be stacked upon the fixed condensers previously described. In addition, large condensers up to 5,100 $\mu\mu$F. are built on the Marconi double-plate system shown diagrammatically in Fig. 69 (*d*).

High-precision condensers are a speciality of Messrs. H. Sullivan, who construct several types based on the designs of Griffiths.† In an ordinary design of condenser the air gaps on the two sides of a moving plate are in parallel, Fig. 70 (*a*). If such a condenser is to retain its calibration over a long period, the greatest care must be taken to ensure that the plates of each system are perfectly parallel and that the moving system rotates truly and exactly midway between the fixed system. The capacitance is proportional to

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* R. F. Field, "Increased accuracy with the precision condenser," *G.R. Exp.*, Vol. 10, pp. 5–6 (Oct, 1935). "A new precision air condenser," *ibid.*, Vol. 10, pp. 1–3 (Jan. 1936). "Building precision into an air condenser," *ibid.*, Vol. 11, pp. 1–6 (Feb. 1937).

† W. H. F. Griffiths, "The accuracy and calibration permanence of variable air condensers for precision wavemeters, *Exp. W. and W. Eng.*, Vol. 5, pp. 17–24, 63–74 (1928). "Further notes on the calibration permanence and overall accuracy of the series-gap precision variable air condenser," *ibid.*, Vol. 6, pp. 23–30, 70–80 (1929). "A new precision variable air condenser," *Journal Sci. Insts.*, Vol. 6, pp. 297–302 (1929).

$$(1/d_1) + (1/d_2) \propto (d_1 + d_2)/d_1d_2 \propto 1/d_1d_2$$

since the sum of the gaps remains constant. Hence if either gap varies slightly through axial play, tilt of the axis of rotation, or wear of bearings there is a marked effect on the permanence of calibration; the effect is least when the gaps are equal. In the principle due to Griffiths, Fig. 70 (*b*), two fixed systems insulated from one another form the two electrodes; the capacitance is varied by interposing between them an insulated moving system. It will be seen that the air gaps are in series, the reciprocal of the total capacitance being proportional to

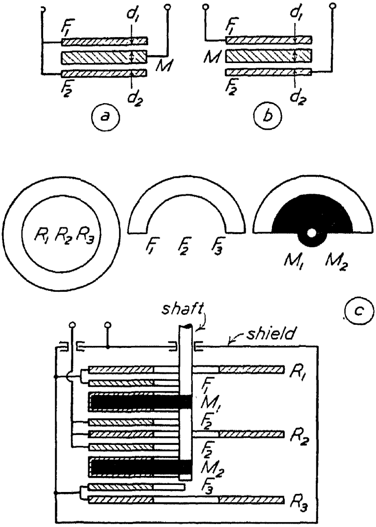

FIG. 70.—THE SERIES-GAP CONDENSER

$$[1/(1/d_1)] + [1/(1/d_2)] \propto d_1 + d_2$$

which is constant. Hence the capacitance is constant no matter how the gaps vary. Fig. 70 (*c*) shows how the principle is applied in a multi-section design, rings R_1, R_2, R_3 being provided to eliminate all parasitic capacitances to the moving plates which, remaining constant despite the relative movement of the fixed and moving systems, tend to prevent the necessary changes in the capacitances of adjacent air gaps. The original papers should be consulted for complete constructional details.

Substandard variable condensers for ordinary bridge balancing purposes are constructed in a great variety of ways, which the reader may study in the catalogues of the well-known instrument makers. Almost always they are of multi-plate design with a vertical axis of rotation, a fixed system insulated by small amounts of good solid insulation, and an uninsulated moving system; the whole is surrounded by a metal shield. Beyond differences of a purely mechanical nature, no new principles are involved.

Variable condensers are usually of the rotary pattern, though other types have been suggested. Lord Kelvin, for example, has described a form in which concentric cylinders are arranged to slide one within the other, and rotary cylindrical designs have also been proposed. Briggs* has made a condenser in which the air spaces are altered in thickness, but neither of these types is so convenient as the rotary pattern.

In some classes of work it is necessary to have variable air condensers of a lower maximum value, say 50 $\mu\mu$F., in order to balance very small capacitance changes in a bridge. Oliver† has designed some shielded low value condensers to give either the smallest minimum capacitance or the largest ratio of maximum to minimum reading. The former condition is illustrated by a condenser with a minimum of 3·5 $\mu\mu$F. and a variation of 20 $\mu\mu$F., while the second condition is attained in a condenser with a variation of 60 $\mu\mu$F. and a minimum of 5·3 $\mu\mu$F. In another design a minimum of 9·1 $\mu\mu$F., a maximum of 67·5 $\mu\mu$F., and a ratio of 7 to 1 is obtained. It was found experimentally that the lowest minimum was secured with the moving system insulated below the top bearing. Accordingly, the fixed system is the low voltage electrode and is attached directly to the brass cover of the metal case; the moving system is insulated below the bearing by a fused quartz rod. Connection is taken from the moving plates by a bronze spring to the high voltage terminal, which is mounted upon a fused quartz washer let into a hole in the cover. For a German design see also p. 343.

A further method of obtaining fine adjustment of capacitance in a bridge network is a device due to Hartshorn.‡ This consists in connecting two air condensers in series, the capacitance of one, C, being large in comparison with that of the other c. If K be the capacitance

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* L. J. Briggs, "A new form of electrical condenser having a capacity capable of continuous adjustment," *Phys. Rev.*, Vol. 11, pp. 14–21 (1900).

† D. A. Oliver, "Small standard air condensers of low minima," *Journal Sci. Insts.*, Vol. 4, pp. 65–71 (1926). "Quartz-metal joints and their application to standard air condensers of low range," *ibid.*, Vol. 5, pp. 9–14 (1928). For other designs of low value condensers with very small or zero initial capacitance see L. Bainbridge-Bell, "A variable capacitive coupling capable of reduction to zero," *Journal Sci. Insts.*, Vol. 7, pp. 162–164 (1930). K. Kuhlmann, "Messkondensator mit einer von exakt Null linear ansteigenden Kapazität," *Arch. f. Elekt.*, Vol. 25, pp. 666–668 (1931). A. C. Bartlett, "A standard of small capacity differences," *Journal Sci. Insts.*, Vol. 8, pp. 260–262 (1931).

‡ L. Hartshorn, "A method of measuring very small capacities," *Proc. Phys. Soc.*, Vol. 36, pp. 399–403 (1924).

The constructional difficulties inherent in the flat plate design, and the difficulty of ensuring perfect shielding of the l.v. electrode, make this type of condenser inferior to the cylindrical type, several examples of which are shown in Fig. 71. Monasch,[*] in 1907, used a variable condenser for 10 kV. consisting of two concentric brass tubes about a metre long, all parts being well-rounded to avoid brush discharges. Petersen[†] in 1911, suggested the use of shielded cylindrical condensers with external h.v. electrode, within which is a l.v. cylinder provided with guard cylinders of the same diameter. A condenser in this form is free from concentration of electric stress likely to lead to brush discharge, while the capacitance is easily calculated. Moreover, the central electrode has a low and definite earth capacitance, which is an important matter in a.c. bridges. If l be the active length of the l.v. electrode in cm., i.e. its actual length plus a small correction for fringing of the electric flux near the gaps separating it from the guard cylinders, which are kept at the same potential as the l.v. cylinder, the capacitance is $C = l/2 \log (D/d)$ cm., D being the internal diameter of the outer cylinder and d diameter of the inner electrode. If now $D/d = \varepsilon$, where ε is the Naperian base (2·718) then $C = l/2$ cm. or $l/1{\cdot}8\ \mu\mu$F. Moreover, with these proportions the condenser will withstand the maximum voltage for a given strength of field on the inner cylinder.

Chubb and Fortescue,[‡] and later Whitehead and Isshiki,[§] have designed condensers on the Petersen plan, but with the l.v. electrode and guard rings outside. This has the defect of much greater earth capacitance, with consequent dependence on surrounding objects and their potentials; these workers, therefore, enclose their condenser in an earthed gauze shield to minimize these effects.

Semm,[||] in 1921, used, at the Reichanstalt, a true Petersen condenser, constructed of metal tubes fitted with spun metal bell ends and beading. His condenser is suitable for 100 kV. Numerous other workers have developed designs for a variety of voltages, following more or less closely the Petersen principle. Thus Rayner,[¶] at the N.P.L., and Churcher and Dannatt,[**] at the Metropolitan-Vickers Co., have described condensers for 30 kV. and 150 kV. respectively. The last-named workers have shown that there are considerable constructional advantages in mounting the condenser with its axis vertical; the heavy electrodes are more easily supported in the vertical position, and there is also greater economy of floor space in comparison with that required for the horizontal arrangement.

The following table gives data for a number of cylindrical condensers, the references being to the papers cited. The example designated A.C.E.C.* is in use at Charleroi ; while that marked C.I. Co. is from data provided by the Cambridge Instrument Co. Horizontal mounting is denoted by H and vertical by V.†

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[*] B. Monasch, *Ann. der Phys.*, Vol. 22, pp. 905–942 (1907).

[†] W. Petersen, *Hochspannungstechnik*, p. 92, 104 (1911).

[‡] L. W. Chubb and C. Fortescue, *Trans. Amer. I.E.E.*, Vol. 32, pp. 739–748 (1913).

[§] J. B. Whitehead and T. Isshiki, *Trans. Amer. I.E.E.*, Vol. 39, pp. 1057–1110 (1920).

[||] A. Semm, *Arch. f. Elekt.*, Vol. 9, pp. 30–34 (1921). See also H. S. Hallo and G. de Zoeten, *Tijd. v. Elect.*, Vol. 7, pp. 27–31, 49–54 (1925).

[¶] E. H. Rayner, "The design and use of an air condenser for high voltages," *Journal Sci. Insts.*, Vol. 3, pp. 33–38, 70–77, 104–106 (1926).

[**] B. G. Churcher and C. Dannatt, "The use of the Schering bridge at 150 kilovolts," *World Power*, Vol. 5, pp. 238–247 (1926).

	D in cm.	*d* in cm.	*D*/*d*	*l* cm.	*C* in $\mu\mu$F.	H.V. Electrode	Voltage
Monasch . . V.	11	9	1·222	100	230	Outside	10 kV.
Rayner . . H.	Not stated	10	—	60	56	Outside	30 kV.
Semm . . H.	60	22	2·730	100	55·6	Outside	100 kV.
C.I. Co. . . V.	60	30·5	1·968	122	100	Outside	100 kV.
Churcher and Dannatt . V.	83·9	38·1	2·200	150	100	Outside	150 kV.
Whitehead and Isshiki . H.	49·3	29·5	1·670	76·2	82·6	Inside	200 kV.
Chubb and Fortescue . H.	162·8	60	2·718	47·7	26·6	Inside	300 kV.
A.C.E.C. . . V.	117	39	3·000	144	80	Inside	300 kV.

The choice of a standard impedance for use in high-voltage bridges lies between a resistor and a condenser. Resistors have been constructed for use up to 132 kilovolts, but involve considerable elaboration to keep the residual phase-angle due to capacitance effects down to a minimum; moreover, they dissipate large amounts of energy as heat. There would, appear to be a limit to the voltage for which such high-voltage resistors could satisfactorily be constructed, as pointed out on p. 374 of the author's book *Instrument Transformers*. In all these particulars the air condenser shows marked advantages. Its construction is simple, it can easily be shielded from extraneous capacitance effects, heating is absent and there is no limit to the voltage for which such condensers may be built, provided only that the necessary space is available to accommodate their considerable bulk.

The primary essentials of a standard high-voltage condenser are accuracy and permanence of capacitance, and freedom from losses with consequent "impurity." The condenser should be arranged so that the current in the l.v. lead is exactly in quadrature with the p.d. between the h.v. and l.v. electrodes; if this is not the case, the condenser is said to be impure, the impurity being measured by the phase-defect from exact quadrature. Investigations‡ have recently been made to determine the factors upon which impurity depends and the conditions necessary to reduce it to a negligible amount. The purity and constancy of capacitance are influenced principally by shielding, humidity and surface dirt. All these factors are considered by Churcher and Dannatt, using an experimental condenser of shielded flat-plate type in which voltage gradients up to 14 kV./cm. at 60 cycles per sec. could be obtained, the relative humidity being carried up to 90 per cent. Kouwenhoven and Lemmon worked at lower gradients, up to 4 kV./cm., confining their attention chiefly to humidity effects up to 95 per cent, though briefly referring to the influence of dust and of atmospheric ionization. Temperature changes influence the capacitance only, the power-factor being unchanged, the alteration being that calculable from the expansion of the electrodes.

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* *Rev. A.C.E.C.*, No. 121, pp. 1–11 (1929).

† For a further example of a vertical condenser for 90 kV. see H. W. Bousman, *Gen. Elec. Rev.*, Vol. 35, pp. 295–298 (1932). The capacitance is 98$\mu\mu$F. and the h.v. electrode is outside.

‡ W. B. Kouwenhoven and C. L. Lemmon, "Phase defect angle of an air capacitor," *Journal Amer. I.E.E.*, Vol. 49, pp. 945–948 (1930). B. G. Churcher and C. Dannatt, "The use of air condensers as high-voltage standards," *Journal I.E.E.*, Vol. 69, pp. 1019–1027 (1931).

It is necessary to provide the l.v. electrode with a guard ring, the electrode and l.v. lead being adequately shielded from capacitance effects with respect to surrounding objects. It is also essential to provide large flares at the electrode extremities and to maintain a parallel portion of the electrodes over the guard sections to ensure freedom from trouble with increasing voltages.

It is found that for any humidity and dust content the capacitance varied with voltage gradient by only a few parts in 100,000. It is, however, clearly established that the deposit of dust on the electrodes is the initial cause of impurity, the variation of power factor with humidity being very small with clean electrodes but quite marked when the electrodes are dusty. The impurity is due to energy loss associated with dust particles set in motion by the electric field, and is likely to be less as the distance apart of the electrodes is increased. Churcher and Dannatt also show that coating the electrodes with ordinary lead-base paint gives an appreciable power-factor; this is not the case when cellulose paint, which is conducting, is used. Cellulose paint is to be preferred, therefore, as a means of protecting the surfaces of the electrodes.

Churcher and Dannatt conclude that a properly designed air condenser is a highly satisfactory standard of impedance for use at high voltages, provided proper care is taken to maintain cleanliness of the surfaces and to avoid high humidities; the shielding necessary is also of the simplest kind. An accuracy of capacitance of 1 in 10^4 can be secured, independent of voltage up to the point of breakdown and invariable with frequency. On the basis of their conclusions they have designed two new vertical cylindrical condensers of the type shown in Fig. 71, but with the high voltage electrode inside, one for 75 kV. (R.M.S.) and the other for 300 kV. (R.M.S.); these have proved quite satisfactory in practice. Their paper should be consulted for complete details of the design and construction of these high precision condensers.

As the preceding discussion shows, there is considerable difficulty in keeping the dimensions of an air condenser for high voltage within reasonable limits, on account of the large clearances required to avoid dielectric breakdown and losses. It has long been known, however, that the dielectric strength of a gas increases proportionally to the pressure and, at the same time, brush discharge and consequent losses at high voltage are very small; thus air at a pressure of 10 atmospheres has a dielectric strength about equal to that of oil. Condensers with compressed gas dielectric have been used much in radio-telegraphy* and have been applied to bridge measurements in the course of the last few years.

Compressed gas condensers are of concentric cylindrical construction. Atkinson† used such a condenser with carbon dioxide under pressure of 100 lb. per sq. in., suitable for 50 kV., with capacitance of 100 $\mu\mu$F. The Cambridge Instrument Co. make a similar condenser for 120 kV., containing carbon dioxide or nitrogen at a pressure of 10 to 12 atmospheres. Schering and Vieweg‡ have constructed a condenser for the Reichsanstalt, illustrated in Fig. 72, which may be taken

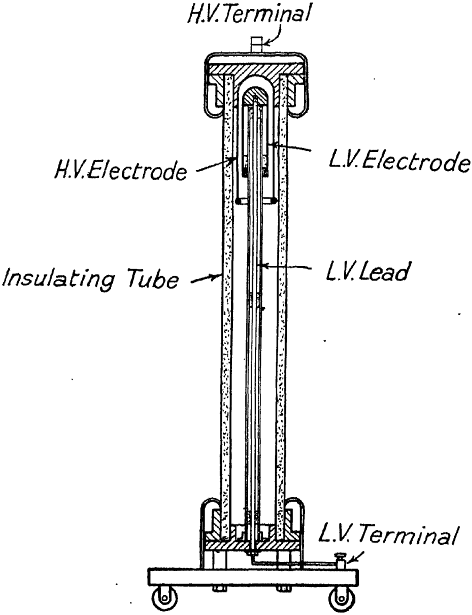

FIG. 72.—SCHERING AND VIEWEG'S COMPRESSED GAS CONDENSER

as a typical example.* The body of the condenser consists of a compressed paper tube provided at each end with substantial steel flanges and lids, by means of which it can be closed and made gas-tight. The high voltage electrode is tubular and is attached to the upper lid; the low voltage electrode is a concentric cylinder supported on a lead passing through the lower lid and insulated therefrom. The l.v. lead is shielded against electrostatic disturbances by a concentric metal tube which, along with the lower cover is earthed; a small gap between the low-voltage electrode and the earthed tube serves as a protective measure against excessive rise in potential of the l.v. cylinder. The upper and lower flanges are covered with well-rounded shields to minimize possibility of glow discharge. The gas is nitrogen or carbon dioxide at 12–14 atmospheres; loss of pressure is less than 1 atmosphere per month. The capacitance of the condenser is about 54 $\mu\mu$F. (49 cm.), the working voltage 250 kV., the overall height 225 cm., and the floor space 92 cm. square, thus occupying much less space than a condenser for the same capacitance and voltage using air at atmospheric pressure.

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* See F. Laws, *Electrical Measurements*, pp. 362–363 (1938); and *Dictionary of Applied Physics*, Vol. 2, p. 123 (1922).

† R. W. Atkinson, *Elect. J.*, Vol. 22, pp. 58–66 (1925). For a condenser of 500$\mu\mu$F. for 25 kV. containing air at 10 atmospheres, see W. B. Kouwenhoven and L. J. Berberich, "A standard of low power factor," *Trans. Amer. I.E.E.*, Vol. 52, pp. 521–527 (1933).

‡ H. Schering and R. Vieweg, "Eine Messkondensator für Höchstspannungen," *Zeits. f. Tech. Phys.*, Vol. 9, pp. 442–445 (1928).

Compressed gas condensers are compact and portable, constant in capacitance, completely shielded, and quite free from losses. The main structural difficulty is the provision of a gas-tight h.v. bushing, but this has been successfully overcome. The pressure must be maintained if the dielectric strength is to be kept up to its proper value; this can readily be done with the aid of a cylinder of compressed or liquefied gas. It should be noted that the capacitance is almost unaffected by changes in gas pressure, even to the extent of 1 or 2 atmospheres.

Some use has been made of modified Leiden jars, of the type designed by Moscicki,† as standard condensers for use in high-voltage testing. These consist of long tubular glass flasks with well-rounded end and neck; the outer and inner coatings are of chemically deposited silver subsequently electroplated with copper. To avoid glow discharges and breakdown at the edges of the coatings the glass of the neck of the flask is made thicker than the body. The high-voltage lead passes through a corrugated porcelain insulator at the mouth of the flask to the inner coating; the outer coating is electrically connected to a metal container, filled with oil to facilitate cooling, in which the flask is mounted.

Jars suitable for voltages up to 25 kV. peak value are constructed by the Jena Glassworks of Schott and Genossen in sizes having about 300, 1,000, 2,000 and 4,000 cm. capacitance, the dielectric being a special glass known as "Minosglas." This material has a high breakdown strength (about 2,500 kV. per cm. on a 0·25 mm. sample at 50 cycles per sec.), a dielectric constant of about 8, high resistivity, and low dielectric loss. Minosflasks have a phase-defect of about 3 minutes practically independent of frequency. The condensers are cheap and occupy only a small space, the largest being about 83 cm. high and 12 cm. in diameter. Schering has used a battery of specially designed Minosflasks for bridge work at 100 kV., using 5 units in series, each of 300 cm. capacitance. There is no doubt that such condensers may be of considerable use in works test rooms where space is limited and the cost of the much superior compressed gas condenser is regarded as prohibitive.

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* For a description of a compressed gas condenser with guard rings applied to the l.v. electrode, having capacitance of 100 $\mu\mu$F. (90 cm.) for use at 140 kV., *see* A. Palm, "Über neuere Hochspannungsmessgeräte und ihre Anwendung," *Elekt. Zeits.*, Vol. 47, pp. 873–875, 904–907 (1926). *See* also the author's *Instrument Transformers* (Pitman) for further particulars and an illustration. For an extension of these designs to 500 kV., see A. Palm, "Die Durchbruchfeldstärke komprimerter Gase und ihre Verwendung zur Hochspannungs-Isolatoren," *Arch. f. Elekt.*, Vol. 28, pp. 296–302 (1934).

† J. Moscicki, "Über Hochspannungskondensatoren," *Elekt. Zeits.*, Vol. 25, pp. 527–532, 549–554 (1904)

15. Earth Capacitance Effects and other Sources of Error in Air Condensers. The capacitance measured across the terminals of an air condenser is not simply the intercapacitance between its electrodes, since, for a given capacitance, the bulk of the condenser is often so large that earth capacitances of the electrodes cannot be neglected. The effects are extremely important when the capacitance is less than 100 $\mu\mu$ F. It is shown on page 13 that the working capacitance of a condenser is $C = c_{12} + (c_1 c_2/(c_1 + c_2)\ )$, where c_{12} is the intercapacitance between the electrodes, c_1 and c_2 being the capacitances of the electrodes to the earthed screen. In practically all air condensers the metal shield or case completely surrounds the electrodes, and can, at will, be joined to one or other set of plates or simply left free. It is usual to connect one electrode to the case, in rotary variable condensers the moving system being so joined and the case arranged to be as near earth potential as possible. By this means the working capacitance of the condenser will be but slightly influenced by the presence of the hand upon the operating knob. In this instance the terminal capacitance is $C = c_{12} + c_1$ simply.

In any particular arrangement of electrodes and shield the effects of earth capacitance can be readily allowed for by Orlich's method of making three measurements from which c_1, c_2 and c_{12} can be found. Let the condenser have three terminals, 1, 2, 3, connected respectively to the two electrodes and the case. Measure the capacitance between 1 and 2, (*a*) with 1 connected to the case, and (*b*) with 2 connected to the case. In (*a*) the measured capacitance is $C_a = c_{12} + c_2$, and in (*b*) is $C_b = c_{12} + c_1$. Now join 1 and 2 together and measure their combined capacitance to the case, i.e. $C_c = c_1 + c_2$. Then from these three measurements, c_1, c_2, and c_{12} are easily calculated, and thence the working capacitance for any method of connection.

In a test recorded by Orlich* the following values were obtained: $C_a = 25{\cdot}6\ \mu\mu$F., $C_b = 24{\cdot}1\ \mu\mu$F., and $C_c = 9{\cdot}0\ \mu\mu$F., from which $c_1 = 3{\cdot}75\ \mu\mu$F., $c_2 = 5{\cdot}25\ \mu\mu$F., $c_{12} = 20{\cdot}35\ \mu\mu$F. With both electrodes insulated from the screen, $C = 22{\cdot}53\ \mu\mu$F., which exceeds c_{12} by over 10 per cent due to the earth capacitances.

This example shows the extreme importance of specifying exactly the arrangement of the connections to the terminals and case of an air condenser, owing to the large effect of earth capacitances.

This point has been developed in detail by Hartshorn.† The capacitance as defined above is a function of c_1 and c_2, which in general may vary as the condenser is moved from place to place. Thus with $q_1 = -q_2$, i.e. $c_1v_1 = -c_2v_2$, the working capacitance is defined on p. 13 as

$$q_1/(v_1 - v_2) = c_{12} + (c_1c_2/(c_1 + c_2)).$$

Again, with $v_2 = 0$, i.e. conductor 2 joined to the earthed shield, and $v_1 = 1$, the capacitance defined as the positive charge on 1 when raised to unit potential with 2 earthed is $c_{12} + c_1$. Likewise, the capacitance defined as the positive charge on 2 when raised to unit potential with 1 earthed is $c_{12} + c_2$. In the last two cases Hartshorn points out that it is better to define the capacitance as the amount of negative charge induced on either plate when this plate is earthed and the other raised to unit potential; in both these cases the capacitance is simply c_{12} and is independent of earth capacitances.

Small condensers should always be enclosed in a metallic shield, which should be as completely closed as possible. The shield may be used in two ways: (1) The shield is connected to one electrode of the condenser and becomes part of that electrode, say 2; then conductor 2 surrounds conductor 1 completely, and hence c_1 is reduced to zero. (2) The shield is earthed but not connected to either plate; this makes c_1 and c_2 quite definite and if the correct potential conditions are imposed the ordinary working capacitance formula applies.

If the shield is not earthed and is insulated from both plates, then the capacitance is not definite, since it depends on the earth capacitance of the shield and on the potentials of the electrodes. It is best, therefore, always to earth the shield and to state completely the way in which the fixed or moving systems are connected to it.

In a.c. bridges we are concerned with currents rather than charges; since it is the custom to regard current as entering one plate of a condenser and leaving the other, take $i_1 = dq_1/dt$ as the entering and $-i_2 = -dq_2/dt$ as the leaving currents. Then, if accenting denote time differentiation, the equations on page 13 give

$$i_1 = c_1v_1' + c_{12}v_{12}',$$
$$-i_2 = -c_2v_2' + c_{12}v_{12}'.$$

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* E. Orlich, *Kapazität und Induktivität*, p. 179.

† L. Hartshorn, "Note on the capacities of small air condensers," *Journal Sci. Insts.*, Vol. 1, pp. 305–309 (1924). D. Oliver, "The screening of small variable air condensers," *Exp. W. and W. Eng.*, Vol. 2, pp. 970–974 (1925).

Clearly these are unequal, due to the earth capacitance terms. Defining the capacitance as the ratio of the current flowing into one of the plates to the rate of change of potential difference between them gives as alternative definitions,

$$i_1/v_{12}' = c_{12} + (c_1v_1'/v_{12}'),$$
$$-i_2/v_{12}' = c_{12} - (c_2v_2'/v_{12}').$$

Thus the value depends in general on the potential distribution and on which current is to be regarded as effective in the rest of the bridge network. In three cases the effective capacitance is definite and independent of the voltages: (i) when 1 is at earth potential, then i_1 is the effective current and c_{12} the effective capacitance. (ii) When 2 completely shields 1, then c_1 is zero and the effective values are i_1, and c_{12}, as before. These conditions are satisfied (1) in Carey Foster's bridge when the detector branch points are maintained at zero potential by the use of a Wagner earth; and (2) in a simple Wheatstone network with a completely shielded condenser in one branch, the shield being joined to one set of plates and to the source, while the other set is maintained at a low potential, preferably zero by connection to a detector branch point. (iii) When i_1 and $-i_2$ are equal, resulting in the usual working capacitance with its accompanying potential condition $c_1v_1 = -c_2v_2$. Also see p. 529.

In precise work, air condensers are found to possess certain small imperfections due to three distinct causes. If it were possible to arrange that the dielectric between the plates were entirely air, the condenser would be quite free from dielectric loss (at ordinary voltages) and would behave as a perfect capacitance. It is necessary to support the plates and to insulate them from one another by pieces of solid dielectric in which, under alternating voltage, some energy loss is bound to occur. Hence the amount of solid dielectric in an air condenser should be kept down to the absolute minimum, and should be made of some material having low loss; it should be so placed that it lies in a weak electric field. Amber or quartz are suitable materials; ebonite less so. The imperfection occurs not only in the dielectric of c_{12} but also in the dielectric interposed in c_1 and c_2, the earth capacitances.*

A second cause of imperfection is faulty insulation resistance. Hence the solid materials employed should be very good insulators. The materials mentioned are excellent for the purpose, but ebonite is rather liable to deteriorate under the action of sunlight and dust, especially when used for the tops of variable condensers; amber or quartz are free from this objection and, in addition, keep their shape more permanently than ebonite, which in time yields to pressure, especially under the action of heat. To avoid faulty insulation in the air between the plates, some drying agent should be kept within the condenser case and the air should be freed from dust.

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* See L. Hartshorn, "Power loss in condensers," *Exp. W. and W. Eng.*, Vol. 3, pp. 225–233 (1926).

The third cause of imperfection lies in the resistance of connections within the condenser. Even if the condenser itself be perfect, internal resistance of the connections will cause losses and produce imperfection. This defect is most frequently encountered in variable condensers in the connection between the terminal and the moving plates. A properly designed flat spring device mentioned above overcomes the difficulty.

The researches of Giebe and Zickner* on the causes of imperfection in condensers have shown that in precise work no air condenser can be considered *absolutely* perfect, but that by proper attention to construction, to insulation and to connection in use, a very close approximation to an ideal condenser can be secured, in which the imperfections are too slight to be taken into account in any but the most refined experiments.

Although it is true that air condensers may be considered to be practically perfect at the frequencies usual in bridge measurements, their imperfections become important at the high frequencies of radio research and have been the subject of many investigations.† Dielectric losses in the air and the solid insulation, and conductor losses in the leads, plates, and internal contacts make an appreciable phase-defect; skin effect in the conducting parts is also important. Again, each bank of plates and its associated supports has a small inductance L which makes the effective capacitance of the condenser equal to $C\ (1 + \omega^2 CL)$ approximately. The correction term, unimportant at low frequencies, may become very large at high values; for example, at 50×10^6 cycles per second the correction may be 50 per cent in a condenser of 150 $\mu\mu$F., corresponding with L about 0·03 μH.

A further effect requiring consideration is the stability of air condensers with change of temperature, a subject exhaustively treated by Thomas.‡ Temperature coefficients of capacitance are between − 65 to

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* E. Giebe and G. Zickner, "Verlustmessungen an Kondensatoren," *Arch. f. Elekt.*, Vol. 11, pp. 109–129 (1922).

† W. Jackson, "An analysis of air condenser loss resistance," *Proc. Inst. Rad. Eng.*, Vol. 22, pp. 957–963 (1934). R. F. Field and D. B. Sinclair, "A method for determining the residual inductance and resistance of a variable air condenser at radio frequencies," *ibid.*, Vol. 24, pp. 255–274 (1936) (contains valuable bibliography). "Residual impedances in the precision condenser," *G.R. Exp.*, Vol. 10, pp. 7–8 (Oct., 1935). W. H. Ward, "The measurement of the self-inductance of variable air condensers," *Journal Sci. Insts.*, Vol. 13, pp. 251–260 (1936).

‡ H. A. Thomas, "The electrical stability of condensers," *Journal I.E.E.*, Vol. 79, pp. 297–335 (1936). W. Gohlke, *E.N.T.*, Vol. 14, pp. 258–264 (1937).

+ 150 parts in 10^6 per degree C. and the thermal behaviour is not cyclic in general. When cyclic behaviour is obtained, the coefficient is 2 to 3 times that of linear expansion of the metals used in construction of the condenser. The principal factors producing large coefficients are unequal expansion of the various parts, residual stress due to mechanical constraint and dissymmetry, and unsuitable solid insulating material. Ebonite and synthetic plastics are non-cyclic and have coefficients that may reach + 2,000 in 10^6; some ceramic materials give negative values up to − 700 in 10^6. Air condensers with practically zero temperature coefficients have been constructed to Thomas's designs at the National Physical Laboratory, using differential expansion of brass and steel to effect the necessary compensation of capacitance change with temperature rise.

16. Liquid Condensers. The capacitance of air condensers is usually small, but can be increased by making the case leak-proof and filling it with a suitable oil. This is frequently done in the case of rotary variable condensers when larger capacitances are required. A suitable oil is paraffin* of high boiling point, thoroughly dried before being put into the condenser. The capacitance is approximately doubled, but there is a small dielectric loss. Castor oil is not so satisfactory, for, although it multiplies the capacitance about four times, the power-factor of the condenser greatly increases at high frequencies. Paraffin is practically free from frequency effects.

17. Condensers with Solid Dielectrics. General. The capacitance obtained in fixed-value air condensers is small, being of the order of a few hundredths of a microfarad or less, and the much larger values frequently required in bridge work cannot be obtained without an enormous increase in the number of electrodes and consequent bulk of the condenser. In order to overcome these defects, high value condensers are constructed with a solid dielectric, having a dielectric constant greater than that of air and of high breakdown strength, so that very thin layers of dielectric can be interposed between the plates, with a consequent increase in capacitance. In practice, many solid materials are used in the manufacture of condensers, e.g. glass, ebonite, waxed paper, mica, but the two materials last named are the only ones used in condensers which are to serve as standards.† Both paper and mica are easily obtainable in the form of very thin sheets and have a high breakdown strength, especially in the case of the latter material.

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* *Dictionary of Applied Physics*, Vol. 2, p. 123. Also S. H. Anderson, "Effect of frequency on the capacity of a condenser with kerosene for the dielectric," *Phys. Rev.*, Vol. 34, pp. 34–39 (1912).

† For a consideration of these and other materials *see Dictionary of Applied Physics*, Vol. 2, pp. 116–120. Also H. A. Thomas, *loc. cit.* and G. Zickner, "Kapazitätsnormale mit festem Dielektrikum (Glimmer und Glas)," *Arch. f. tech. Mess.*, Z131–2 (April, 1934).

Condensers made with solid dielectrics show certain imperfections when used with alternating current, due to energy losses produced by dielectric hysteresis, imperfect insulation, and internal resistance. If a condenser be charged with a certain quantity of electricity and is then discharged, the discharge is found to consist of two parts: (i) the so-called free charge, which is given up during the first instants of the discharge period; and (ii) the bound charge, which is only given up very slowly as the short-circuit between the condenser plates is continued. This bound charge may be a considerable fraction of the original charge in the condenser and may take days to be completely removed. The phenomenon has its seat in the physical nature of the dielectric and is referred to as *absorption*; many theories have been advanced in explanation of absorption, but it cannot be said that the laws governing it are very well understood.

If, now, an absorptive condenser be taken through a cyclic charge and discharge, as in an a.c. circuit, the absorption occasions a dissipation of energy, since the charge admitted into the condenser in one interval of time is not entirely returned in the next discharge interval. If the instantaneous value of the charge throughout a cycle be plotted as a function of the instantaneous charging voltage, a closed curve will result, the area within the curve representing energy lost per cycle due to absorption effects. By analogy with a similar effect in ferromagnetism the phenomenon is referred to as *dielectric hysteresis*. The importance of the effect will increase at high frequencies.

Energy is also dissipated in condensers due to leakage conductance through the dielectric or over the insulation between the terminals. Again, ohmic resistance in the internal connections to the plates, or, at high frequencies with thin plates,* in the electrodes themselves, will also produce losses.

In a general way, all these losses are taken together and are referred to as the condenser energy loss. In consequence, the current through such an *imperfect condenser* will not lead on the applied voltage by $\pi/2$, but will differ therefrom by an angle θ, called the *phase-difference* or *loss angle* of the condenser.

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* J. G. Coffin, "The effect of frequency upon the capacity of absolute condensers," *Phys. Rev.*, Vol. 25, pp. 123–135 (1907). The effect is usually only important at radio frequencies.

The angle θ is a measure of the imperfection of the condenser, since it depends upon the losses therein; the power-factor is clearly $\sin\theta$ and will depend on the frequency. A *perfect condenser* is one in which there is no energy loss, so that $\theta = 0$; this ideal is very nearly attained in a condenser with air or other gas for the dielectric.

So far as its effect in an a.c. circuit is concerned, an imperfect condenser can be represented by a perfect capacitance in combination with a resistance. The equivalent capacitance and resistance can be arranged either in series or in parallel. The resistance is chosen to dissipate the same power as in the imperfect condenser, and in combination with the capacitance to give the same loss angle. The equivalent series arrangement is usually more convenient in practice, but the parallel arrangement is sometimes useful; both are illustrated in Fig. 73.

In the case of the series circuit, Fig. 10 gives $z = \rho - (j/\omega C_s)$; from which, numerically,

$$\tan\theta = \omega\rho C_s$$

For small values of θ, as in good condensers,

$$\cos\phi = \sin\theta \doteqdot \tan\theta$$

In the parallel circuit, Fig. 10 shows that

$$z = W(1 - j\omega C_p W)/(1 + \omega^2 C_p{}^2 W^2);$$

so that, $$\tan\theta = 1/\omega W C_p.$$

The two systems are very simply related*; clearly the effective resistances and reactances of the two circuits must be equal, i.e.

$$\rho = W/(1 + \omega^2 C_p{}^2 W^2) \text{ and } 1/\omega^2 C_s = C_p W^2/(1 + \omega^2 C_p{}^2 W^2).$$

so that

$$W = \rho\left(1 + \frac{1}{\omega^2 C_s{}^2 \rho^2}\right) \text{and } C_p = C_s/(1 + \omega^2 C_s{}^2 \rho^2).$$

With good condensers, $\tan\theta$ is small, so that

$$C_p \doteqdot C_s \text{ and } W \doteqdot \rho/\omega^2 C_s{}^2 \rho^2 \doteqdot 1/\omega^2 C_s{}^2 \rho;$$

hence the equivalent capacitance is then approximately the same whichever circuit be assumed, and W is a very large resistance.

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* For a full discussion of the physical meaning of these relationships see L. Hartshorn, "A critical resume of recent work on dielectrics," *Journal I.E.E.*, Vol. 64, pp. 1152–1190 (1926).

The importance of the loss angle, θ, as an index of the perfection of a condenser cannot be over-emphasized, since it is the best single test which can be made of the suitability of a condenser to serve as a standard. In a good condenser, e.g. a mica standard, θ is a very small angle ; in a poor condenser θ may be large, even several degrees. A large loss angle is usually accompanied by instability of the condenser and by large variations of capacitance with frequency and temperature. The measurement of θ involves certain difficulties and necessitates special methods, which are dealt with in Chapter IV (*see* also p. 502).

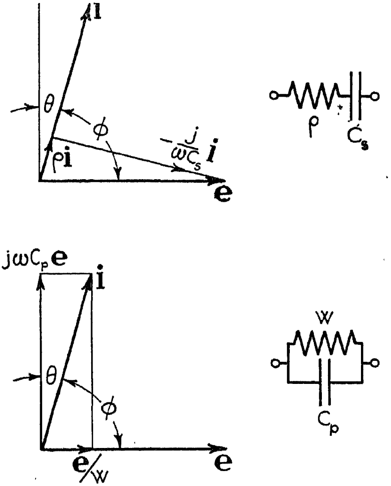

FIG. 73.—ILLUSTRATING THE CIRCUITS EQUIVALENT TO AN IMPERFECT CONDENSER

18. Paraffined-paper Condensers. Condensers of large capacitance, say up to 20 microfarads, are conveniently made with paper for the dielectric. A thin, tough paper, free from loading materials, is used, and after being thoroughly dried is impregnated with paraffin wax. Other materials, such as beeswax, oil, resin, and shellac are sometimes employed, but are usually less satisfactory.* The plates of the condenser are sheets of tin-foil, alternate sheets being joined together to form the two electrodes. Between each pair of tin-foils, two or three sheets of paraffined paper about 0·025 mm. thick are put, the completed condenser being compressed to form a solid mass, and frequently confined between wood or metal clamping plates.

Large numbers of condensers are made on the principle introduced by Mansbridge† in which the electrodes consist of strips of paper metallized on one surface by a deposit of tin. The condenser is made by winding up two such strips with two strips of plain paper interposed. After assembly the condenser is vacuum dried, and impregnated with hot wax. The condenser may be used in the cylindrical form, or pressed flat; a suitable number of units are assembled in a metal case, and sealed by semi-plastic wax. For further information on the properties of rolled paper-insulated condensers at voltages up to 500 an article by Schäfer‡ may be cited. The general investigation, though directed to quite other ends, serves to emphasize the unsatisfactory nature of paper condensers as standards of capacitance.

While paraffined paper condensers are very useful in bridge work, they suffer from certain serious defects which render them unsuitable as reference standards of capacitance, condensers with mica dielectric being vastly superior, both from the point of view of permanence and of freedom from dielectric imperfections. If frequently calibrated they serve quite well in ordinary testing work, but are particularly useful in cases where accuracy and perfection are not requisite, e.g. in wave-filters, oscillator circuits, etc., especially when large capacitance is required, since large mica condensers are relatively costly.

On account of the fact that paraffined paper condensers are less constant than mica condensers, and have much larger loss angles and frequency errors, Grover§ concludes that paper condensers form poor standards of capacitance. Moreover, it does not seem that any definite rules can be formulated to say, in general, how paper condensers behave with changes of frequency and temperature. The following facts are, however, fairly well established for condensers used in bridge work, though commercial condensers may depart very considerably from these rules.

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* It is sometimes stated that the addition of resin to the paraffin wax reduces the temperature coefficient, but this has been shown by D. C. Gall, "Some tests on non-temperature coefficient paper condensers," *Journal Sci. Insts.*, Vol. 7, pp. 157–162 (1930) to be incorrect. The resin addition causes a temporary increase in the losses when the temperature is rising or falling.

† G. F. Mansbridge, "The manufacture of electrical condensers," *Journal I.E.E.*, Vol. 41, pp. 535–564 (1908). *British Patent*, No. 19,451.

‡ F. A. Schäfer, "Über das dielektrische Verhalten von Niederspannungskondensatoren mit geschichteter Papierisolation," *Arch. f. Elekt.*, Vol. 23, pp. 351–381 (1930).

§ F. W. Grover, "The capacity and phase-difference of paraffined paper condensers as functions of temperature and frequency," *Bull. Bur. Stds.*, Vol. 7, pp. 495–578 (1911).

(i) The changes of capacitance with frequency and temperature are greater than for mica condensers.

(ii) The capacitance usually diminishes with increase of frequency, the variation with frequency being greater at higher temperatures.

(iii) At constant frequency the capacitance generally falls with rise of temperature. In a condenser of small loss angle the temperature coefficient of capacitance is usually negative and is nearly constant, not exceeding about -5 parts in 10,000 per °C. In condensers of large power-factor the temperature coefficient may be positive, or may change from negative to positive, its value increasing considerably at high temperatures and low frequencies.

(iv) The loss angle, and hence the power factor, depends very much on the materials used in the condenser, and particularly on the dryness of the paper dielectric. Grover found values of the loss angle ranging between 6′ and 22° in good and in poor condensers respectively, corresponding to power-factors between 0·0017 and 0··37. The values in poor condensers rise rapidly at low frequencies and high temperatures.

In Fig. 74 curves are drawn to show the comparative behaviour of good and poor paraffined paper condensers over a range of temperature and frequency, the values being taken from Grover's paper just cited.

19. Mica Condensers. Although fixed-value air condensers form the ultimate reference standards in bridge measurements, mica condensers are the usual working standards, since they can be made of adequate capacitance with small bulk. They can be constructed in such a way as to have very small imperfections, and their variations with temperature and frequency are regular and definite.

Mica* has the advantage that it can be split into very thin plates of very great crushing strength, and has very large electrical constants. Specimens 0·1 mm. thick have a dielectric strength of about $1{\cdot}2 \times 10^6$ volts/cm.; the dielectric constant lies between 4 and 8, and the insulation resistivity is exceed-

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* *Mica* is a mineral form of silicate [illegible] sh (Muscovite) or magnesia (Biotite). It is found in the for[illegible]-hexagonal) crystals which cleave readily into very thi[illegible]

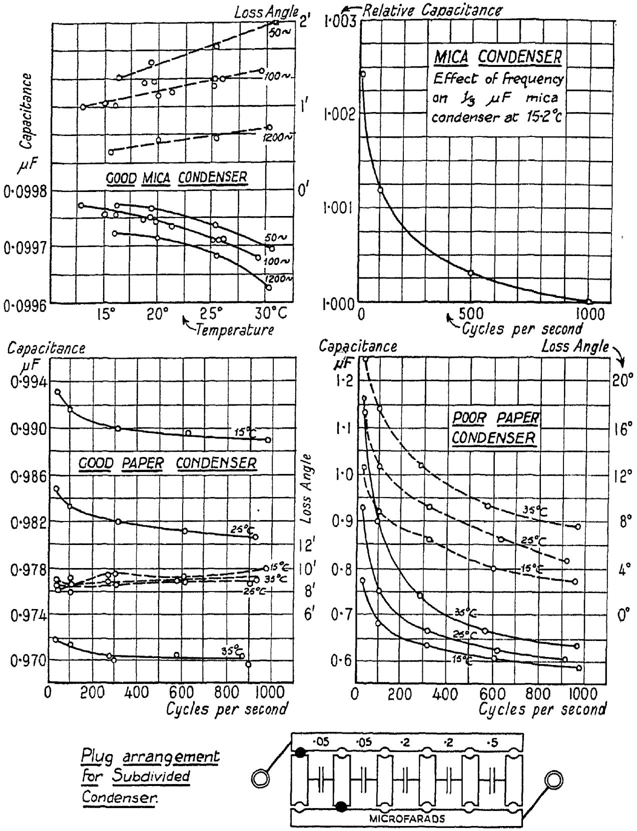

FIG. 74.—ILLUSTRATING THE PROPERTIES OF PARAFFINED-PAPER AND MICA CONDENSERS. ARRANGEMENT OF SUBDIVIDED CONDENSER

(Capacitance shown by full lines ; loss angle by dotted lines)

'The capacit
ons of ter
1911)

ingly high. The mica used for the construction of standard condensers is of the best clear ruby variety and is split into very thin sheets. As it is not possible to obtain very large sheets, a considerable number of thin laminae are required to make even a small capacitance ; for example, a 0·1 μF. condenser would have 20 sheets of mica, if each were 8·0 cm. × 6·5 cm. and 0·05 mm. thick. Moreover, the material is expensive, so that mica condensers are not usually made of greater capacitance than 1 μF.

The sheets of mica are arranged in a pile with alternate sheets of tin-foil, the operation being carried out under melted paraffin wax. All trace of moisture must be carefully removed, and air bubbles must be excluded. The operator must have a very dry skin, so that perspiration of the hands does not take place when they are immersed in the molten wax, i.e. at 60° to 65° C. The assembled pile is then taken out of the wax and is squeezed so that much of the wax is removed, the magnitude of the temperature coefficient and other properties of the condenser depending very considerably on the amount of paraffin left between the plates. The completed condenser is then adjusted to its approximately correct value and is firmly clamped between stout brass castings or plates, since unclamped condensers do not remain permanent in value. The two sets of electrodes are soldered to suitable connecting leads, and the condenser is sealed up in an air-tight case to prevent it from absorbing moisture from the atmosphere. See Fig. 75 (*a*).

Another process of construction is sometimes adopted in which the electrodes are formed of a coating of silver deposited chemically upon the mica laminae. Such condensers, however, show somewhat erratic changes of capacitance, especially when tested at different voltages. These instabilities have been traced to deposits of silver upon flakes of mica not properly attached to the main deposit.

A very thorough investigation of the properties of mica condensers has been made by Curtis,* in order to determine the conditions under which they can be used as reliable working standards of capacitance. He shows that, although several condensers may have different properties from one another, any individual condenser behaves in quite a regular and definite way under specified physical conditions. His conclusions are summarized below—

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* H. L. Curtis, "Mica condensers as standards of capacity," *Bull. Bur. Stds.*, Vol. 6, pp. 431–488 (1910) ; also F. W. Grover, "Simultaneous measurement of the capacity and power factor of condensers," *Bull. Bur. Stds.*, Vol. 3, pp. 371–431 (1907).

(i) The capacitance diminishes with increase of frequency, the rate of decrease becoming small at high frequencies. Curtis calls the limiting value to which the capacitance tends as the frequency approaches infinity, the *geometric capacitance*, since it depends only on the dimensions of the condenser and the dielectric constant. In a good condenser, with a small loss angle, the change with frequency* from 50 up to about 2,000 cycles per second may be about 1 part in 1,000; but in poor condensers it may be three or four times as much.

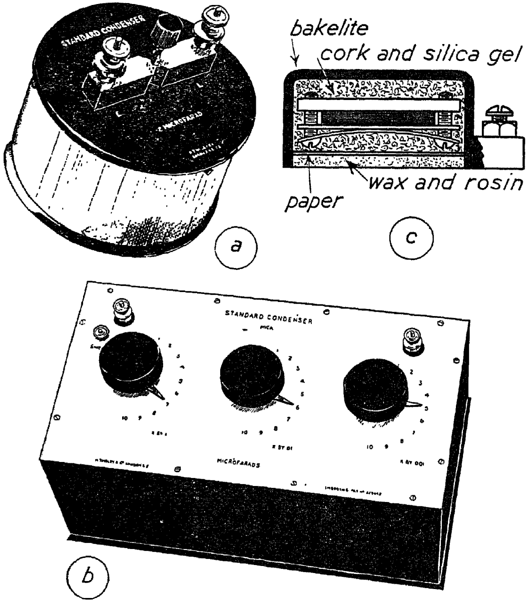

Fig. 75.—Mica Condensers

(ii) The temperature coefficient of capacitance† at constant frequency

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[*] *See* also B. V. Hill, *Phys. Rev.*, Vol. 26, pp. 400–405 (1908); and S. Butterworth, "On the use of Anderson's bridge for the measurement of the variations of the capacity and effective resistance of a condenser with frequency," *Proc. Phys. Soc.*, Vol. 34, pp. 1–7 (1922).

[†] *See* E. M. Terry, *Phys. Rev.*, Vol. 21, pp. 193–197 (1905); also D. Owen, *Proc. Phys. Soc.*, Vol. 27, p. 52 (1915).

is generally negative, the capacitance falling from 1 to 3 parts in 10,000 per °C. rise. If much paraffin be left between the plates, abnormal values as high as 10 in 10,000 may occur; Curtis has shown that by squeezing out much of the paraffin, the temperature coefficient can be reduced, and may be made zero or even caused to become positive. Since the dielectric of mica and wax is a poor conductor of heat, a condenser will only slowly attain the temperature of its surroundings; for this reason the N.P.L. keep mica condensers in a constant temperature room for 24 hours before making measurements. With silvered mica condensers the temperature coefficient is small and positive.

(iii) The loss angle of a good condenser at normal temperatures and frequencies may lie between 1′ and 5′, corresponding to a power factor between 0·0003 and 0·0015. Values as low as 0·0001 are sometimes met with in the very best condensers. The loss angle increases with rise of temperature and with fall of frequency. Abnormal values of θ are not infrequently found in small condensers and are a sure indication of poorness of quality.

(iv) Finally, a mica condenser forms a very permanent standard; its capacitance will remain constant within 1 or 2 parts in 10,000 for years. The temperature and frequency variations are definite and regular, and can be made quite small. Mica condensers should always be firmly clamped; the tests made by Curtis show that unclamped condensers are unstable and are affected by changes of atmospheric pressure. Clamped standards are but little affected by pressure changes.

In Fig. 74, characteristic curves for a good condenser are shown, from which the reader may see how nearly the above rules are followed in an actual example.

It is admitted, even by experienced makers, that the manufacture of a satisfactory mica condenser is a rather troublesome matter, the power factor being considerably affected by a number of important, and often puzzling, considerations. The use of the highest grade material is not, in itself, sufficient to ensure success; the technique of assembly and design must also be chosen with great care. Two questions of the greatest importance are perfect dryness and cleanliness, and certainty of the clamping arrangements. Thiessen* has discussed some of these problems in connection with the mica condensers made by the General Radio Co. Mica readily absorbs a thin surface film of moisture which greatly increases the dielectric loss. Before use, the mica and tin-foil are maintained for a considerable time at about 150° C. and are kept hot until assembled. To maintain dryness of the finished condenser it is sealed with resin and beeswax into a bakelite case containing a mixture of ground cork and silica gel; see Fig. 75 (*c*). Another important cause of increased power factor is mechanical damage of the mica, such as scratches, imperfect cleavage, loose flakes, and the like; careful inspection is the only guard against this source of trouble.

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* A. E. Thiessen, "Recent developments in mica condensers," *G.R. Exp.*, Vol. 7, pp. 1–4 (Jan., 1933).

The capacitance varies considerably with pressure, but Thiessen shows that the changes are small after a pressure of 2,000 lb. per sq. in. is exceeded; he recommends that the clamps be tightened up to give at least this figure if temperature changes are to be small. Even so, well-clamped condensers do sometimes show high loss angles, which are attributed to the fact that the clamping plates are appreciably bent by the pull of the screws at their edges. Consequently, although the condenser may be very tight at the edge it may be relatively slack in the middle, because of the slight concavity of the stressed clamps; the condenser plates may, therefore, vibrate under the alternating electric forces and thereby increase the losses. Some makers overcome this defect by using substantially ribbed cast clamps; others use plates to which an original convexity is given so that they pull up flat. The General Radio Co. use a stiff convex backing plate on one clamp as shown in Fig. 75 (*c*).

20. Grouping of Condensers in Working Standards. Standard condensers are arranged in a variety of ways according to the uses to which they are to be put. Single value mica standards are frequently used, a mica condenser being sealed up in a box provided with suitably insulated terminals. In most bridge work a subdivided condenser is very convenient; several mica condensers, totalling 1 microfarad, are assembled in a box. A system of blocks and plugs enables the sections to be combined in any desired fashion, one such arrangement being shown in Fig. 74. In this the condensers may be grouped in series or parallel, or any section can be discharged. Simpler plug arrangements are sometimes provided by which the parallel grouping only is possible, the same end being also served by a suitable system of dial switches and contact blocks. An example of a decade box is shown in Fig. 75 (*b*). Some makers construct a condenser consisting of a set of mica decades together with a variable air condenser, all in the same case; this is an exceedingly convenient arrangement for use in bridge circuits.

The plug arrangement has a disadvantage when applied to small condensers, that the capacitance of the blocks is included in the measurements and the error is different for various settings of the condenser. Such small condensers are frequently mounted together in a box, each having its own pair of terminals, so that connection capacitances are reduced to the least possible amount.

SOURCES OF ALTERNATING CURRENT

21. General Remarks. On page 6 it has been shown that the modern alternating current bridge is developed from the older ballistic bridge to which some rotating commutator, such as a "secohmmeter," has been applied in order to increase the sensitivity of adjustment. Such a commutator supplies the bridge with a current which is periodically interrupted or reversed, and, together with its battery, may be looked upon as a crude form of periodic or alternating current source.

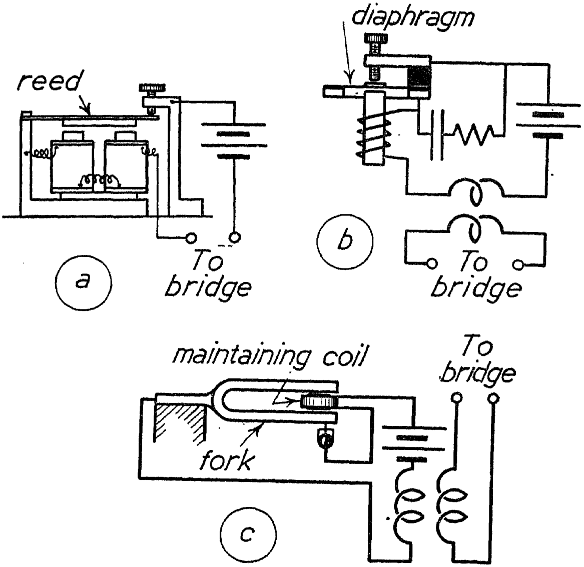

FIG. 76.—SIMPLE INTERRUPTERS

Another simple device, which is much used in rapid laboratory tests, is the "buzzer," shown in Fig. 76 (*a*). This piece of apparatus is the same in principle as an ordinary trembler bell, a rapidly vibrating armature making and breaking the current supplied to the bridge.* This current may also act as the maintaining current of the buzzer, as the diagram shows, or two independent currents may be used by interposing a transformer, as shown in Fig. 76 (*b*). Sparking is reduced by the condenser and resistance shunted across the break. The frequency of interruption is not usually very steady, and the armature is easily put out of adjustment. Hence, a buzzer is unsuitable in any case where the bridge balance conditions or the sensitivity of the detector depend upon constancy of frequency. The buzzer emits a loud note and must be enclosed in a well-padded sound-proof box to reduce interference with the use of telephones.

In all precise measurements with modern bridge methods it is usual practice to use a vibration galvanometer tuned to a definite frequency. Moreover, in many bridges the conditions of balance depend upon the frequency of the applied current. Hence, an ideal source of alternating current should have a constant and known frequency. The wave-form of the current should preferably be sinusoidal, so that methods in which balance depends upon frequency can be used with telephones, and also in order that tests may be carried out under simple, known wave-form conditions. Absolute purity of wave-form is not so very essential if the source is to be used in conjunction with a tuned detector, owing to the enormous sensitivity of such an instrument to the frequency for which it is tuned and its insensitiveness to all others.

22. Interrupters. A very common source of current with a constant, known frequency is a small induction coil or transformer in combination with an interrupter. The bridge current is taken from the secondary winding; the primary circuit contains a battery and an interrupter working at a constant frequency.†

A simple interrupter is an electrically maintained tuning fork* of suitable frequency, operating a contact by means of which the primary circuit is regularly opened and closed (*see* Fig. 76 (*c*)). The frequency is fixed by the pitch of the fork and is, therefore, very constant; the wave-form at the secondary terminals is not sinusoidal. Frequencies lying between 5 and about 1,000 cycles per second are obtainable.

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* *See* K. Perlewitz, "Elektrische Signalhuppen," *Elekt. Zeits.*, Vol. 29, pp. 445–450 (1908).

† D. Dvorák, "Ueber verschiedene Arten selbthätiger Stromunterbrecher und deren Verwendung," *Zeits. f. Inst.*, Vol. 11, pp. 423–439 (1891).

Instead of using a tuning fork as the source of constant frequency oscillations, a stretched wire is often employed. The arrangement is then called a "string" interrupter, and was first extensively used by Max Wien.† In its simplest form, a string interrupter consists of a thin iron or steel wire stretched over two bridges which determine its vibrating length. The natural frequency of the wire is adjusted by varying the distance between the bridges and by altering the tension of the wire by means of a screw (*see* Fig. 77 (*a*)). Near the middle of the wire two platinum contacts are attached so that, when the wire is at rest, they just touch the surface of mercury contained in two adjustable cups. Over the centre of the wire an electromagnet is fixed, its exciting winding being connected in series with a battery through the left-hand half of the wire and the corresponding mercury cup. The right-hand mercury cup is used to act as an interrupter in the primary circuit of the induction coil.

The action of the apparatus is simple. Current flowing in the winding of the electromagnet magnetizes the latter and attracts the iron wire, pulling it upward. This breaks contact at the left-hand cup, whereupon the wire flies back and re-makes the maintaining circuit. The monocord is thus maintained in continuous vibration, in the course of which the contact at the right-hand cup is made and broken, so that the primary of the induction coil carries an interrupted current. Sparking at the surface of the mercury is minimized by the use of the shunt resistances shown in the diagram, or by covering the

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* For various methods of electrically maintaining a tuning fork, *see* S. P. Thompson, "Note on a mode of maintaining tuning forks by electricity," *Proc. Phys. Soc.*, Vol. 8, pp. 72–76 (1887); W. G. Gregory, "On a method of driving tuning forks electrically," *Proc. Phys. Soc.*, Vol. 10, pp. 288–290 (1890); W. H. Eccles, "The use of the triode valve in maintaining the vibration of a tuning fork," *Proc. Phys. Soc.*, Vol. 31, p. 269 (1919); S. Butterworth, "The maintenance of a vibrating system by means of a triode valve," *Proc. Phys. Soc.*, Vol. 32, pp. 345–360 (1920). T. G. Hodgkinson, "Valve maintained tuning forks without condensers," *Proc. Phys. Soc.*, Vol. 38, pp. 24–34 (1926); "Synchronous alternating current motor and mechanical vibrating system maintained by a thermionic valve at the frequency of the vibrating system," *ibid.*, Vol. 39, pp. 203–222 (1927). See also p. 229. Also H. M. Dadourian, "On the characteristics of electrically operated tuning forks," *Phys. Rev.*, 2nd series, Vol. 13, pp. 337–359 (1919).

† F. Niemöller, "Ueber einen neuen Stromunterbrecher," *Ann. der Phys.*, Vol. 6, pp. 302–304 (1879). Max Wien, "Das Telephon als optischer Apparat zur Strommessung," *Ann. der Phys.*, Vol. 42, pp. 593–621 (1891), Vol. 44, pp. 681–688 (1891); "Messung der Inductionsconstanten mit dem 'optische Telephon,'" *Ann. der Phys.*, Vol. 44, pp. 689–712 (1891). For the theory of maintained oscillation of a wire, *see* S. Butterworth, "On electrically maintained vibrations," *Proc. Phys. Soc.*, Vol. 27, pp. 410–424 (1915). For a description of the N.P.L. form of the apparatus, see the book by Campbell and Childs previously cited.

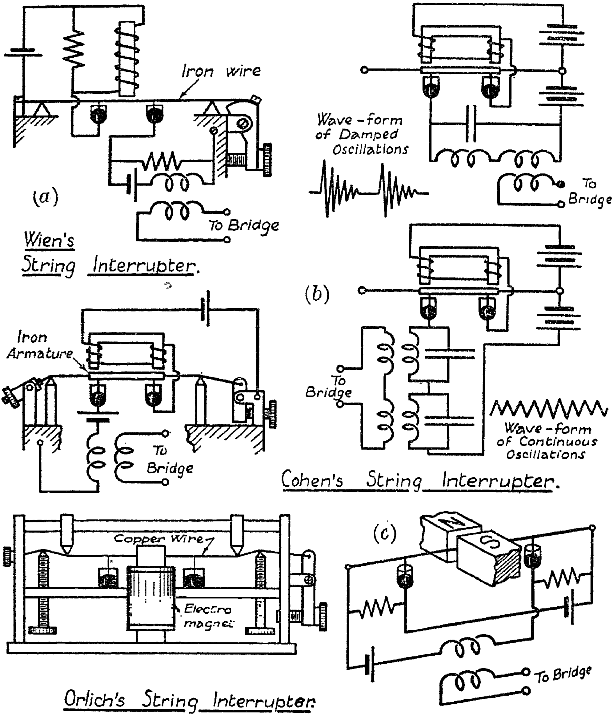
Iron wire
Wave-form of Damped Oscillations
To Bridge
(a)
To Bridge
Wien's String Interrupter.
(b)
Iron Armature
To Bridge
To Bridge
Wave-form of Continuous Oscillations
Cohen's String Interrupter.
Copper Wire
Electro magnet
(c)
N
S
To Bridge
Orlich's String Interrupter.

FIG. 77.—STRING INTERRUPTERS

mercury with a layer of paraffin oil. A range of frequency from 50 to 500 cycles per second can be obtained.

Cohen* has described a slight modification of the string interrupter which enables it to be used for the production of alternating as distinct from interrupted current. The apparatus is similar to that just described. A steel wire, the tension and length of which can be varied, carries at its centre a soft iron armature and the two platinum contacts. The maintaining magnet is of horse-shoe form and can be adjusted relatively to the wire until steady vibrations are produced.

In the left-hand diagram of Fig. 77 (*b*) the device is used as an interrupter, the frequency being adjustable between 20 and 500 cycles per second by altering the position of the bridges and the tension of the wire.

The upper diagram shows how the interrupter may be used to produce damped high frequency oscillations. The wire is maintained in steady vibration, so that it makes and breaks the circuit of a battery connected to an oscillatory circuit composed of an inductance and condenser in parallel. Each time the interrupter breaks contact a train of damped waves is produced in the oscillatory circuit, the frequency of the waves depending only on the value of inductance and capacity used. There is no oscillation on making the contact, since the maintaining battery acts as a short-circuit to the oscillating portion of the circuit. The wave form of the current in the secondary consists of a series of damped oscillations of a known frequency, the trains of waves following one another at the frequency of the interrupter. The particular instrument described can be used to produce oscillations varying between 560 and 2,500 cycles per second by simple adjustment of the inductance and capacity of the oscillatory circuit, the interrupter working at a constant rate.

By arranging two oscillatory circuits tuned to the same frequency, and suitably proportioning the inductances, capacitances, and resistances, the secondary oscillations can be made to be nearly continuous and approximately sinusoidal, as the third diagram shows.

The reader is referred to Cohen's paper for a detailed description of the uses of the apparatus in telephonic research, and for the excellent series of wave-form oscillograms taken with the interrupter working under various conditions.

The disadvantage of the string interrupters hitherto described is that the maintaining circuit which is made and broken by the vibrating wire is highly inductive, since it contains the windings of a powerful electromagnet. Hence there will be troublesome sparking at the surface of the mercury in the contact cups. In order to avoid this trouble, Arons* has introduced a type of interrupter in which the wire is made of copper and lies in the field of a strong permanent magnet. The mercury contacts are only called upon to break the slightly inductive circuit of the wire itself. Orlich† has improved this instrument, in the pattern used at the Physikalische Reichsanstalt, by the use of an electromagnet and various devices by means of which the frequency may be accurately adjusted (Fig. 77 (c)). The direction of the maintaining current in the wire should be such that the force acting on the wire tends to move it upward out of the magnetic field ; the circuit is then broken and the wire flies back, and so on, so that the oscillations are maintained. Any small spark at the contacts can be suppressed by the connection of resistances (or condensers) across the cups, as shown. A frequency of about 500 cycles per second can be obtained.

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* B. S. Cohen, "The production of small variable frequency alternating currents suitable for telephonic and other measurements," *Proc. Phys. Soc.*, Vol. 21, pp. 283–297 (1910). For another variant see *Dictionary of Applied Physics*, Vol. 2, pp. 395–396.

The output of string interrupters is not very large, but is usually sufficient for a bridge measurement. The frequency is easily controlled and is reasonably constant. In the Orlich pattern, care should be taken not to pass an excessive current along the wire, since the consequent heating and slackening of the string will cause the frequency to alter considerably. This will be particularly troublesome when sharply-tuned vibration galvanometers are in use, causing an enormous fall in sensitiveness. The main defect of string interrupters is their fairly large stray magnetic field, which may interfere with bridge circuits; to avoid this trouble the interrupter should be situated in another room at some distance from the bridge.

23. Microphone Hummers. The microphone, which is so much used in telephony for the production of speech currents, forms a useful source of alternating current for bridge work. In principle the microphone consists of a hollow chamber or capsule containing granules of carbon in loose contact with one another. Mechanical vibrations, even of very small amplitude, acting upon the microphone produce very large alterations in the electrical resistance of the contacts between the granules. Hence, by subjecting a microphone to suitable mechanical vibrations and connecting it in series with a battery, the current through it will be made to vary. This varying current can be arranged to act upon the source of vibrations in such a way that the oscillations are maintained. Such a device is then called a "microphone hummer," and forms a convenient source of alternating current.

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* L. Arons, "Ein neuer electromagnetischer Saitenunterbrecher," *Ann. der Phys.*, Vol. 66, pp. 1177–1178 (1898).

† E. Orlich, "Über Selbstinduktiononormale und die Messung von Selbstinduktionen," *Elekt. Zeits.*, Vol. 24, pp. 502–506 (1903). Also, "Saitenunterbrecher," *Zeits. f. Inst.*, Vol. 24, pp. 126–127 (1904).

In its simplest form, a hummer can be made by placing a microphone transmitter opposite the diaphragm of an ordinary magnetic telephone receiver (*see* Fig. 78 (*a*)). The microphone is connected in series with a battery and the primary of a transformer; the secondary of the transformer is connected to the receiver. The action of the apparatus is then as follows: An external disturbance—a stray sound, or some change in the battery circuit—acting on the microphone will change the current through it. This change of current will produce a transient current in the secondary circuit, and will cause a movement of the telephone diaphragm. Sound waves will be emitted and will fall upon the microphone, producing further transient changes in its resistance and in the current through it, these reacting on the telephone *via* the transformer; and so on. The system, once disturbed, will continue in action and will emit a loud note, the frequency of which will depend upon the length of the air column between the telephone and microphone, and upon a variety of other circumstances.* An additional winding on the transformer will provide current for the bridge.

Unfortunately, the frequency of a simple hummer is not very constant, and is much affected by external effects. Larsen† has shown (Fig. 78 (*b*)) that the action can be made much more definite by joining the microphone to the telephone by means of a tube. The air column is now confined, so that external effects are small; the action can be adjusted until the best conditions are found by means of a tuning tube sliding upon the connecting pipe. Still further improvement can be secured by tuning, in addition, the column of air behind the diaphragm of the telephone receiver. Frequencies lying between 600–1,100 cycles per second are obtainable.

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* F. Gill, "Note on a humming telephone," *Journal, I.E.E.*, Vol. 31, pp. 388–395 (1902).

† A. Larsen, "A new high frequency generator," *Elecn.*, Vol. 67, p. 827 (1911). See also A. E. Kennelly, *Electrical Vibration Instruments*, Ch. xxiii (1923).

Dolezalek* has greatly improved the microphone hummer by combining in one piece of apparatus the telephone and microphone elements (*see* Fig. 78 (*c*)). In this instrument a telephone diaphragm carries at its centre a small microphone consisting of a silk bag filled with carbon granules. The diaphragm is made of thin sheet iron and stands above a cylindrical,

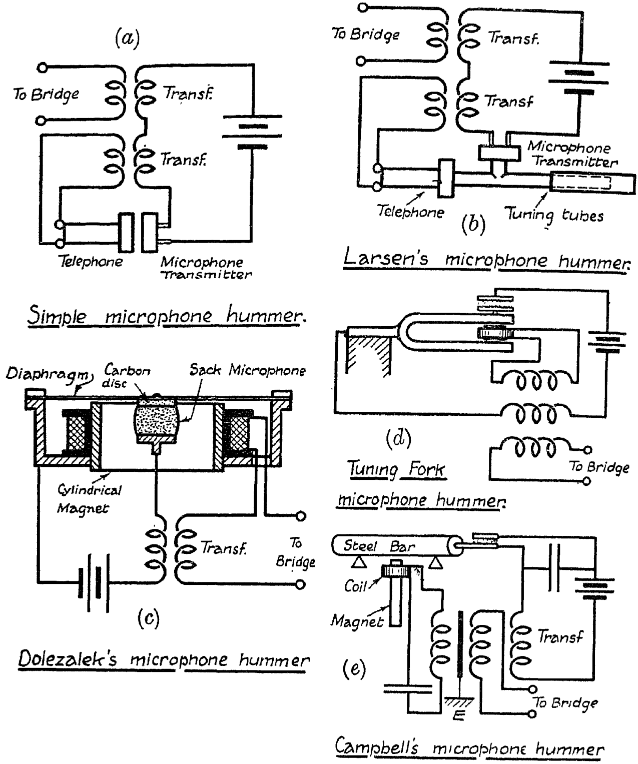

FIG. 78.—MICROPHONE HUMMERS

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* F. Dolezalek, "Messeinrichtung zur Bestimmung der Induktionskonstanten und des Energieverlustes von Wechselstromapparaten," *Zeits. f. Inst.*, Vol. 23, pp. 240–248 (1903).

hollow, permanent magnet surrounded by a coil. When the diaphragm is set in motion, the resistance of the microphone is changed and the current from the battery varied. By the action of the transformer, currents will flow in the coil and will affect the magnet in such a way that the motion of the diaphragm is maintained. The frequency is very constant and can be adjusted by substitution of diaphragms of different thicknesses; a range of from 300 to 1,000 cycles per second is covered. The secondary circuit is frequently tuned by means of a condenser so that the maximum effect is obtained. The resulting oscillations are very nearly sinusoidal.

In the hummers just described, the microphone currents are used to maintain in vibration the diaphragm of a telephone receiver; they can, however, be used to act in a similar way upon quite other dynamical systems.* For example, a common arrangement is a tuning fork maintained in oscillation by the microphone current. The fork† acts on the microphone, in series with which is the primary of a transformer. The secondary winding is joined in series with the maintaining magnet, and an additional winding provides the bridge current. The fork‡ usually has a frequency of 800 or 1,000 cycles/second and forms a very constant source of current. The output is a fraction of a watt (*see* Fig. 78 (*d*)).

A further development of the tuning fork hummer is the vibrating bar hummer of A. Campbell.§ In this (Fig. 78 (*e*)) the vibrating element is a steel bar 2·5 cm. diameter and 23·7 cm. long, supported upon knife-edges at two nodal points. At one end the bar carries a microphone button, the microphone being connected in series with a battery and the primary of a small transformer. Beneath the centre of the bar a magnet is fixed, a coil wound upon it carrying current from the secondary of the transformer and serving to maintain the vibrations of the bar when once started. Tuning condensers are connected in these circuits as shown. Current for the bridge is taken from a third winding on the transformer, an earthed screen being put between the two secondaries so that errors due to earth capacitance are avoided when the hummer is used on a bridge.

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* L. H. Stauffer, "String controlled alternating current source," *Rev. Sci. Insts.*, Vol. 4, pp. 483–485 (1933).

† For other arrangements, *see* R. Appleyard, "A new method of electrically driving funing forks," *Tel. J. and Elec. Rev.*, Vol. 26, p. 57 (1890). J. E. Taylor, *Journal I.E.E.*, Vol. 31, pp. 395–398 (1902). Fork oscillators on this principle are made by the General Radio Co. *See* C. E. Worthen, "A tuning fork audio oscillator," *G.R. Exp.*, Vol. 4, pp. 1–4 (April, 1930). H. W. Lamson, "An improved audio oscillator," *G.R. Exp.*, Vol. 9, pp. 1–4 (May, 1935). "A precision tuning fork," *G.R. Exp.*, Vol. 10, pp. 1–3 (May, 1936).

‡ For precautions necessary to attain success with low frequency (50 to 100 cycles/second) hummers, *see* A. Campbell, "A note on low frequency microphone hummers," *Proc. Phys. Soc.*, Vol. 31, p. 84 (1919).

§ A. Campbell, "On the electric inductive capacities of dry paper and of solid cellulose," *Proc. Roy. Soc. A.*, Vol. 78, pp. 196–211 (1907).

With a bar of the dimensions given, a frequency of about 2,000 cycles/second is obtained; other bars can be substituted and a range of frequency from 500 to 3,000 cycles/second, or higher, covered. The frequency depends only on the physical properties of the bar and is constant within 1 or 2 parts in 1,000; the wave-form is practically pure, though the output is small. By resonating the circuit which supplies the bridge it is possible to obtain 50 to 100 volts on the supply terminals, which is useful when high impedance networks are used.

Another simple use of the microphone is found in the reed hummer, used in the British Post Office. This instrument is identical with an ordinary hummer, the telephone diaphragm being replaced by a vibrating steel reed carrying the microphone button. The action is just the same as that of the telephonic device. Hummers on this principle are made by Messrs. H. Sullivan and by the Cambridge Instrument Co.

24. Alternators. As will have been gathered from the above remarks, the output required from a source of current for bridge work is not very great, the interrupters and hummers described above usually supplying sufficient output for most purposes. For much laboratory work, especially when outputs somewhat larger than those given by these devices are required, a small motor-driven alternator forms a convenient source of current. Many types of alternator have been devised to work at telephonic, or even higher, frequencies and to produce a wave form reasonably free from harmonics; the principle and action of some of these machines will now be considered. For tests at ordinary commercial frequencies of 25 to 60 cycles per second, as in cable testing, etc., normal alternators used in conjunction with a vibration galvanometer usually suffice, but generally require special means to maintain constant frequency; see p. 212.

Inductor Alternators. Some of the earliest alternators designed for bridge work were arranged to act on the inductor principle. A magnetizing coil is arranged to produce a flux in a magnetic circuit in which the rotating element of the machine is included. The latter is shaped in such a way that, as it rotates, the reluctance of the magnetic circuit is periodically varied. The flux is thus caused to change harmonically and links a secondary or armature winding in which an electromotive force is induced.

The simplest type of inductor alternator* is the so-called "alternating current syren" shown in Fig. 79(a). In this a disc of wood, ebonite, or brass is caused to rotate at a high speed between the poles of an electromagnet. Holes are drilled at equidistant intervals round a circle near the rim of the disc, soft iron plugs or studs being driven into the holes; two or more rows of studs are sometimes provided, together with means to move the electromagnet to face any desired row. The pulsating flux links small coils wound upon the pole tips, these coils acting as an armature winding from which the bridge current is taken. Frequencies up to about 3,000 cycles/second are easily obtained, the disc being direct-coupled to the shaft of a small shunt motor.

In a modification† the studs are replaced by slots cut in the rim of the disc, filled with packets of transformer iron. Wien‡ has constructed such a machine to give 8,500 cycles/second and an output of 20 volts, 0·2 amps. The wave form is not pure, and by resonating the second harmonic Wien was able to obtain a frequency of 17,000.

A common type of inductor alternator uses as the rotating element a packet of thin soft iron stampings clamped tightly together upon a shaft. The periphery of the iron disc is cut into the form of teeth, an electromagnet being placed opposite the rim of the disc so that the teeth, when moving past the poles, will produce fluctuations in the magnetic field.

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* Max Wien, "Ueber die Magnetisirung durch Wechselstrom," *Ann. der Phys.*, Vol. 66, pp. 859–953 (1898). E. Behne, "Beiträge zur Frage der günstigsten Verhältnisse in Erregerstromkreise der Wienschen Hochfrequenzsirene," *Arch. f. Elekt.*, Vol. 2, pp. 181–207 (1914).

E. Orlich, "Ueber Selbstinduktionsnormale und die Messung von Selbstinduktionen," *Elekt. Zeits.*, Vol. 24, pp. 502–506 (1903).

B. V. Hill, "The variation of apparent capacity of a condenser with the time of discharge and the variation of capacity with frequency in alternating current measurements," *Phys. Rev.*, Vol. 26, pp. 400–405 (1908).

† J. Puluj, "Bestimmung des Koëfficienten der Selbstinduktion mit Hülfe des Elektrodynamometers und eines Induktors," *Elekt. Zeits.*, Vol. 12, pp. 346–350 (1891).

‡ Max Wien, "Ueber die Erzeugung und Messung von Sinusströmen," *Ann. der Phys.*, Vol. 4, pp. 425–449, 1901.

Small armature coils are wound on the poles of the magnet, and have a high frequency e.m.f. induced in them by the varying flux. Frequencies up to 6,600, with an output of 7 watts, have been attained by the device as designed by

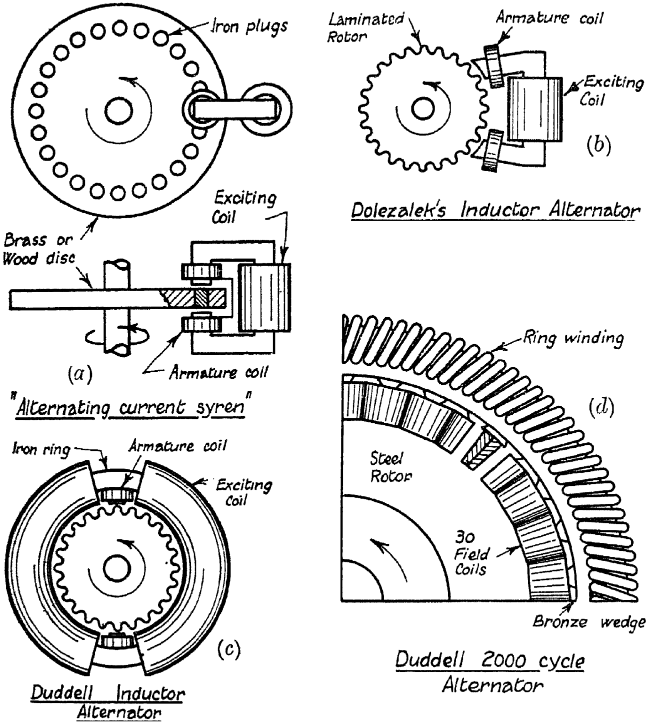

Fig. 79.—Types of Alternators

Dolezalek* and shown in Fig. 79 (*b*). The disc is direct driven by a small shunt motor run at a constant speed.

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* F. Dolezalek, "Messeinrichtung zur Bestimmung der Induktionskonstanten und des Energieverlustes von Wechselstromapparaten," *Zeits. f. Inst.*, Vol 23, pp. 240–248 (1903). *See* also *Elecn.*, Vol. 64, p. 449, (1910), for an illustration of a similar machine.

Duddell* has adapted the toothed disc for the production of extra-high frequency currents, up to 120,000 cycles/second, his arrangement being illustrated in Fig. 79 (*c*). In his machine the electromagnet takes the form of a ring concentric with the disc, the exciting coils being connected in such a way that the flux passes along a diameter. The disc was specially driven by means of a belt, speeds as high as 35,000 r.p.m. being attained. The armature coils were wound upon the inwardly-projecting pole pieces from which the flux issued.†

Heteropolar Alternators. Heteropolar alternators are machines of the ordinary type in which a system of alternate north and south poles is moved relatively to an armature winding. For low frequencies an ordinary alternator is sufficiently good up to about 200 cycles/second. Much attention has been given to the design of machines which will give a sinusoidal wave form, even when heavily loaded, at frequencies lying between 200 and 10,000 for measurements within the telephonic range.

A typical machine of this class was designed by the late Mr. W. Duddell‡ to give an output of about $\frac{1}{2}$ kilowatt at a frequency of 2,000 cycles per second (*see* Fig. 79 (*d*)). The rotor forms the field magnet system and runs at a speed of 8,000 r.p.m. The rotor disc is of steel, 20 cm. diameter, with 30 polar projections milled out of the solid; the field coils are slipped over the poles and secured in place by bronze wedges. The stator is a smooth ring provided with a Gramme winding. Slots are avoided, so that the wave form is free from tooth ripples; also the necessarily long air-gap makes the effect of armature reaction very small, so that the wave form remains pure when the machine is loaded. The rotor runs in long bronze bearings and is driven by a motor and link belt.

The laboratories of the City and Guilds (Engineering) College contain two small high-frequency alternators which were specially designed§ for bridge work. The first works up to 250 cycles per second and has 6 poles; the second gives 1,000 cycles per second and has 24 poles. The field systems of both machines are stationary, the field coils being wound on every other pole. The armatures of both machines are wound on the smooth rotor. In the low frequency machine a simple Gramme ring winding is used. The high-frequency machine has a smooth rotor with former-wound coils held on its surface by steel binding wire. The alternators are mounted with their axes vertical, and are direct coupled to $\frac{1}{4}$ h.p. shunt wound motors supplied from a 200 volt battery. The wave forms are very nearly sinusoidal.

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* W. Duddell, "A high frequency alternator," *Proc. Phys. Soc.*, Vol. 19, pp. 431–442 (1905). A list of references to other papers on high frequency alternators is given.

† For another form of high frequency inductor alternator, *see* A. Franke. "Die elektrischen Vorgänge in Fernsprechleitungen und Apparaten," *Elekt, Zeits.*, Vol. 12, pp. 447–452, 458–463 (1891).

‡ W. Duddell, "A 2,000 frequency alternator," *Proc. Phys. Soc.*, Vol. 24, pp. 172–180 (1912).

§ H. F. Haworth, "The measurement of electrolytic resistance using alternating currents," *Trans. Far. Soc.*, Vol. 16, pp. 365–391 (1921).

For another high frequency machine, *see* F. Addey, "The experimenting room," *Journal, P.O.E.E.*, Vol. 3, pp. 1–16 (1910–11).

25. Methods of Maintaining Alternators at Constant Speed. In modern bridge measurements the balance detector usually employed is the vibration galvanometer. This instrument, as will be shown on page 254, is tuned to respond to currents of one definite frequency and is insensitive to currents with frequencies even slightly removed from the critical value. Moreover, in some of the methods of measurement which are now in regular use, the frequency must be known as one of the conditions for balance. For both these reasons it becomes necessary to maintain strictly constant the frequency of the current supplied to the bridge. It is proposed now to show how this can be done when an alternator is the source of current.

In order to avoid any effects due to backlash or slip, the alternator is best driven by a direct current shunt motor, the shafts being rigidly connected. The motor should be run from a well-charged battery of ample output, so that variations of voltage of the battery, and consequent changes of speed of the motor, are small. In order that the small changes of load on the alternator, produced by adjusting the bridge to obtain balance and by changes of bearing friction, shall not affect the speed, it is frequently arranged that the motor shall at the same time drive a small shunt generator. This acts as a permanent constant load which can be arranged to swamp the effect of the alternator (Fig. 80 (*a*)).

While these precautions are in many cases sufficient, it is necessary in very exact work with sharply tuned detectors, to have some means of checking even the smallest casual fluctuations of speed. One simple way is to apply a pair of telephones to the alternator, and to compare the note emitted by them with that of a standard tuning fork, regulating the motor speed by means of its field rheostat, or by light pressure of the hand upon the rotating shaft, until constant beats (or unison, as the case may be) are maintained. The frequency is thus known and is controlled.

A device of much greater sensitiveness is the Maxwell condenser bridge shown in Fig. 80 (*b*). In the use of this, a commutator, by means of which a condenser can be charged and discharged in a branch of a Wheatstone bridge, is mounted on the motor shaft. If the condenser be charged and discharged n times per second, then very nearly $n = R/SPC$. Hence with given resistances the auxiliary bridge is balanced for a certain value of n, i.e. for a certain speed of the motor shaft.[*] The observer merely has to watch the galvanometer G and keep it at zero by small adjustments of the motor field rheostat or otherwise. This method is much used at the Bureau of Standards Laboratory[†]; the speed, and hence the frequency, can be kept constant on the average to about 1 part in 10,000, and is known from the balance resistances of the Maxwell bridge.

Both the above methods have the disadvantage that two observers are necessary, one to keep the speed constant by the use of the auxiliary telephone or bridge, while the other makes the tests on the alternating current bridge. To bring the work within the scope of a single observer, automatic speed regulators have been devised.

One of the simplest devices was developed from the centrifugal governor by Giebe,[‡] following a suggestion of von Helmholtz. In Fig. 80 (*c*) the connections are shown; R is a small resistance in the field circuit of the motor, the speed of which is to be kept constant. The governor is mounted on the motor shaft and is shown diagrammatically in the figure. M is a mass which is supported upon a guide wire W stretched tightly in the frame F; spiral springs S, of which the tension can be varied, draw the mass inward. When the shaft is running, centrifugal force tends to cause the mass to move outward along W against the tension of the springs. At a sufficiently high speed the mass moves out so far that C_1 and C_2, a pair of light platinum contacts joined to the slip-rings ss, make contact and cut out of circuit the resistance R. The speed of the motor driving the shaft then falls, since its field is strengthened, the contacts open again. The speed thereupon rises and the whole cycle of operations is repeated. The driven shaft thus fluctuates about its average speed, and with proper adjustment great

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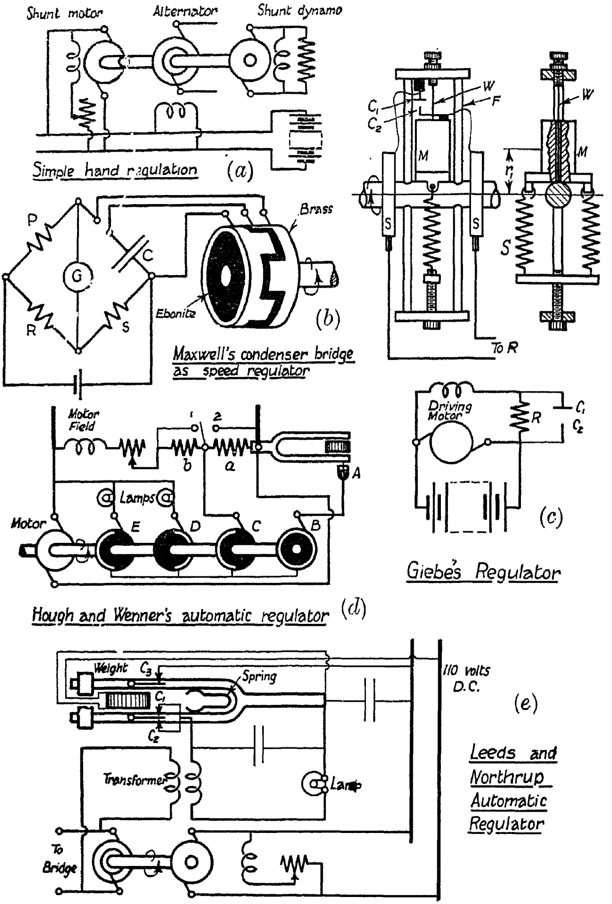
[*] The device was introduced at the National Physical Laboratory about 1902, and is widely used for the maintenance of constant speed. *Dictionary of Applied Physics*, Vol. 2, p. 127.

[†] E. B. Rosa and F. W. Grover, "The absolute measurement of inductance," *Bull. Bur. Stds.*, Vol. 1, pp. 128–129 (1905). E. B. Rosa and F. W. Grover, "The absolute measurement of capacity," *idem*, pp. 175–181 (1905). E. B. Rosa and N. E. Dorsey, "A new determination of the ratio of the electromagnetic to the electrostatic unit of electricity," *idem*, Vol. 3, pp. 557–561 (1907). F. W. Grover, "The capacity and phase difference of paraffined paper condensers as functions of temperature and frequency," *idem*, Vol. 7, pp. 504–505; also pp. 505–508 (1911).

[‡] E. Giebe, "Ein empfindlicher Tourenregler für Elektromotoren," *Zeits. f. Inst.*, Vol. 29, pp. 205–216 (1909). For modifications giving still higher sensitivity *see* H. Boas, "Drehzahlregler," *Elekt. Zeits.*, Vol. 52, pp. 1558–1559 (1931). E. Giebe, "Ein empfindlicher Drehzahlregler für Elektromotoren," *Zeits. f. Inst.*, Vol. 52, pp. 345–348 (1932).



Simple hand regulation (a)

Maxwell's condenser bridge as speed regulator (b)

Giebe's Regulator (c)

Hough and Wenner's automatic regulator (d)

Leeds and Northrup Automatic Regulator (e)

FIG. 80.—METHODS OF MAINTAINING ALTERNATORS AT CONSTANT SPEED

constancy is attained. Tests show that the variations from the mean speed do not exceed 1 part in 10^5.

The theory of the regulator is very simple. With the shaft at rest, let r_1 be the distance of the centre of gravity of M from the axis of rotation and T_1 the initial pull of the springs. When the device is running at angular velocity ω, the mass will fly out to a radius r and the equation of equilibrium will be $c(r-r_1) + T_1 = Mr\omega^2$, where c is the force per unit extension of the springs. Writing $e_1 = T_1/c$ for the original extension of the springs, it is easy to prove that equilibrium is stable if $r_1 \geqq e_1$; and the change of r for a small change in ω tends to infinity as $r_1 = e_1$. Hence if the springs are given such a tension as would bring the centre of gravity of the mass to the centre of the shaft—were its motion in that direction unimpeded—the apparatus is extremely sensitive to small changes of angular velocity. Putting this condition in the equation makes the critical velocity at which the most sensitive adjustment occurs $\omega = \sqrt{c/M}$. At speeds above this the apparatus is unstable, the mass flying out to the end of its travel and remaining there until the speed falls to the critical value. To change the speed at which the device will satisfactorily regulate it is necessary to change either the mass or the springs.

Other devices are developed from an arrangement described by Lebedew* in which the motor-driven shaft is made to run in synchronism with an electrically maintained tuning fork of suitable frequency. As modified by Hough and Wenner,† the apparatus is arranged in the manner illustrated in Fig. 80 (*d*). The switch is, in the running position, put on contact 1. The maintained tuning fork and the contact rings B, C mounted on the motor shaft are then arranged so that the resistance a is short-circuited during a greater or lesser fraction of a revolution, according as the speed of the shaft is above or below the normal value. To do this, the fork is provided with a contact A which is closed during half a vibration of the prongs. Contact B is a slip-ring in electrical connection with the half-ring C. Since C and A are in series, the resistance a is shorted only when both the contacts are simultaneously closed. For normal speed the contact C is arranged to close a quarter period later than contact A, so that a is cut out for a quarter of a revolution and is in circuit for $\frac{3}{4}$. If now the shaft runs too quickly the lag of C is reduced, a is shorted for a greater time and the motor speed will fall. On the other hand, when the speed falls too low, a is cut out for a shorter time, the motor field is weaker, and the speed rises. Two other half-rings, D, E, in electrical connection with B, serve to show when the arrangement is synchronized. To do this, the switch is thrown over to contact 2, putting b in place of a; b is about $\frac{3}{4}$ of a. The motor is then started and run up to such a speed that the lamps glow equally brightly; the switch is then put on to 1 and the apparatus continues to run in synchronism, regulating the speed as described.

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* P. Lebedew, "Ueber die ponderomotorische Wirkung der Wellen auf ruhende Resonatoren," *Ann. der Phys.*, Vol. 59, pp. 118–119 (1896).

† R. H. Hough and F. Wenner, "The operation of a direct current shunt wound motor in synchronism with a tuning fork," *Phys. Rev.*. Vol. 24, p. 535 (1907).

The Leeds & Northrup Co. have an ingenious modification of this device, shown in Fig. 80 (*e*), in which the fork is synchronized with the alternator instead of with a half-ring commutator. The tuning fork is of a special design arranged to work over a range of 50 to 75 vibrations per second. The variation is secured by sliding weights on the prongs and by springs pressing on them. The contact C_3 operates the maintaining circuit of the fork. A small transformer is connected to the alternator, the secondary winding being joined to a lamp through the contacts C_1 and C_2. If the fork and the alternator be running synchronously, two contacts per vibration will be made at the fork, two impulses of alternating voltage being applied to the lamp. Clearly, if the speed of the alternator be constant, these impulses will always take place at the same points in each voltage cycle.

Suppose now that the speed of the machine were to change, tending to put the alternating current and the fork out of synchronism. The voltages impressed on the lamp in successive cycles would thus be different, since contacts are not now made at the same points in each cycle. If the speed had increased, the contacts would be made at higher values of the voltage curve, tending to increase the current through the lamp ; the increased load then slows down the motor until it runs in step again. Should the speed decrease, the voltage impulses are lower, the current in the lamp is less, and the machine tends to increase its speed.

To avoid sparking, condensers are shunted across the contacts ; the transformer serves to isolate the lamp-load current from the maintaining current. The speed can be kept constant to about 2 parts in 10,000. Variations of alternator load or of the voltage on the motor up to 10 per cent can be tolerated without synchronism being destroyed.

Messrs. Muirhead & Co. use the method illustrated in Fig. 81 (*a*). The a.c. output is obtained from an inverted rotary converter deriving its input from d.c. mains, the converter being held in synchronism with a maintained fork by the use of a small centre-tapped transformer. The fork rectifies the a.c. supplied by the transformer and this rectified current is passed into an auxiliary field winding in the converter. Conditions are so arranged that a fall in the converter speed relative to the fork frequency causes this winding to weaken the total flux and restore synchronism. With an elinvar fork a constancy better than 1 part in 10^4 can be obtained; an overload of 400 watts on a 150 watt machine or a voltage change of 5 per cent with full load applied can be endured without losing synchronism.

Other methods, devised independently by Bearden and Shaw and by Orkney,* make use of gas-filled thermionic relay tubes of the "Thyratron" type. Such tubes consist of a glass envelope within which are contained a hot filament or kathode, an anode and an interposed grid exactly as in a normal thermionic tube, but instead of being evacuated to a very high degree the tube contains a small amount of inert gas (such as argon) or of mercury vapour. In the normal highly evacuated or "hard" triode the thermionic current passing between the kathode and the anode, when the latter is given a positive potential relative to the former, can be completely and continuously controlled by the potential of the intermediate grid. This is not the case, however, in the gas-filled or "soft" triode. If the anode be given a positive potential, no anode current will flow if the grid be maintained sufficiently negative. On reducing the grid potential a critical value will be reached at which current instantaneously starts to flow to the anode as an arc; the grid is surrounded by a positive ion sheath and has no further control on the current, which can only be stopped by removing the anode voltage, even for so short a time as a few microseconds, whereupon the grid again takes control. The device acts, therefore, as an inertialess relay; a few microwatts supplied to the grid releases quite a considerable flow of energy in the anode circuit, but has no control over its

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* J. A. Bearden and C. H. Shaw, "A constant speed d.c. motor control," *Rev. Sci. Insts.*, Vol. 5, pp. 292–295 (1934). J. C. Orkney, "Precision frequency control," *Elec. Rev.*, Vol. 117, pp. 669–670 (1935).

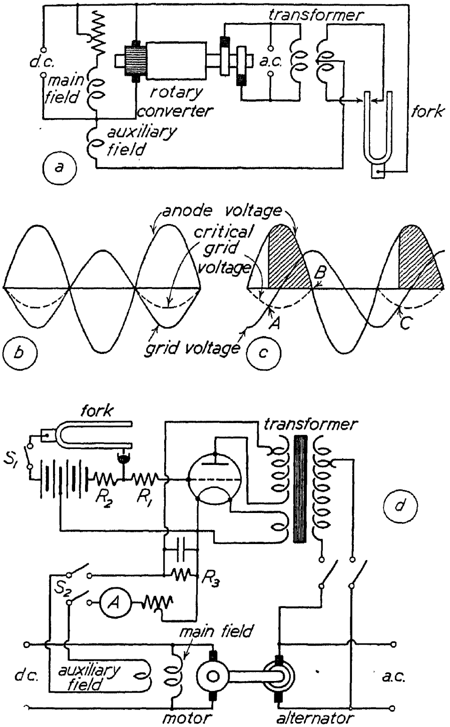

FIG. 81.—METHODS FOR MAINTAINING CONSTANT SPEED

magnitude. When the anode voltage alternates, as in Fig. 81 (*b*), to each value of the instantaneous voltage there corresponds a critical grid voltage at which the arc would be formed. Now suppose an alternating voltage is applied to the grid synchronous with that on the anode, in opposition thereto and greater than the critical value; no anode current can flow. If the grid voltage is shifted in phase, as in Fig. 81 (*c*), no current will flow until the point A is reached, when the arc is formed. The current continues throughout the shaded area up to the point B; here the anode voltage reaches zero and reverses, the arc is extinguished, the grid resumes control and current ceases from B to C when the process is repeated. The amount and duration of the anode current depend on the relative phase of the anode and grid voltages, and the current is unidirectional; this principle of phase control, introduced by Hull,* underlies the methods of speed control now to be described.

In Orkney's method the reference frequency is obtained from a maintained tuning fork which impresses impulses upon the grid of the tube, Fig. 81 (*d*), the alternator frequency being supplied to the anode. If the alternator runs too slowly, the phase difference between the grid and anode impulses releases anode current which is passed into the auxiliary field winding of the motor in such a sense as to weaken the total flux and raise the speed. The resistors R_1 and R_2 are about 0·1 megohm and limit the grid and fork currents; R_3 serves to pass a few milliamperes while the set is running up to speed before the switch S_2 is closed. Synchronism is obtained by running the set about 5 per cent too slow and then closing S_1 and S_2; it will pull into step and maintain perfect constancy at 50 cycles per second for hours at a time. Bearden and Shaw's method is rather more complicated. The reference standard is a rotating contact driven by a synchronous clock motor receiving its current from a triode oscillator, the frequency of which is stabilized by a maintained fork; this contact applies impulses to the grid of a gas-filled triode. A second similar triode receives grid impulses from a circuit containing a rotating contact operated by the alternator shaft. The two triodes are connected in such a way that the anode current of the second is controlled by the phase difference between their grid impulses; the anode current is used to modify the field of the driving motor as in Orkney's method. Constancy within 0·1 per cent is claimed at 60 cycles per second.

26. Thermionic Oscillators.

All the sources of current hitherto described depend for their action, in part at least, upon certain movements, e.g. the vibrations of a fork or bar, the rotation of a shaft or the movement of a microphone, and the use of purely mechanical devices to derive alternating currents as a consequence of these movements. In the class of generators now to be considered, the frequency is set either by an oscillatory electric circuit or by a mechanical system, such as a tuning fork.

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* A. W. Hull, "Hot cathode thyratrons," *Gen. Elec. Rev.*, Vol. 32, pp. 213–223, 390–393 (1929). Also see E. W. Golding, *Elec. Times*, Vol. 84, pp. 673–676 (1933).

This standard or "monitor" is maintained in vibration by purely electrical means, using some kind of thermionic device. Such arrangements are called thermionic oscillators and are widely used in bridge work.

Consider first a circuit with inductance L and capacitance C. If the resistance of the circuit be sufficiently small, the circuit will possess a natural frequency given approximately by $f = 1/2\pi\sqrt{LC}$. That is, if the circuit be disturbed and then left to itself, a sinusoidal current of this frequency will flow in it, this natural oscillation being gradually damped out by the resistance of the circuit. The frequency of these lightly damped oscillations is fixed entirely by the properties of the circuit, and is therefore constant. If some means can be found to maintain these natural oscillations, i.e. to make up for the dissipation of energy in the circuit, a sinusoidal current of fixed frequency and amplitude will be produced. This condition is realized in an oscillator by the devices described below. Similar considerations apply to certain mechanical oscillatory systems.

The action of an oscillator has a mechanical analogy in that of an ordinary pendulum clock. Here the pendulum is a mechanical system having a definite natural period, and is acted upon when in motion by certain frictional damping forces; it is the mechanical parallel of the oscillatory circuit. The energy required to run the driving train of the clock corresponds to the energy which the oscillatory circuit has to supply to the bridge. The source of energy is the clock spring. The action is then as follows: the pendulum being disturbed from its position of rest is maintained in steady motion by receiving impulses at the right instant from the escapement wheel, the energy of these impulses being drawn from the mainspring. The same escapement regulates the passage of energy from the spring to the driving train at each beat of the pendulum. Hence the oscillations of the pendulum are maintained and regulate the flow of energy to the gears which operate the hands, turning the latter at a regular rate fixed only by the vibration constants of the pendulum. The student will find that these processes are followed, by analogy, in a thermionic oscillator, the current in an oscillatory circuit or the vibrations of a mechanical system being maintained by a kind of escapement action, so that energy absorbed in damping and in the doing of external work is automatically supplied.

Vreeland's Oscillator. In Vreeland's oscillator* the oscillations in a circuit are maintained by means of a special type of mercury arc lamp shown diagrammatically in Fig. 82. A large, exhausted, glass bulb has at its lower part a tube containing a mercury kathode *K*, and two horizontal side tubes each containing a carbon anode, *AA*. A battery supplies current for the mercury vapour arc which, if everything is symmetrical, spreads equally from the kathode upon the two anodes. Across *AA* an oscillatory circuit is connected, consisting of an adjustable condenser in series with two coils, one placed behind and the other in front of the bulb.

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* F. Vreeland, "A sine wave electrical oscillator of the organ pipe type," *Phys. Rev.*, Vol. 27, pp. 286–293 (1908). B. Liebowitz, "Electrical oscillations from mercury vapour tubes," *Phys. Rev.*, 2nd series, Vol. 6, pp. 450–477 (1915). A. Tobler and K. Schild, "L'oscillateur Vreeland et son emploi dans les mésures à courant alternatif," *Journal Tél.*, Vol. 40, pp. 121–124, 145–148,

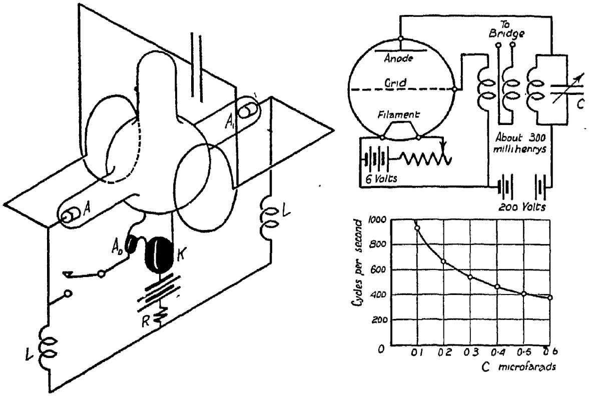

Fig. 82.—Vreeland's Mercury Tube Oscillator. The Simple Triode Oscillator

If the arcs be slightly disturbed so that one is stronger than the other, the potential of the anode emitting the stronger arc will be lower relatively to that which emits the weaker arc. There will thus be a difference of potential across the oscillating circuit and a current will start to flow in it. If the coils be wound in such a direction that their magnetic field causes the arc to be still further deflected toward the stronger side, the potential difference across the oscillatory circuit will increase until the condenser is fully charged. The condenser then commences to discharge, the arc is deflected to the other side,

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169–172, 193–196 (1916). See also W. A. Taylor and S. F. Acree, *Journal Amer. Chem. Soc.*, Vol. 38, pp. 2396–2403 (1916). J. L. R. Morgan and O. M. Lammert, *ibid.*, Vol. 48, pp. 1220–1233 (1926).

and the oscillations are maintained by the impulses given by the arcs to the two anodes, these impulses being controlled by the oscillations themselves.

The working circuit is taken from a third coil placed close to one of the deflecting coils; this is omitted from the figure. In order to keep the high frequency oscillations out of the battery, choking coils, L, L, are inserted; a resistance R serves to adjust the arc. The arcs are started by means of the auxiliary mercury anode A_0; by closing the key and tilting the bulb, the mercury in A_0 and that in K are brought into contact. A current flows between these electrodes and on restoring the tube to the vertical this current starts an arc which spreads to the anodes AA.

With the coils in a fixed position, the frequency can be adjusted over a range of from 160 up to 4,000 cycles per second merely by adjustment of the condenser. For any given capacitance, the frequency is steady and the wave form of the working current is very closely sinusoidal. An output of about 3 watts is obtainable.

Triode Oscillators. The Vreeland oscillator is now practically obsolete as a means of producing alternating current of a steady frequency, its place being taken by the thermionic triode tube or valve. The triode forms about the best obtainable source of current for bridge work; the frequency is quite constant, the wave form is very nearly sinusoidal and the output is ample. The apparatus is easily procured and assembled, and is quite cheap.

It is not proposed here to enter into the theory of the action of the triode, this being adequately treated in books on wireless telegraphy.* Attention will be directed merely to the description of triode oscillators and their mode of working.

The three-electrode tube or triode consists of a highly evacuated glass bulb containing within it three elements: (i) a tungsten filament heated by the passage of electric current through it; (ii) a plate of tungsten or molybdenum, known variously as the "anode" or "plate"; (iii) interposed between the filament and the anode is a third electrode, called the "grid," consisting of a piece of metal gauze, perforated foil, or merely of a wire spiral. The negative terminal of the filament, and the terminals of the anode and grid constitute the three electrodes.

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* For example, E. Mallett, *Telegraphy and Telephony including Wireless* (Chapman & Hall, 1929). L. S. Palmer, *Wireless Engineering* (Longmans, Green, 1936). Also see particularly E. V. Appleton, *Thermionic Vacuum Tubes and their Applications* (Methuen, 1933).

A simple method of connection is shown diagrammatically in Fig. 82, this figure having reference to a small oscillator used in many laboratories. The filament is kept at white heat by current from an accumulator. Between the anode and the filament an oscillatory circuit, consisting of an inductance and a condenser in parallel, is connected through a battery, so that the anode is strongly positive with respect to the filament. Inductively coupled to this circuit is a coil joining the grid to the filament.

The action is then, roughly, as follows. A stream of negative electrons steadily emitted from the hot filament crosses the bulb and impinges on the anode, a steady current thus flowing through the oscillatory circuit from the high voltage battery. If the current in the oscillatory circuit be disturbed, e.g. by switching on, oscillations will be set up in this circuit, having a frequency fixed by the inductance of the anode coil and the capacitance of the condenser in parallel with it. These oscillations act inductively on the grid and modify its potential. Now the function of the grid is to act as a control electrode upon the electronic emission from filament to anode, the unique property of the triode being that small variations of grid potential will produce changes in anode current. If, therefore, the relative directions of winding of the grid and anode coils be correct, this amplifying property of the tube may result in such modifications of the anode potential that the oscillations are maintained, energy being drawn from the battery at the right instant to make up for the losses in the oscillatory circuit and for the energy taken by the working circuit. The working circuit is coupled inductively to the other coils by a third winding, as shown in the diagram.

In the apparatus mentioned the triode is of the "French" or "R" type, taking a filament current at 0·75 ampere at 6 volts. The anode coil has an inductance of about 300 millihenrys and is shunted by a good paper condenser which can be varied in steps of 0·01 microfarad. The 200 volt battery consists of small storage cells. With the cells well charged, it is easy to obtain an output of 30 milliampères through 60 ohms at 400 cycles per second. By adjustment of the condenser C the frequency can be varied from about 350 cycles per second upwards, being almost exactly proportional to $1/\sqrt{C}$.

The Type *A* oscillator of the Cambridge Instrument Co. is of this kind; it has a range of 60 to 7,000 cycles per second and gives an output of 0·2 watt into a load of 1,000 ohms, with good wave form.

By suitable design, a triode oscillator can be made to give a pure wave form and to cover an enormous range of frequency. From 300 cycles per second up to the values used in radio telegraphy—millions per second—the apparatus is compact and easy to construct. For the lower frequencies, however, very large coils and condensers are necessary, though oscillators have been constructed to work at values as low as 5 cycles per second. In modern bridge work, with the use of highly-sensitive, sharply-tuned, vibration galvanometers, the triode oscillator becomes almost a necessity and is by far the best source to use for such work.

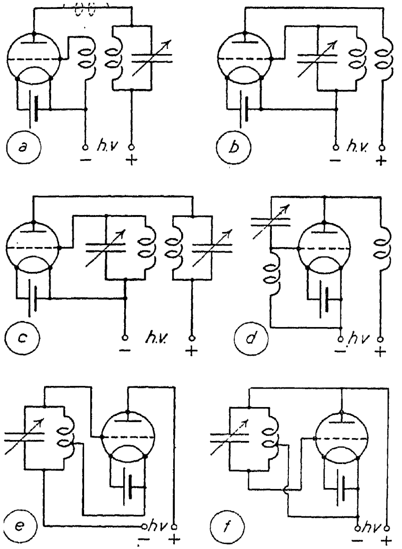

FIG. 83.—FUNDAMENTAL TYPES OF SIMPLE TRIODE OSCILLATORS

Many other fundamental circuits have been used in triode oscillators designed as bridge sources, some being shown in Fig. 83. In these diagrams a number of omissions are made for simplicity, e.g. (i) the output circuit, which is usually taken from a coil inductively coupled to the oscillating circuit; (ii) grid bias, the negative voltage often applied to the grid to reduce the grid current to zero, thereby reducing damping of the oscillations and improving the frequency stability of the device; (iii) choking coils to keep oscillatory currents out of the batteries and similar arrangements improving the general performance. Fig. 83 (*a*) shows the common type of simple tuned-anode oscillator. In Fig. 83 (*b*) the grid circuit is tuned, an arrangement only used in low-power oscillators since large outputs can only be obtained by using large voltage variations on the grid; these will set up saturation long before the anode circuit develops the full output of which it is capable, with resulting distorted wave form. Fig. 83 (*c*) combines both methods, as in an oscillator described by Campbell and Childs (*loc. cit. ante*). In all these circuits reaction between the anode and grid circuits, with consequent maintenance of the oscillations, is obtained by inductive coupling. An oscillator used by Adams and Hall* in some electrochemical tests uses capacitive coupling, the coils being independent; the circuit is shown in Fig. 83 (*d*). The so-called Hartley† circuit, Fig. 83 (*e*), combines inductive and capacitive coupling; this circuit is equivalent to Fig. 83 (*b*) with autotransformer between grid and anode. A modification, Fig. 83 (*f*), is used in the Cambridge Instrument Co. Type B oscillator described below.

These simple oscillators have certain defects which become very important in precise testing, the most notable of these being frequency drift or instability and impurity of wave form. Frequency changes may arise from the triode itself, either by changes in the anode, grid, or filament voltages or in the spacing of the electrodes with temperature; alternatively, they may be due to alterations in the capacitance and inductance of the oscillatory circuit, contributory causes being rise of temperature of both elements, and change of effective inductance by variations in the load. Mallett‡ has shown that frequency drift arising from the variation of tube characteristics may be removed by adding series inductance of suitable amount to the tuned anode circuit as shown dotted in Fig. 83 (*a*). Drift due to the oscillatory circuit can be overcome by (i) the use of low temperature-coefficient inductors and condensers; (ii) using the oscillator tube only as a "master" in the manner to be described below. Change in wave form with load is caused by operation of the tube on a non-linear part of its characteristic.

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* L. H. Adams and R. E. Hall, *Journal Amer. Chem. Soc.*, Vol. 41, pp. 1515–1525 (1919).

† M. G. Scroggie, "The Hartley circuit. Evolution of the single coil oscillator," *Wireless World*, Vol. 40, pp. 486–487, 510–512 (1937).

‡ E. Mallett, "Frequency stabilization of valve oscillators," *Journal I.E.E.*, Vol. 68, pp. 578–582 (1930). F. B. Llewellyn, "Constant frequency oscillators," *Proc. Inst. Rad. Eng.*, Vol. 19, pp. 2063–2094 (1931). Also see R. W. King, "Thermionic vacuum tubes and their applications," *Journal Opt. Soc. Amer.*, Vol. 8, pp. 77–147 (1924) for another method of stabilizing an oscillator.

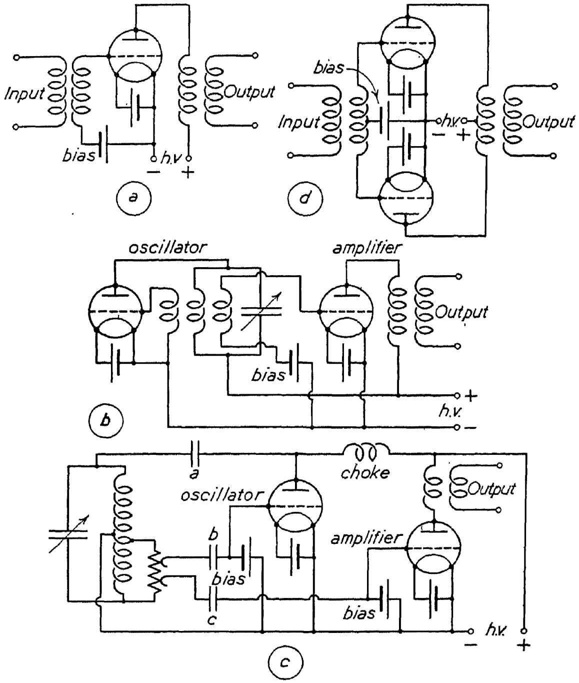

FIG. 84.—TRIODE OSCILLATORS WITH AMPLIFIERS

The frequency drift and change of wave form due to the loading of the oscillator can be avoided by using one triode tube to maintain the oscillations as its sole duty. The load is taken from a second triode arranged as an amplifier, receiving voltage impulses from the oscillatory circuit. Fig. 84 (*a*) shows the simplest type of amplifier, in which an alternating voltage from the oscillator is impressed in the grid circuit, sufficient bias being applied to ensure that the grid is always negative. In these circumstances no grid current can flow, and the oscillator is unloaded. The grid voltage fluctuations are amplified by the triode and liberate current in the anode circuit, to which the output circuit is coupled. Fig. 84 (*b*) shows the essential elements of a generator consisting of a tuned-anode unloaded oscillator combined with a simple amplifier. By sharing the operation between two tubes, one maintaining the oscillations and the other delivering the output, a very pure wave form can be obtained and it is possible to draw a much greater output without fear of distortion than is possible in a single-tube generator. Final harmonic traces can be eliminated by an output filter and by choking coils in the h.v. circuit. Apparatus on this principle is made by a number of firms.

The Type *B* generator of the Cambridge Instrument Co. has some interesting features, its essential circuits being shown in Fig. 84 (*c*), which should be compared with the modified Hartley circuit of Fig. 83 (*f*). The h.v. is supplied directly to the anode of the oscillator tube through a smoothing choke, and the oscillating circuit is put in parallel with the anode circuit; a blocking condenser *a* excludes direct current from the coil. The grid voltages for the maintenance of the oscillations and for the production of output are derived from a resistance divider tapped across a portion of the coil; the amount of grid voltage can be independently regulated for each tube, and bias is provided to reduce the grid currents to zero. Blocking condensers *b* and *c* prevent current flowing in the conductive parts of the circuit, from the bias batteries. The range of frequency is 60 to 30,000 cycles per second the output being about 0·3 W. at 100 cycles per second, 2 W. at 350 cycles per second and 0·2 W. at 30,000 cycles per second. The condenser, coil, and tube circuits form three separate units.

In other types of generator, e.g. those of the Ericsson Aktiebolaget (Type ZD200) and Salford Electrical Instruments (Types 111X and 112X), a simple tuned-anode oscillator feeds a *push-pull* amplifier illustrated in Fig. 84 (*d*). This device consists of two tubes so connected that their grids receive similar but opposite voltages, in consequence of the centre tapping. Hence the curvature of the characteristics of the tubes will be compensated; considerable grid swing can be used, therefore, giving large output with very pure wave form.

A type of oscillator due to Gunn* uses two tubes in an ingenious way to stabilize the frequency, amplify the output and purify the wave form; the essential circuit is shown in Fig. 85 (*a*). Two similar, sharply tuned anode circuits are used, each with its own tube; the parallel impedance of each circuit

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* R. Gunn, "A source of constant frequency oscillations," *Journal Opt. Soc. Amer.*, Vol. 8, pp. 545–547 (1924). "A new frequency stabilized oscillator system," *Proc. Inst. Rad. Eng.*, Vol. 18, pp. 1560–1574 (1930). Also see M. Lattmann and H. Salinger, *E.N.T.*, Vol. 13, pp. 133–142 (1936) for an arrangement without tuned circuits.

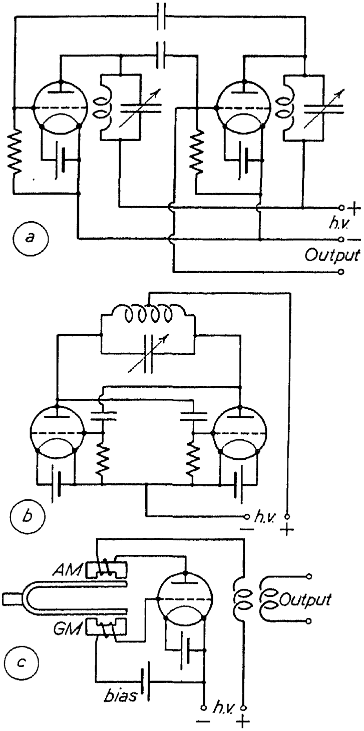

FIG. 85.—GUNN'S OSCILLATOR. ECCLES AND JORDAN'S OSCILLATOR.· TRIODE-MAINTAINED FORK

is high for the tuned frequency and low for all others. The grid of the first tube is capacitively coupled to the anode of the second and conversely, forming a cascade arrangement. A potential variation on the first grid is amplified, reversed in phase and passed on to the second grid, the process being repeated by the second tube; the twice amplified potential arrives at the first grid in phase with the initial variation and tends to keep the arrangement oscillating. If the couplings are correct, the oscillations with frequency corresponding to the high parallel impedance are amplified and passed through the system again and again; those of other frequencies are attenuated and gradually dissipated. Hence the arrangement simulates a filter circuit and oscillates strongly at the tuned frequency. In a 1,000 cycle oscillator the frequency change was only 0·1 per cent for a 50 per cent change of anode voltage.

When large outputs are required, two or more triodes may be operated in parallel, but it is difficult to attain success unless they are exactly similar and the grid voltages are accurately adjusted. Moreover, if they are too closely coupled, excessive oscillation of the grid voltage may be produced, carrying the tubes off the straight parts of their characteristics and greatly distorting the wave form. To avoid these troubles, several workers with a.c. bridges have devised *push-pull* oscillators on the principle described by Eccles and Jordan,* (Fig. 85 (*b*). The centre tapping causes the grid potentials to vary in opposite directions, increasing the anode current of one tube and decreasing that of the other; the phase of the change can be arranged, by the cross-coupling condensers, to maintain the oscillations. Thus one tube is responsible for one half-cycle and the other for the second; considerable grid swing can be used since the effect of curvature of the characteristic cancels out. The generator oscillates very freely and with good wave form; the grid resistors are adjusted by trial to give the best value of grid potential. The output is taken from a coil coupled to the oscillatory circuit. In some cases the arrangement is combined with an appropriate stage of amplification.

The normal types of oscillator are not very convenient for the generation of low frequencies, since coils and condensers necessarily become very large. To avoid this and also to give a long range of frequency with a single control many workers have designed *beat-tone* or *heterodyne* oscillators. In these, two independent stable oscillators generate very high frequencies, e.g. 100,000 to 200,000 cycles per second; one of these, the master oscillator, works at a fixed frequency and the other may be varied by a single condenser control. The outputs of these oscillators are applied to a suitable triode amplifier or output stage, the output from which will thus contain the low-frequency corresponding with the difference in the oscillator frequencies; the summation frequency can be filtered out if desired, but as it is supersonic is usually unimportant in bridge work. Thus if one oscillator works at 100,000 and the other at 100,010, a frequency of 10 cycles per second has been generated by the use of small value coils and condensers corresponding with these high frequencies. It is essential that the oscillators have a high degree of frequency stability,* and that they should not tend to pull into step until their frequency difference is very small, much less than 1 cycle per second. It is usual to provide some check, such as a tuned reed, by means of which a point on the scale can be calibrated. Beat-tone oscillators for bridge work are made by Messrs. Muirhead, Salford Electrical Instruments Ltd., and General Radio Co., among others; a typical range is 10 to 20,000 cycles per second.

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* W. H. Eccles and F. W. Jordan, "A method of using two triode valves in parallel for generating oscillations," *Elecn.*, Vol. 83, p. 299 (1919). J. A. Stratton, "A high-frequency bridge," *Journal Opt. Soc. Amer.*, Vol. 13, pp. 471–494 (1926). T. Shedlovsky, "A screened bridge for the measurement of electrolytic conductance," *Journal Amer. Chem. Soc.*, Vol. 52, pp. 1793–1805 (1930). J. M. Hudack, "A low-frequency oscillator," *Bell Lab. Rec.*, Vol. 10, pp. 378-380 (1932).

All the oscillators that have been considered use an oscillatory electric circuit as the triode-maintained element. In another class are oscillators in which a suitable mechanical system is used as the vibrating element.† The simplest example is the triode-maintained tuning fork due to Eccles, Fig. 85 (*c*), made by the Cambridge Instrument Co. (*see* references on p. 201). Here *AM* and *GM* are permanent magnets, such as telephone receiver magnets, with coils joined respectively in the anode and grid circuits of a tube. When the fork vibrates, an e.m.f. of the fork frequency is induced in the grid coil and is impressed on the grid, causing oscillations of the same frequency in the anode current. If the connections of *AM* are correct, the vibration of the fork will be maintained. Output to the bridge is taken from a transformer in the anode circuit.

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* For stabilization by means of a piezo-electric crystal, *see* A. Hund, "Generator for audio currents of adjustable frequency with piezo-electric stabilization," *Bur. Stds. Sci. Papers*, No. 569, pp. 631–637 (1928). E. G. Lapham, "An improved audio-frequency generator," *Bur. Stds., Journal of Res.*, Vol. 7, pp. 691–695 (1931).

† W. S. Stuart, "A valve-maintained stretched wire vibrator," *Phil Mag.*, Vol. 18, pp. 566–571 (1934). C. W. Oatley, "A low-power audio-frequency current supply for general laboratory use," *Journal Sci. Insts.*, Vol. 6, pp. 217–220 (1929).

For high precision oscillators the vibrating system may be a bar of nickel alloy set into oscillation by magneto-striction or, as is now more usual, a quartz crystal vibrating under piezo-electric forces; these elements are entirely confined to high-frequency work, and do not seem to have been applied to ordinary bridge measurements.

The coils of a triode oscillator set up a considerable stray field, which may be a serious cause of interference with the bridge circuit. This trouble can be overcome in several ways; (i) by removing the oscillator to a distance; (ii) more conveniently, by using toroidal windings or enclosing the oscillator in a suitable shield. The d.c. supplies for filament, bias, and anode voltages, are best taken from batteries, but the high voltage battery for the anode is often troublesome to maintain. A useful alternative is to take the anode supply from a thermionic full-wave rectifier connected to a.c. mains, many types of which are on the market.

27. Wave-Filters and Interbridge Transformers. *Wave-Filters.* In precise work it is necessary to use some "filtering" device in all cases where absolute purity of wave form is essential to sharp balance. Three types of filter are in use: (i) in which all harmonic components of the impure wave, except one, are *partially* suppressed; (ii) in which a certain range of harmonic components is suppressed, and the remainder accepted; and (iii) in which one component is *entirely* suppressed.

(i) Fleming and Dyke* have used the first. Their alternator, the wave form of which contained very pronounced harmonics, supplied a variable inductance and a condenser in series, the inductance being adjusted until a chosen harmonic was in resonance. To enhance the harmonic still further, this first circuit was coupled by means of a transformer to a second filter circuit adjusted to resonance. A transformer in the second circuit supplied the bridge. The chosen harmonic was thus exaggerated to such an extent that the other components were of negligible importance. All the coils were free from iron.

(ii) Wide use is made of filter circuits consisting of "chain" or "ladder" networks, three standard forms of which are shown in Fig. 86; each consists of an assembly of similar T-shaped units or links of series and shunt elements. In Fig. 86 (*a*) the series element is inductive and the shunt capacitive. For frequencies below a critical value this network offers a low impedance, the impedance rising rapidly as this frequency is passed. The critical value is given by $\omega_c = 2/\sqrt{(LC)}$, called the cut-off pulsatance, where $\omega_c = 2\pi f_c$. Currents of all frequencies below f_c pass readily through the filter, but those of frequencies above this value are greatly attenuated; the network is called a *low-pass -filter*. In Fig. 86 (*b*) the network consists of series capacitances and shunt inductances; it offers low impedance to frequencies above a critical value derived from $\omega_c = 1/2\sqrt{(LC)}$, and high impedance to all other frequencies. Thus currents with frequencies above f_c pass through the filter with ease and those of frequencies below this are attenuated; the arrangement is a *high-pass filter*. The network of Fig. 86 (*c*) has two critical pulsatances, $\omega_c' = 1/\sqrt{(LC_1)}$ and $\omega_c'' = \sqrt{[(4/LC) + (1/LC_1)]}$

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* J. A. Fleming and G. B. Dyke, "On the power factor and conductivity of dielectrics when tested with alternating currents of telephonic frequency at various temperatures," *Journal, I.E.E.*, Vol. 49, pp. 323–431 (1912).

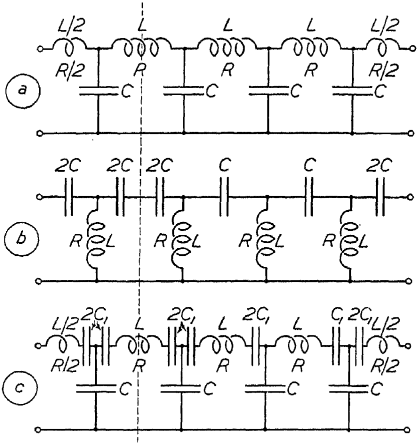

FIG. 86.—LOW-PASS, HIGH-PASS, AND BAND-PASS FILTERS

giving frequencies f_c' and f_c'' respectively. Frequencies below f_c' and above f_c'' are attenuated; frequencies between these values are readily passed. The arrangement is a *band-pass filter*.

In all filters the resistance should be low, ensuring sharp cut-off and rapid rise of attenuation of the rejected frequencies. Also, the filter must be designed to avoid terminal reflections, i.e. it must be properly terminated at its input and output terminals. The reader is referred to the relevant books* for details of filter circuit theory and design.

(iii) In many alternators or hummers there is often one particular harmonic which causes trouble and which it is desired to suppress. Campbell† has described a number of ways in which this suppression may be carried out. One of these is shown in Fig. 169 (*a*). Suppose the telephone in the figure is replaced by the primary of a small transformer from the secondary of which the bridge is to be supplied. Then if M and C be such that $2\pi f = 1/\sqrt{MC}$, no current of frequency f—which is that of the undesirable harmonic—can flow in the transformer. This method is suitable only for higher frequencies owing to the inconveniently large value of MC at low frequency. For frequencies of the order of 100 or less, Campbell describes several other types of sifter (see p. 465).

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* A. E.Kennelly, *ArtificialElectric Lines* (McGraw-Hill, 1917). G. W. Pierce, *Electric Oscillations and Electric Waves* (McGraw-Hill, 1920). E. Mallett, *Telegraphy and Telephony* (Chapman & Hall, 1929). A. T. Starr, *Electric Circuits and Wave Filters* (Pitman, 1934).

† A. Campbell, "On wave-form sifters for alternating currents," *Proc. Phys. Soc.*, Vol. 24, pp. 107–111, 158–159 (1912). For a more perfect type of filter *see* G. A. Campbell, *Phil. Mag.*, Vol. 5, p. 313 (1903).

Interbridge Transformers. It is always best to connect the source, whether it be alternator, interrupter, hummer, or oscillator, to the bridge *via* a small transformer whose windings are well insulated from one another. Between the two windings an earthed metallic shield is put, by means of which capacitance effects between the source and the bridge are reduced to a minimum. Such capacitance effects are readily detected if, after the bridge has been balanced, the connections to the transformer secondary are reversed. In general, small adjustments of the branches will be required to restore balance ; the mean of the two values represents the true balance, provided that the capacitance effects are small. For a further discussion see p. 537.

An interbridge transformer should have a very small leakage field so as not to interfere inductively with the bridge. Various windings should be provided so that the working voltage may be varied as desired. A closed iron ring wound with uniformly distributed toroidal windings with an earth shield to separate the primary coils from the secondary is very suitable.*

The choice of a suitable ratio of transformation has an important effect on the sensitivity of the bridge. Suppose Z to be the impedance of the source, t_1 the number of turns in the primary winding to which it is joined, the secondary coil of t_2 turns being connected to the bridge. Then, if $t_2/t_1 = T$ be the ratio of transformation, the effective impedance of the source expressed in terms of the secondary winding of the transformer is T^2Z. This artifice is of great service when measuring small condensers by the methods shown on pages 327 to 374. For sensitivity it is pointed out that, since the branches of the network have a high impedance, a high impedance source and detector are requisite. This can be artificially produced by connecting a source of normal impedance to the bridge through a step-up ($T > 1$) transformer of appropriate ratio. A precisely similar method can be used to adapt detectors to high or low impedance bridges, *see* p. 494.

DETECTORS

To determine when an alternating current bridge is balanced, some detecting instrument capable of giving an indication with a small alternating current or voltage is required. In most cases the detectors usually employed are either telephones or vibration galvanometers, but there are many other devices which are frequently of service. In this section detectors will be classified and described, their special fields of usefulness being indicated. Broadly speaking, two main classes will be distinguished. The first class has phase-selectivity, i.e. it enables discrimination to be made between reactance and resistance adjustments when balancing a bridge. The second class has frequency-selectivity, i.e. high sensitivity to currents of one frequency and low response to all others.

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* For the design of a suitable transformer see *Dictionary of Applied Physics,* Vol. 2, p. 393.

28. Mechanical Rectifiers. On page 6 and in Fig. 1 (*d*) it has been pointed out that one of the earliest applications of alternating current to bridge work is found in the secohmmeter. In this method a crude type of a.c. is supplied to the bridge by means of a reversing commutator and battery, a second commutator successively reversing the connections to a ballistic galvanometer so that the transient impulses in the instrument act in the same direction and are additive, producing a steady deflection. Owing to the sensitiveness of the modern moving-coil ballistic galvanometer, several devices have been introduced to enable this instrument to be used as the detector in bridges to which a sinusoidal current is supplied. Obviously, it is necessary to have some commutating device in the galvanometer circuit working synchronously with the alternating supply and reversing the galvanometer connections at each half-wave of the supplied current. For use with a two pole alternator, a simple two-part, four-brush commutator mounted on the alternator shaft is all that is required, as in Fig. 1 (*d*).

Fig. 87 (*a*) illustrates a rather more elaborate arrangement designed by Dr. H. F. Haworth for application to a 6-pole alternator. The commutator is driven by the alternator shaft ; contact is made with it by five small carbon brushes, these being mounted at 60° apart on a fibre ring surrounding the commutator so that they may be turned into the most advantageous position. When used at low frequencies, with a good moving coil ballistic galvanometer, very precise balance settings can be secured.

Contacts between stationary brushes and a rotating commutator are frequently troublesome. Sharp and Crawford* have designed, therefore, a special reversing key in which the contacts are operated by a cam driven in synchronism with the alternator.

By adjustment of the position of the brushes, or the cam, relatively to the alternator, the reversals of the galvanometer can be made to take place at any desired part of the current wave; the galvanometer can be set, therefore, to be sensitive to one component of the current and to be insensitive to the component in quadrature therewith. It is easy with a synchronous commutator, therefore, to make sensitive independent adjustments of the bridge for reactance and then for resistance, an advantage shared with many other instruments.

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* C. H. Sharp and W. W. Crawford, "Some recent developments in exact alternating current measurements," *Trans. Amer. I.E.E.*, Vol. 39, part 2, pp. 1518–1521 (1911).

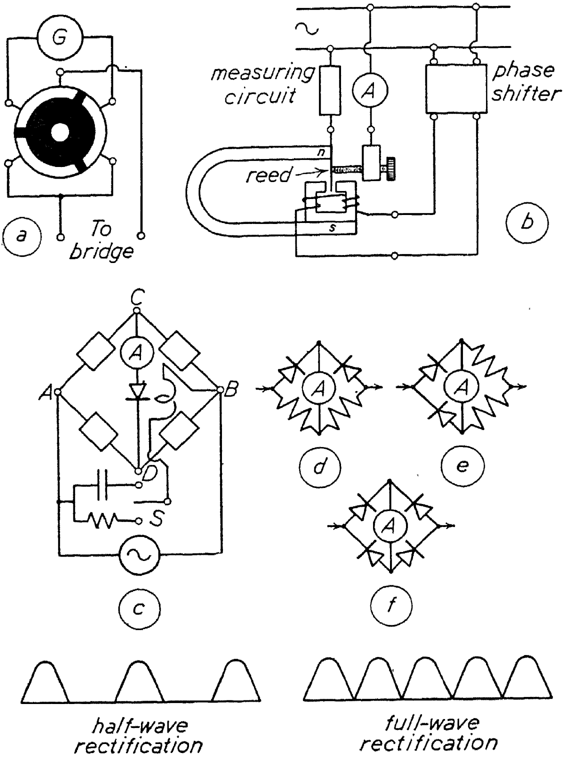

Fig. 87.—Mechanical Rectifiers

Small vibrating rectifiers have been successfully developed by the Siemens & Halske A.G., and applied to many classes of a.c. measurements, including bridge methods. The principle* is shown in Fig. 87 (*b*). A small steel reed or tongue is set into forced vibration by means of a polarized magnet excited from a phase-shifting device; the reed interrupts the measuring circuit, which is connected to the same source as the phase-shifter. When the reed is at rest the contact is closed; during one half-cycle, therefore, the contact is opened and during the next remains closed, so that the device acts as a half-wave rectifier, suppressing alternate half-cycles. A d.c. ammeter indicates the average value of the current. The phase of the rectification can be changed by adjusting the phase-angle γ between the voltage applied to the measuring circuit and that applied to the magnet. Let R be the total resistance of the measuring circuit, and $v = v_1 \sin\theta$ the voltage applied to it, where $\theta = \omega t$. Suppose the contact closed from $\theta = 0$ to $\theta = \gamma$, opened for the next half-period $\theta = \gamma$ to $\theta = \pi + \gamma$, and then reclosed from $\theta = \pi + \gamma$ to $\theta = 2\pi$. The currents in these three intervals will be v/R, 0, and v/R, and the average value read on the moving coil ammeter will be

$$i_{av} = \frac{v_1}{R} \cdot \frac{1}{2\pi}\left[\int_0^{\gamma} \sin\theta \, . \, d\theta + \int_{\pi+\gamma}^{2\pi} \sin\theta \, . \, d\theta\right]$$
$$= -\frac{1}{\pi} \cdot \frac{v_1}{R} \cos\gamma = -\frac{\sqrt{2}}{\pi} I \cos\gamma,$$

I being the r.m.s. current if the rectifier were removed. The angle γ gives to the arrangement its phase-selective properties. In its simplest application, Fig. 87 (*c*), the rectifier is put across the bridge branch-points C and D, the magnet winding across A and B in series either with a condenser or a resistor. The current through the magnet winding can thereby be shifted in phase by about 90° relative to the supply voltage. Approximately null reading is obtained, with S in the upper position, by operating one of the two essential bridge adjustments; then with S in the lower position a deflection will be obtained that can be reduced nearly to zero by means of the other bridge adjustment. By repeating the process until zero reading occurs in both positions, the current in the detector is finally annulled and the bridge is balanced. These rectifiers work satisfactorily up to 1,000 cycles per second.

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* H. Pfannenmüller, "Mechanische Gleichrichter für Mess-Zwecke. Grundbegriffe, Bezeichnungen und Formeln," *Arch. f. tech. Mess.*, Z. 540–1 (Feb., 1932). J. Krönert, "Neuere Messmethoden mit Schwingkontakt-Gleichrichter," *Zeits. f. tech. Phys.*, Vol. 14, pp. 474–477 (1933). H. Pfannenmüller, "Überblick über die Messvefahren mit Gleichrichtern," *Arch. f. Elekt.*, Vol. 28, pp. 356–384 (1934). Also see *Instrument Transformers* (Pitman), p. 400.

This simple arrangement opens the detector circuit each half-cycle and may give rise to trouble, especially in current measurements. Many methods* have been devised, using two or more rectifiers, to give full-wave rectification, i.e. inversion of the negative half-cycle instead of its suppression. Figs. 87 (*d*) and (*e*) show two ways using a centre-tapped resistor (or transformer secondary winding); Fig. 87 (*f*) is the so-called Graetz connection requiring four rectifiers. In all these diagrams current flows through the rectifier only in the direction shown by the open arrow head and is zero in the opposite direction.

29. Copper Oxide Rectifiers. Unilateral conductors of various kinds have been used as rectifiers, and some have been applied to bridge methods. Anderson† used a rectifying crystal and reflecting galvanometer as far back as 1912, but it is only quite recently that electronic rectifiers have been developed capable of dealing with sufficient current to operate a d.c. pointer instrument. Of these the selenium-iron and cuprous oxide-copper rectifiers are the most noteworthy, and the second alone has been used in bridges. As shown in Fig. 88 (*a*), the rectifier introduced by Grondahl‡ consists of a copper disc with a layer of cuprous oxide about 0·001 in. thick upon one face. This oxide must be "grown" on the copper by thermal or other means; oxide pastes, etc., merely applied to the surface are almost useless. Contact is made with the oxide by a lead or soft foil disc, the whole being clamped up by an insulated bolt. Fig. 88 (*b*) shows in double logarithmic co-ordinates how the resistances across an oxide-copper interface of 1 sq. cm. area varies with the applied voltage for each direction of current. It will be seen that (i) at low voltages, less than 0·1 volt, the resistances are nearly equal, so that rectification is almost impossible; (ii) in the range 4 to 6 volts the resistance from copper to oxide is very much higher than that from oxide to copper, the ratio being at least 7,000, so that the device here behaves as an efficient half-wave rectifier. Rectification is good even at 20,000 cycles per second. These rectifiers work at about 6 volts and can carry about 2 A. per sq. in. without artificial cooling; series or parallel grouping can be arranged for higher voltages or currents.

Strobel§ has described a bridge detector, consisting of four rectifiers connected as in Fig. 87 (*f*), to supply a 3,500 ohm pointer d.c. microammeter reading to 0·4 μA. with divisions of 0·005 μA. Each rectifier consists of 0·1935 in. dia. discs in an insulated hole drilled in a brass block and has a zero-voltage resistance of 50,000 ohms. It is found that the sensitivity in the audio range is as good as that of a telephone, with the advantage that the rectifiers can be used down to zero frequency,

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* H. Pfannenmüller, "Mechanische Gleichrichter. Die Messkreisschaltungen mit einem oder mehreren einheitlich erregten Gleichrichtern," *Arch. f. tech. Mess.*, Z. 540–3 (Apr., 1932). "Erregerschaltungen für eine noder mehrere einheitlich erregte Gleichrichter, insbesondere zur Vektoren-Bestimmungen," *idem.*, Z. 540–4 (Dec., 1932).

† S. H. Anderson, *Phys. Rev.*, Vol. 34, pp. 34–39 (1912).

‡ L. O. Grondahl, "A new type of rectifier," *Phys. Rev.*, Vol. 27, p. 812 (1926). L. O. Grondahl and P. H. Geiger, "A new electronic rectifier," *Trans. Amer. I.E.E.*, Vol. 46, pp. 357–364 (1927).

§ C. K. Strobel, "A sensitive visual detector for alternating current bridge methods," *Rev. Sci. Insts.*, Vol. 6, pp. 319–320 (1935).

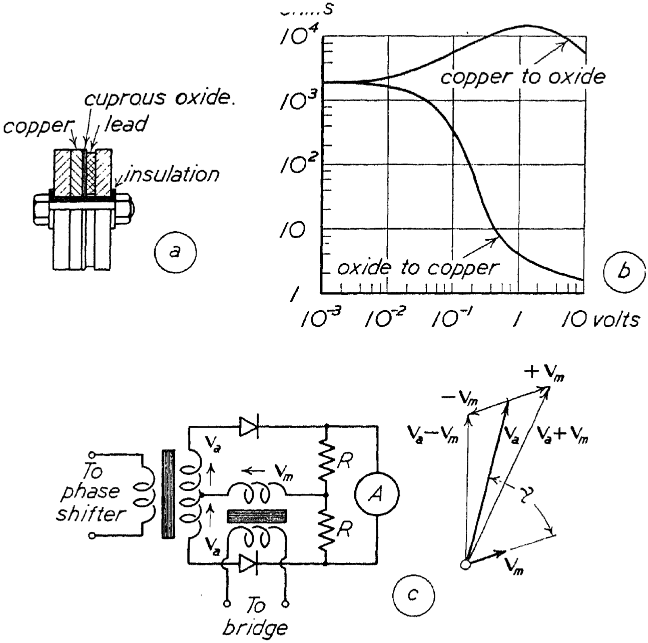

FIG. 88.—COPPER OXIDE RECTIFIERS

or well above the audio limit, with little change of sensitivity. He shows that for high sensitivity it is necessary that the zero-voltage resistance of the rectifier assembly, i.e. of a single element when four are used in Graetz-connection, be high compared with the impedance of any bridge arm; by this means the highest possible voltage can be maintained on the rectifier for a given percentage unbalance of the bridge.

Copper oxide rectifiers are now incorporated in many standard pointer instruments of robust construction; many workers have designed bridge detectors in which commercial instruments of this class are operated by a thermionic amplifier. Some of these will be noted in the next section.

Simple electronic rectifiers are not inherently phase selective, but can be given this characteristic by suitable methods of connection. Fig. 88 (*c*) shows an arrangement due to Walter,* in which $\mathbf{v}_a$ is an auxiliary voltage of adjustable phase supplied by a centre-tapped transformer, $\mathbf{v}_m$ is the voltage to be measured, taken from a transformer joined to the detector branch-points of the a.c. bridge; γ is the angle between them, and $\mathbf{v}_a$ is large compared with $\mathbf{v}_m$, so that the momentary sign of $\mathbf{v}_a$ determines whether one or the other rectifier will operate. Currents will flow in the two resistors during alternate half-cycles only; if the current in the d.c. instrument is small, the drop over the upper resistor during the first half-cycle is proportional to the average value of $\mathbf{v}_a + \mathbf{v}_m$ and that over the lower during the next half-cycle to the average of $\mathbf{v}_a - \mathbf{v}_m$. As these two drops are in opposition across the detector and the auxiliary voltage is large compared with that to be measured, the detector current is approximately proportional to the average value of $2\mathbf{v}_m$, i.e. to $V_m \cos \gamma$, V_m being the r.m.s. value of $\mathbf{v}_m$. The arrangement is, therefore, phase selective and has been applied to a number of bridges.

30. Thermionic Amplifiers and Detectors. AMPLIFIERS. It is now common practice in bridge technique to obtain an increase in sensitivity, especially when using rectifier instruments, a.c. galvanometers, or telephones, by interposing between the bridge and the detector some type of thermionic amplifier. In such an amplifier a small power input, impressed in the grid circuit of a thermionic tube, liberates a much enhanced output in the anode circuit, this output being drawn from the anode-battery and depending in amount upon the amplifying characteristic of the tube. The principles of some standard types of amplifiers will now be considered.

Since amplification is usually regarded as a change in power levels—because work has to be expended to operate most types of detectors—a suitable unit is needed for its concise expression. Assuming the approximate truth of the Weber-Fechner law, which states that the difference between two energy stimuli is expressed by the logarithm of the ratio of their energies, a logarithmic scale is indicated. Thus if the power delivered to an amplifier is P_i and that given out by it is P_o, the level rises by $\log_{10} (P_o/P_i)$ bels or $10 \log_{10} (P_o/P_i)$ decibels (abbreviation db.); this is often called the *gain* of the amplifier. The ear can detect a change of about 1 db., corresponding with an increase of 26 per cent in power.

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* C. H. Walter, "Über eine neue Gleichrichtermessanordnung," *Zeits. f. tech. Phys.*, Vol. 13, pp. 363–367 (1932). "Die Anwendung der Gleichrichterbrücke in der Messtechnik," *idem.*, Vol. 13, pp. 436–441 (1932). *See also* K. Karandejew, "Eine Detektorbrücke für Wechselstrom," *Tech. Phys. U.S.S.R.*, Vol. 3, pp. 356–360 (1936). E. Mittelmann, "Eine einfache Impedanzmessbrücke mit Nullgalvanometer," *E.u.M.*, Vol. 55, pp. 321–322 (1937). C. Morton, "Visual balance detectors for conductance bridges," *Trans. Far. Soc.*, Vol. 33, pp. 474–479 (1937).

For many purposes at audio frequencies a single-stage amplifier like Fig. 84 (*a*) is quite adequate* and gives ample increase of sensitivity of a telephone or rectifier ammeter. Using a Ferranti rectifier microammeter (750 μA. range) with an Osram P240 triode, Banner† has constructed a useful visual detector. The input transformer has a step-up ratio of 1 : 40; the output transformer steps down 2 :1. The filament voltage is 2 V., grid bias —24 V. and anode supply 150 V. Messrs. Muirhead make a push-pull amplifier, Fig. 84 (*d*), suitable for bridge work at 40 to 10,000 cycles per second; the amplification is constant over this range, within a limit of 0·5 db., and 2 watts can be supplied into 6,000 ohms.

For the best conditions of working the input source and output detector, usually the bridge and a telephone respectively, should be about equal to the tube impedance (see L. S. Palmer, *loc. cit.*, p. 368). Since this is very large compared with ordinary bridge and telephone values, these are best adapted by using transformers stepping up from the input terminals and stepping down to the detector.

When larger gains than can be obtained from a single tube are desired, two or more triodes are joined in cascade; the output from the first tube is amplified by the second, and so on, the arrangement being a multi-stage amplifier. With a large number of stages it is difficult to avoid instability, retroaction of the later on the earlier stages, and self-oscillation; for this reason bridge amplifiers usually contain two and rarely exceed three stages and give ample gain for all purposes. Fig. 89 (*a*) shows a two-stage amplifier in which the two tubes are coupled by a combination of resistance and capacitance. The input from the bridge terminals is applied either directly or, better, through a shielded transformer to the grid of the first tube. The amplified voltage appearing across its output resistor R_1 is impressed on the grid-filament circuit of the second tube via the condenser C. This condenser isolates the second grid from the positive potential of the h.v. battery; it should have an impedance low compared with the internal grid-filament impedance of the tube. Such a completely isolated grid would quickly become strongly negatively charged by the electrons bombarding it; to allow this charge to leak away a grid leak of about a megohm is provided. The detector, telephone, or rectifier ammeter, is either

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* L. H. Adams and R. E. Hall, "Application of the thermionic amplifier to conductivity measurements," *Journal Amer. Chem. Soc.*, Vol. 41, pp. 1515–1525 (1919). C. Günther, "Eine neue Messbrücke zur Bestimmung von Selbstinduktionen," *Zeits. f. Inst.*, Vol. 46, pp. 623–631 (1926). Both use a telephone.

† E. H. W. Banner, "Visible null indicator for a.c. bridges," *Journal Sci. Insts.*, Vol. 9, p. 236 (1932).

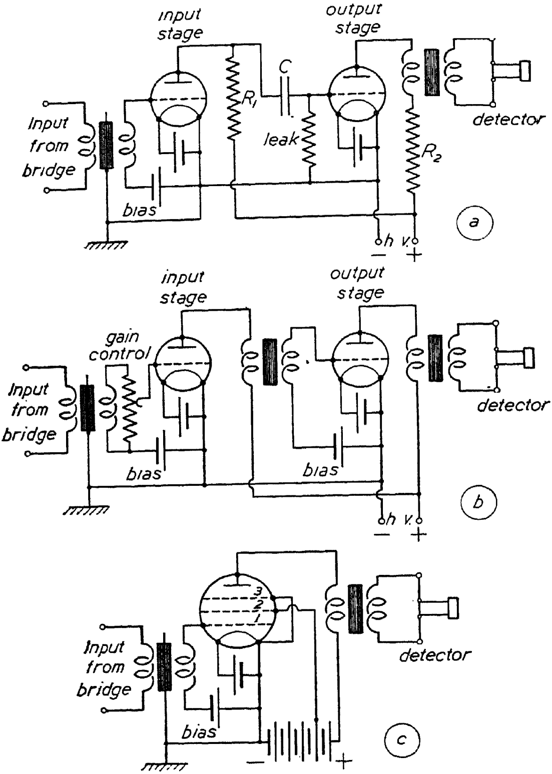

FIG. 89.—THERMIONIC AMPLIFIERS

included in, or transformer coupled to, the anode resistance R_2. Provided R_1 and R_2 are not too large, resistance-capacitance coupled amplifiers give almost constant amplification over a wide range of frequencies. An alternative, commonly-used arrangement is the transformer-coupled amplifier of

Fig. 89 (*b*). By using step-up bridge and interstage transformers, the overall amplification per stage greatly exceeds the magnification of the tube itself; on account of the ease with which high gains are obtained by this means, most commercially-available bridge amplifiers are of this class. The iron-cored transformers have frequency characteristics and have been very fully studied to produce a design capable of giving, over a wide frequency range, a uniform amplification per stage; the transformation ratio seldom exceeds 1 : 6. Commercial two-stage amplifiers give a gain of the order of 80 db. (power ratio of $10^8 : 1$) and a reasonably flat response over the audio range, say 40 to 10,000 cycles per second. While such enormous amplification is an invaluable aid in making final precise adjustments of a bridge balance, it may be a considerable nuisance during the earlier stages, a telephone emitting a continuous loud note, and a rectifier ammeter giving full-scale deflection until the balance condition is nearly attained. Gain control may conveniently take the form of a high resistance (2 megohms or so) potentiometer supplying the first grid. Alternatively, a jack may be provided to put the detector (i) direct on the bridge, (ii) after one stage, and (iii) after two stages, as balance is approached. To avoid difficulty due to stray fields and also to ensure stable operation, especially in high-gain amplifiers, it is essential that amplifiers should be carefully shielded and appropriately earthed.

Another type of high-gain amplifier that has had some application in bridge work uses the pentode tube shown diagrammatically in Fig. 89 (*c*). This has the usual filament and anode, with three grids interposed between them; grid 1 is the normal control electrode. Grid 2 is the screen grid usually maintained at a positive potential a little less than that of the anode. It acts as an electric shield between the anode and grid 1, preventing reaction between the output and input circuits, and thereby reducing considerably any tendency to instability and self-oscillation with high amplification. Grid 3, the earth grid, is joined to the filament and drives back into the anode any secondary electron emission between the anode and screen grid; its effect is to make the anode current chiefly under the control of grid 1 and not much influenced by anode potential changes. A pentode permits a large grid voltage variation to be used with high amplification, freedom from distortion, and absence of retroaction and instability.

Ford and Bousman* have discussed the design of bridge amplifiers in some detail. They point out that the earth admittances at the bridge branch-points may affect the amplifier design, and they make several practical recommendations. When the amplifier is connected to the bridge through an input transformer this should be shielded and the tube kathodes earthed; this applies either when the detector branch-points are brought to earth potential by a Wagner earthing device, or when one or other of these points is directly earthed. A three-stage resistance-capacitance coupled amplifier with shielded input transformer is described. When the bridge circuit has a strong magnetic field this transformer may be affected, and considerable interference with the amplifier can occur. In such cases the amplifier must be multiply shielded and the transformer omitted, a resistance-capacitance coupling to the bridge being used instead; an amplifier of this type for a Schering bridge earthed at one of the source branch-points is described. Their designs cover an audio range from 60 to 10,000 cycles per second.

Adams† gives details of a transformer-coupled, shielded, two-stage amplifier using pentode tubes; the detector is a 500 μA. Weston rectifier ammeter. Several manufacturers list standard bridge amplifiers. The Ericsson Aktiebolaget make a portable two-stage transformer-coupled arrangement using small screen-grid triodes operated from dry cells; over a range of 200 to 5,000 cycles per second the gain is about 45 db. Messrs. Muirhead have a two-stage unit with a gain of 80 d.b. in the range 40 to 10,000 cycles per second. The General Radio Co. have given particular attention to bridge amplifiers. An early‡ two-stage, transformer-coupled model using triodes takes its filament currents from a transformer connected to 60 cycle mains, the same transformer feeding the anodes *via* a double-diode rectifier; the gain is 40 db. from 90 to 6,000 cycles per second. The modern§ pattern uses dry batteries for all supplies, and has three pentode tubes with resistance-capacitance coupling; the gain is 90 db. from 18 to 10,000 cycles per second. By adding a parallel-resonant circuit in the grid circuit of the third tube high amplification is given at a single selected frequency‖ instead of over a wide band; the amplifier and its detector then have a tuned characteristic like that of a vibration galvanometer.

THERMIONIC DETECTORS. The arrangements just described make use of the amplifying properties of a thermionic tube as a kind of relay device, by means of which a small input, derived from the detector branch-points of the bridge, releases a very much larger output through the detecting instrument. This may be a telephone, either a headset or a loudspeaker; the latter is particularly convenient when a long series of measurements are to be made. Alternatively, a visual indication of balance is often preferred, and the high gain of amplifiers enables a pointer-type d.c. microammeter to be operated by some class of rectifier, usually of the electronic or thermionic class. Unfortunately, most rectifying devices operate with a square-law and greatly lose in sensitiveness near the balance point. Several devices have been designed to overcome this defect by means of suitably connected thermionic tubes.

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* W. A. Ford and H. W. Bousman, "Applifiers for alternating current bridges," *Gen. Elec. Rev.*, Vol. 37, pp. 224–226 (1934). J. D. Crawford, "Amplifiers for alternating current bridges," *G.R.Exp.*, Vol. 9, pp. 7–8 (July–Aug., 1934).

† N. I. Adams, "A sensitive audio-frequency null indicator'" *Rev. Sci. Insts.*, Vol. 7, pp. 180–181 (1936).

‡ A. E. Thiessen, "An audio amplifier for the laboratory," *G.R.Exp.*, Vol. 4, pp. 5–7 (Apr., 1930).

§ H. H. Scott, "A general purpose amplifier," *G.R.Exp.*, Vol. II, pp. 6–7 (July-Aug. 1936).

‖ H. H. Scott, "Tuning the type 814-A amplifier," *G.R.Exp.*, Vol. II, pp. 1–3 (Mar., 1937).

McNamara* uses a modification of the push-pull amplifier of Fig. 84 (*d*) in the way shown by Fig. 90 (*a*). In this, *ab* and *cd* are high-resistance voltage dividers joined across the branch-points *AB* and *CD* respectively. Suppose first that slider *e* is at *d* and that the two tubes are identical; then if their grids are disconnected from the bridge, the steady anode currents of the tubes are equal and in the direction of the arrows; no current flows in the microammeter. By reconnecting the grids and moving slider *f* to *a*, the grid-filament circuits of the tubes have the voltages across z_1 and z_4 impressed in them. These voltages are amplified by the upper and lower tubes respectively, the anode currents are changed and the unbalance causes the ammeter to read; when the drops are equal at every instant the reading becomes zero. Shifting *f* to *b* the voltages across z_2 and z_3 are similarly compared. When the ammeter does not read whether *f* is at *a* or at *b*, then *C* and *D* are at the same potential. High sensitivity is obtained because the detector is operated by the difference of the squares of two quantities, instead of by the square of their difference. Thus if the p.d. across z_1 is 5·1 volts and across z_4 is 5 volts a simple thermionic rectifier across *CD* operates on the square of their difference, namely, 0·01. With the circuit shown, the upper tube measures $(5{\cdot}1)^2$ and the lower 5^2, the ammeter recording the difference, namely, 1·01, which is 100 times as great. In practice it is difficult to obtain two identical tubes and the divider *cd* is used to compensate for their difference. The contact *e* is set at *c* and *f* at *a*, impressing equal voltages, the drop over z_1, on both grids. The reading of the meter is observed and used as a false zero; on moving *e* gradually towards *d* the bridge is adjusted until the reading remains unchanged. The process is then repeated with f at b and the bridge is balanced when both sliders can be moved from end to end without changing the reading. This detector has been applied in bridges with 100,000 ohm arms at frequencies up to 50,000 cycles per second.

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* F. T. McNamara, "A new type of bridge balance indicator," *Rev. Sci. Insts.*, Vol. 2, pp. 343–347 (1931).

McNamara's device is not phase selective, i.e. it does not discriminate between reactance and resistance bridge adjustments, and the false zero method is slow and inconvenient. Both defects are removed in the detector designed by Cosens,* shown in Fig. 90 (*b*), which is based upon the principle that if the anode current of a triode be supplied not from a battery but from an alternating source, half-wave rectification occurs and the current can be measured by a d.c. microammeter in the anode circuit; if the potential of the grid be varied, the anode current will be changed. If the anode be fed from the bridge source and the grid is connected to the detector branch-points, the rectified current will increase or decrease according as the grid potential is in phase or in opposition to that on the anode, and the magnitude of the change varies linearly with the grid potential. The sensitivity can be increased by balancing out the steady deflection of the ammeter, which is most easily done by using two triodes T_1 and T_2, the whole resembling a full-wave rectifier. Assume first that the grids are disconnected. The anodes are supplied from the bridge source with voltages in phase opposition, so that during one half-cycle the anode of T_1 is positive and that of T_2 negative; current flows in R_1 from c to a and none in R_2. During the next half-cycle T_2 is positive and T_1 negative, current flowing only in R_2 from c to b. If the two tubes are identical and the resistors equal, the average drops in ca and cb will be the same, no average p.d. appears across ab and the ammeter is undeflected; if the tubes are not quite identical, regulation of one resistor will ensure this null condition. Now connect the grids to the detector branch-points of the bridge, and suppose the voltage applied to them to be in phase with that on the anode of T_1. During the half-cycle when the grid and the anode of T_1 are positive a larger current will flow in R_1 than before, but no current flows in

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* C. R. Cosens, "A balance detector for alternating current bridges," *Proc. Phys. Soc.*, Vol. 46, pp. 818–823 (1934). *British Patent*, No. 385, 982 (1933). The instrument is made by the Cambridge Instrument Co. The arrangement is the thermionic analogue of Walter's rectifier bridge, Fig. 88 (*c*), with which it should be compared. Also see C. Morton, *loc. cit.* on p. 238.

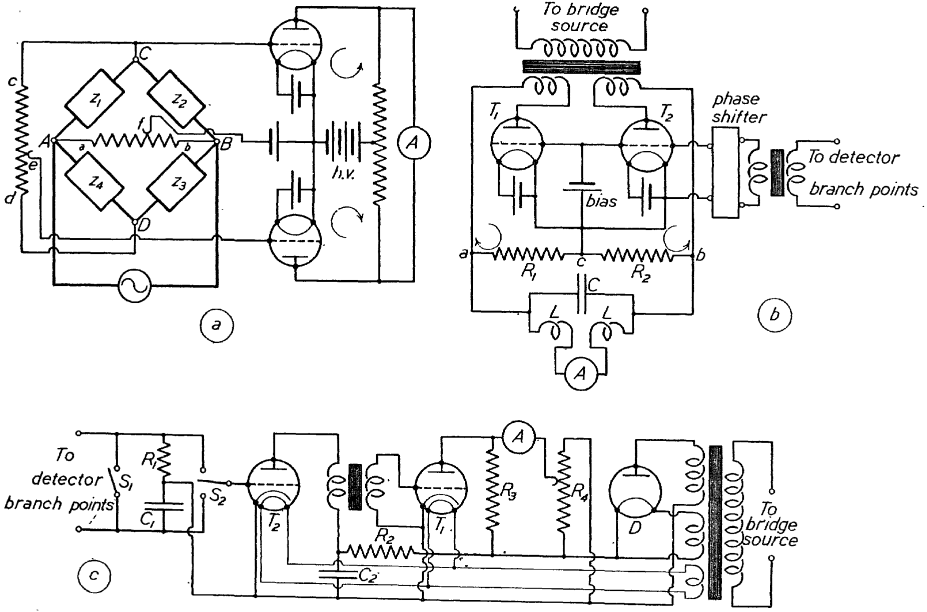

FIG. 90.—THERMIONIC DETECTORS

R_2 since the anode of T_2 is negative. During the next half-cycle the anode of T_1 is negative and so also is the grid, so that this tube is inoperative, and the current in R_1 is zero; the anode of T_2 is positive, but, since its grid is negative, less current flows in R_2 than before. Hence T_1 passes more and T_2 less current than in the original state, an average p.d. exists between a and b, and the ammeter deflects. If the phase of the grid voltage be reversed, so also is the ammeter reading; a centre zero microammeter is used. In general, if the first anode and the grid voltages differ in phase by γ, it is easy to show that the reading is proportional to $\cos \gamma$. The condenser C and chokes $L\,L$ are to prevent pointer oscillation at low frequencies. The arrangement is phase selective, and for bridge use the grids are supplied via a phase-shifting network, a suitable form being suggested by Cosens (see for example the single-phase device shown in Fig. 201 of *Instrument Transformers*).

Another thermionic rectifier device, Fig. 90 (c), is due to Stuart* and consists of a half-wave rectifier diode, D, energized by the bridge source and supplying current to the anode circuit of a triode T_1. A voltage from the detector branch-points of the bridge is impressed on the grid of T_1, via an amplifying triode T_2, and modifies the current in R_3; the amount of anode current change depends on $\cos \gamma$, γ being the angle between the grid and anode voltages of T_1. The effect of the original current component in R_3 is balanced out by removing the grid voltage with the short-circuiting switch S_1 and then adjusting the tapping on R_4 until the centre-zero microammeter is undeflected. The phase selectivity in balancing the bridge for resistance and reactance adjustments is obtained by the simple quadrature phase-shifter, consisting of R_1 and C_1 in series, with $R_1 \fallingdotseq 1/\omega C_1$. The switch S_2 enables the drop over one or the other to be applied to the grid of T_2; at 50 cycles per second R_1 is 796 ohms and C_1 is 4 μF. The elements C_2, R_2 (10 μF., 50,000 ohms) form a smoothing arrangement for the amplifier anode voltage. The triodes are of medium impedance (12,000 ohms) and have indirectly heated filaments, though this is not essential.

Another interesting device† makes use of the small kathode ray tubes frequently used in radio sets to indicate sharpness of tuning. In one pattern (the Westinghouse 6E5 tube) the kathode beam gives a circular sectorial area on the fluorescent screen, the amount of the area being modified by the grid potential. Using a pentode amplifier with one of these tubes Breazeale claims a sensitivity of bridge adjustment ten times greater than with unaided telephones.

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* W. S. Stuart, "A thermionic valve detector for supply frequencies," *Journal Sci. Insts.*, Vol. II, pp. 121–123 (1934).

† D. Ulrey, "New electron tubes and new uses," *Physics*, Vol. 7, pp. 97–105 (1936). W. M. Breazeale, "A vacuum tube alternating current bridge detector," *Rev. Sci. Insts.*, Vol. 7, pp. 250–251 (1936). R. L. Garman, *ibid.*, Vol. 8, pp. 327–329 (1937).

HIGH-FREQUENCY DETECTORS. In bridges used at radio frequencies, thermionic devices are exclusively used; although their detailed discussion lies outside the scope of this volume, the general principles must be mentioned in view of the fact that many bridges, especially in carrier telephony measurements, are designed for continuous operation from the audio- into the super-sonic range of frequencies and incorporate such h.f. detectors. Examples will be cited in Chapter V.

By far the best detector is a heterodyne-operated telephone. The detector branch-points of the bridge are connected to a h.f. amplifier, the output from which is caused to produce a beat note of audible frequency by means of an auxiliary radio-frequency oscillator. The result of the heterodyne is passed through an audio-frequency amplifier and impressed on a telephone. The difference between the bridge and auxiliary oscillator frequencies is adjusted to about 1,000 cycles per second, at which the ear is most sensitive and also the telephone diaphragm comes into resonance. It is essential that the amplifiers be very efficiently shielded; examples will be found in the papers mentioned on p. 242.

31. Electrodynamometers. Any sensitive electrodynamometer can be used as a bridge detector. In use, the instrument can be connected in various ways, e.g. the fixed coils may be joined to the source and the moving coil across the detector branch points. Then, if i_1, i_2 be the instantaneous values of the currents in the fixed and moving elements, the average torque and the deflection of the moving coil will be proportional to $\int i_1 i_2 \, dt$ taken over a period, i.e. to $I_1 I_2 \cos \phi$, where I_1 and I_2 are the r.m.s. values of the currents and ϕ is the phase-difference between them. Assuming I_1 not to be zero, the moving coil will not deflect if either (i) $I_2 = 0$, as in the ordinary null adjustment of the bridge; or (ii) $\cos \phi = 0$, $\phi = \pi/2$, the fixed and moving coils carrying currents in quadrature. Rowland* has used a dynamometer in both these ways, giving various methods of connecting the fixed and moving coils to the bridge, though other experimenters had employed the instrument before him.†

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* H. Rowland, *Amer. J. Sc.*, 4th series, Vol. 4, pp. 429–448 (1897).

† A. Oberbeck, *Ann. der Phys.*, Vol. 17, pp. 816–841, 1040–1042 (1882). J. Puluj, *Elekt. Zeits.*, Vol. 12, pp. 346–350 (1891); O. Troje, *Ann. der Phys.*, Vol. 47, pp. 501–512 (1892); H. Martienssen, *Ann. der Phys.*, Vol. 67, pp. 95–104 (1899); A. Blondel, *Écl. Élect.*, Vol. 21, pp. 138–141 (1899).

The dynamometer must be very sensitive,* having a light moving coil hanging from a long suspension to give a very weak control, and provided with a mirror to enable deflections to be observed. Electrostatic forces between the fixed and moving coil are minimized by means of a tin-foil screen surrounding the latter and connected to one of its terminals. The inductance of the moving coil is preferably compensated by connecting in series with it a shunted condenser; the condenser and its shunt are adjusted until the entire combination has zero effective inductance.†

32. Alternating Current Galvanometers. The sensitiveness of an electrodynamometer can be greatly increased by providing the fixed coils with an iron core built up of laminations, since the strength of the magnetic field in which the moving coil lies is then much greater. The instrument then resembles an ordinary moving-coil galvanometer except that the field is produced by an alternating current electromagnet with laminated core; a laminated core may also be inserted within the moving coil. The arrangement is then called an alternating current galvanometer and was first introduced by Stroud and Oates,‡ who showed that the instrument is capable of very great sensitiveness. In use, the electromagnet is excited by the alternator supplying the bridge, and therefore takes a current approximately $\pi/2$ out of phase with the alternator voltage. A resistance R, Fig. 91 (*a*), connected between the bridge and the source ensures that the bridge current is nearly in phase with the supply voltage. With the moving coil connected across the detector points, the instrument will be very sensitive to components of current in quadrature with the alternator voltage, i.e. it is in the best condition to measure reactances. If the instrument is to be sensitive to resistance adjustments in the network, the magnet current and the bridge current must be brought more nearly into phase.

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* *See* A. Palm, "Spiegel Elektrodynamometer mit besonders hoher Empfindlichkeit," *Zeits. f. Inst.*, Vol. 33, pp. 368–373 (1913). Also *Dictionary of Applied Physics*, Vol. 2, p. 402.

† For a design of a sensitive instrument for bridge work *see* H. Mukherjee, "Ein Wechselstromgalvanometer," *Zeits f. Phys.*, Vol. 64, pp. 286–291 (1930). The choice of the best instrument for null tests is discussed by W. Spielhagen, "Die Empfindlichkeit der Nullinstrumente bei Brückenschaltungen zum Messen von Wechselspannungen," *Zeits. f. Phys.*, Vol. 77, pp. 346–351 (1932).

‡ W. Stroud and J. H. Oates, "On the application of alternating currents to the calibration of capacity boxes, and to the comparison of capacities and inductances," *Phil. Mag.*, 6th series, Vol. 6, pp. 707–720 (1903).

A more sensitive form of instrument was later designed by Abraham* in which the inductance of the moving coil was compensated by the addition of a shunted condenser in series with it, thereby improving its performance.

In practical working, these instruments possess a troublesome feature due to the strong electromagnetic control exerted on the moving coil when on closed circuit, by transformer action from the main field. Stroud and Oates noticed the trouble but did not explain it; they showed, however, that it could be overcome in practice by use of a false zero. The theory of the control was briefly given by Taylor,† but a complete theory of the instrument was not supplied until much later by Weibel. Instruments of the Stroud and Oates type have been used, very largely in America, by other observers.‡

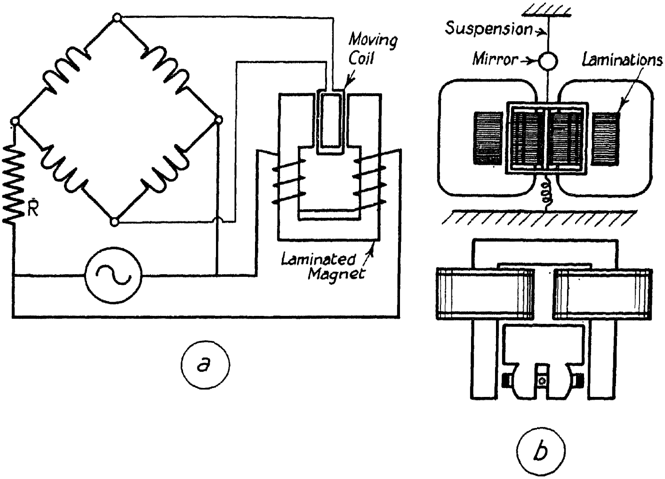

FIG. 91.—A.C. GALVANOMETERS

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* H. Abraham, "Galvanomètre à cadre mobile pour courants alternatifs," *Comptes Rendus*, Vol. 142, pp. 993–994 (1906).

† A. H. Taylor, "Theory of control of the alternating current galvanometer," *Phys. Rev.*, Vol. 25, pp. 61–63 (1907).

‡ E. M. Terry, *Phys. Rev.*, Vol. 21, pp. 193–197 (1905), describes a means of overcoming the control difficulty; A. Trowbridge and A. H. Taylor, *Phys. Rev.*, Vol. 23, pp. 475–488 (1906), use Terry's instrument as a differential galvanometer.

Sumpner* has made a thorough investigation of the design and construction of iron-cored a.c. instruments, and has described a galvanometer† suitable for bridge work which is diagrammatically shown in Fig. 91 (*b*). In this, a strong alternating current electromagnet of special construction is used, so arranged that the flux is proportional to and in quadrature with the applied voltage. The moving coil is suspended in the specially shaped air-gap and has a low inductance. The control-effect of the alternating field on the moving coil, which in this instrument is small, can be allowed for by adjustment of the bridge balance until reversal of the moving coil produces no deflection; alternatively, Abraham's compensation or Stroud and Oates' false zero method are applicable. Owing to the phase-selectivity of the instrument, bridges can be independently balanced for inductance or capacitance with an accuracy of 1 part in 10,000 of the measured values when the p.d. over the coils or condensers concerned is only 1 or 2 volts. Sumpner and Phillips described numerous bridge tests showing the high sensitiveness of the galvanometer.

More recently, Weibel‡ has given an exhaustive study of high-sensitivity alternating current galvanometers. He has obtained their equation of motion, and shows that their performance is similar to that of d.c. moving coil galvanometers since there may be any degree of damping, chiefly due to the circuit with which the instrument is connected; the period and damping depend on the circuit, the former being shortened when there is inductance present and lengthened by capacitance. The deflection is proportional to the applied p.d. on the moving coil, and depends on the component of the p.d. in phase with the exciting current; hence the phase-selectiveness of the galvanometers. He has described the design procedure and the methods of measuring the intrinsic constants of the instruments; galvanometers having a sensitiveness at low frequencies exceeding that of the telephone and the vibration galvanometer, comparable with the best d.c. moving coil instruments are described, one being highly sensitive at 2,100 cycles per second. He points out the necessity for shielding the moving coil from electrostatic effects, as mentioned on page 248.

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* W. E. Sumpner, "The use of iron in alternate current instruments," *Journal I.E.E.*, Vol. 34, pp. 144–170 (1905); "New iron-cored instruments for alternate current working," *Journal, I.E.E.*, Vol. 36, pp. 421–467 (1906).

† W. E. Sumpner and W. C. S. Phillips, "A galvanometer for alternate current circuits," *Proc. Phys. Soc.*, Vol. 22, pp. 395–409 (1910).

‡ E. Weibel, "A study of electromagnet moving coil galvanometers for use in alternating current measurements," *Bull. Bur. Stds.*, Vol. 14, pp. 23–58 (1919).

Other forms of a.c. galvanometer, working on the moving needle principle,* have been proposed but have not found much application.

33. The Telephone. The detector most used in a.c. bridge work is the ordinary telephone receiver and at the higher audio frequencies, say about 800 cycles per second, is also the most sensitive. In principle the modern telephone is little different from the well-known type introduced by Bell in 1876. In its simplest form it consists of a thin iron diaphragm clamped by its rim so that it is close to the pole of a permanent bar-magnet. A coil wound near the pole of the magnet carries the current to be detected ; the modification of the permanent field by the alternating field of the coil produces periodic attractions of the diaphragm, thus setting it into forced vibration and causing it to emit an audible note. In the more modern form, the permanent magnet is of horse-shoe form so that both poles act on the diaphragm, the whole device being contained in a flat watch-like case. In detail, the watch pattern telephone has been somewhat modified with a view to obtaining greater sensitiveness.† Two such flat receivers are fixed to a head band so that one can be applied to each ear.

The sensitivity of the telephone is very high, but is far from being constant for currents of differing frequencies, due to the fact that the diaphragm possesses definite normal modes of vibration. When the frequency of the current approaches one of these natural frequencies, the amplitude of vibration of the diaphragm increases enormously‡ owing to resonance, and the sensitivity is correspondingly high ; hence the sensitivity/frequency curve is sharply peaked. Wien§ records results for a Bell telephone having resonances at 1,100, 2,800, and 6,500 cycles per second, and for a Siemens instrument at

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* W. S. Franklin and L. A. Freudenberger, "A new type of alternating current galvanometer of high sensibility," *Phys. Rev.*, Vol. 24, pp. 37–41 (1907); also Max Wien, *Ann. der Phys.*, Vol. 4, p. 445 (1901).

† K. W. Wagner, "Ueber die Verbesserung des Telephons," *Elekt. Zeits.*, Vol. 32, pp. 80–83, 110–112 (1911).

‡ *See* A. E. Kennelly and H. A. Affel, *Proc. Amer. Acad.*, Vol. 51, p. 421 (1915).

§ Max Wien, "Die akustichen und elektrischen Constanten des Telephons," *Ann. der Phys.*, Vol. 4, pp. 450–458 (1901).

750, 2,350, 5,400. In general, most telephones used in connection with speech transmission show a maximum of current sensitivity somewhere between 700 and 1,200 cycles per second.

The sensitiveness of a telephone is not entirely determined by the amplitude of oscillation of the diaphragm, but depends also on the acuteness of hearing of the experimenter, and is, therefore, different for different persons. The combined sensitiveness of telephone receiver and observer is best expressed by the least current which will cause an audible sound. Lord Rayleigh* and Max Wien† have given results for a number of receivers, the results of the latter observer for the Siemens telephone cited above being as follows—

Frequency . .	64	128	256	512	720	1,024	1,500	2,030	2,400	4,000	16,000
Current in Micro-amps. . .	12	1·5	0·135	0·027	0·008	0·0135	0·024	0·03	0·01	0·3	17

Since a telephone is more sensitive at certain frequencies than at others, care must be taken to select a telephone suitable for the work in hand. The resistance of the telephone should also be chosen to correspond with the resistance of the network; or its resistance should be adapted to the bridge by the interposition of an interbridge transformer‡ of suitable ratio (*see* p. 232). For ordinary bridges of medium impedance a telephone of 150 ohms resistance is suitable. For use in bridges with high impedance branches such a telephone can be combined with a step-up transformer, the winding with few turns being joined to the telephone, while that with many turns is connected to the bridge branch-points. Conversely, the use of a step-down transformer will adapt the instrument to a low impedance bridge. When choosing a telephone, it should be remembered that it is the *effective* resistance and reactance of the instrument at the frequency of the test which must be considered. Capacitance effects between the observer and the telephone are sometimes disturbing and can best be allowed for by some device such as the Wagner earth point (*see* p. 542). Stray magnetic fields from the bridge may also cause trouble; in this case the instrument may be set up out of inductive range, arranged in the position of minimum interference, and connected to the observer by flexible ear-tubes. A loudspeaker is often a great convenience, especially in lengthy tests where continued use of a head set or of ear tubes becomes uncomfortable and fatiguing. The use of a telephone with a thermionic amplifier gives enormously increased sensitiveness and has been discussed in Section 30; an amplifier is practically essential when a loudspeaker is used.

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* Lord Rayleigh, *Phil. Mag.*, Vol. 38, p. 294 (1894).

† Max Wien, *loc. cit.*, p. 456; *see* L. W. Austin, *Bull. Bur. Stds.*, Vol. 5, pp. 153–157 (1909). E. W. Washburn, *Phys. Rev.*, 2nd series, Vol. 9, pp. 437–439 (1917).

‡ G. Chaperon, *Comptes Rendus*, Vol. 108, p. 799 (1889).

34. Differential Telephones. In the early days of a.c. measurement some useful work was done with the aid of a differential telephone, an instrument devised by Chrystal* only four years after the invention of the telephone by Graham Bell. An ordinary receiver is rewound with bifilar conductor in such a way that two coils of exactly equal resistance, inductance and turns, are provided; equal currents in opposition in the two windings cause no sound. The device can be used to compare resistances, inductances, or capacitances by joining the unknown and an adjustable standard in parallel to an a.c. supply, a telephone winding being included in each branch in such a way that the coils act in opposition. When the reactance and resistance of the standard are equal to those of the unknown, the telephone will be silent. Although the apparatus has been independently reintroduced from time to time,† it is now obsolete, but it has given rise to other devices which even to-day have some application.

Trowbridge,‡ reinventing an earlier arrangement due to Elsas,§ pointed out that it is unnecessary to make a special telephone. The comparison of the two branch currents can be easily made by passing them into two balanced windings on an iron core, equality being indicated by silence in an ordinary telephone joined to a tertiary winding on the same core. A ring core with toroidally wound, uniformly distributed windings is best. These methods are then simple cases of a class of mutual inductance bridges related to the modern "hybrid coil," and will be again referred to on p. 421.

Other differential detectors have been suggested, notably a differential electrometer,‖ but they have not received any extensive application.

35. Tuned Detectors. It has been pointed out in the section 334 that a telephone shows preferential sensitivity for currents of certain frequencies, namely, those which will set the diaphragm into resonant vibration in one of its normal modes of oscillation. It would appear advantageous to have a telephone or other piece of apparatus which would be capable of being *tuned* to resonance with the current to be measured, so that the greatly magnified oscillation and increase of sensitivity can be secured. Such instruments are called *tuned detectors* and are very widely used.

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* G. Chrystal, "On the differential telephone," *Trans. Roy. Soc. Edin.*, Vol. 29, pp. 609–636 (1880).

† H. Ho, "The use of differentially wound telephone receiver in electrical measurements," *Elec. World*, Vol. 41, pp. 884–885 (1903). W. Duane and C. A. Lory, "On the differential telephone," *Phys. Rev.*, Vol. 18, pp. 275–279 (1904).

‡ A. Trowbridge, "On the differential transformer," *Phys. Rev.*, Vol. 20, pp. 65–76 (1905).

§ A. Elsas, "Ueber Widerstandsmessungen mit dem Differentialinductor," *Ann. der Phys.*, Vol. 35, pp. 828–833 (1888); Vol. 42, pp. 164–177 (1891).

‖ G. Athanasiadis, "Bestimmung von Selbstinduktionskoeffizienten mittels Differentialelektrometer," *Phys. Zeits.*, Vol. 8, pp. 606–607 (1907). J. Krönert, "Differential-Galvanometer und ihre Anwendung," *Archiv f. tech. Mess.*, J. 726–1 (July, 1932).

A. Campbell* has shown that an ordinary telephone can be tuned by pressing on the diaphragm at an eccentric point by means of an adjusting screw. A more accurate and sensitive device is the monotelephone of Abraham,† in which the diaphragm is replaced by two stretched strips carrying an armature which is arranged to face the magnet/coil system. Tuning is effected by varying the tension of the strips. Mercadier‡ has introduced a form of telephone tuned to a fixed frequency; ordinary sensitive telephones made to respond to fixed frequencies are made by the Cambridge Instrument Co. A highly sensitive telephone is that of S. G. Brown, in which the usual diaphragm is replaced by a vibrating iron reed carrying a light conical aluminium diaphragm; this instrument is not tunable in the sense considered here.

Wien§ has shown that the ordinary telephone can be rendered enormously more sensitive by acoustically tuning it. This is done by causing the sound emitted by the telephone to pass directly into a suitable Rayleigh acoustic resonator. Electrical tuning, by means of a condenser in series with the telephone, has also been occasionally used.

The most important tuned detectors are the so-called *vibration* or *resonance galvanometers*. These are really galvanometers, working either on the moving coil or moving magnet principle, in which the natural frequency of the moving system can be tuned to be in resonance with the alternating current to be measured. The forced vibration of the moving system will be much greater when currents of the resonating frequency pass through the instrument than when currents of any other frequency are used. By the use of the principle of resonance, a great increase in sensitiveness is produced and the instrument shows marked frequency selectiveness. Hence when the instrument is tuned to the fundamental of a given source of current, harmonics in the wave form produce little effect, so that when a vibration galvanometer or other tuned detector is used in an a.c. bridge, the balance obtained is the same as if the harmonics were absent. The source, therefore, need not have a pure wave form ; it must, however, have constant frequency, since the high sensitivity is confined to a narrow range of frequency near the resonance value.

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* *Dictionary of Applied Physics*, Vol. 2, p. 404.

† H. Abraham, "Rendement acoustique du téléphone," *Comptes Rendus*, Vol. 144, pp. 906–908 (1907).

‡ E. Mercadier, *Comptes Rendus*, t. 130, pp. 1382 (1900).

§ Max Wien, "Ueber die Anwendung von Luftresonatoren bei Telephontönen," *Phys. Zeits.*, Vol. 13, pp. 1034–1037 (1912). *See also*, J. W. Birnbaum, *Ann. der Phys.*, Vol. 49, pp. 201–228 (1916) where such acoustically tuned telephones are used in tests on the acuteness of hearing; and E. W. Washburn and K. Parker, *Journal Amer. Chem. Soc.*, Vol. 39, pp. 235–245 (1917) where they are used in bridge tests on electrolytes.

Vibration galvanometers are constructed to cover a range of frequency from about 1,000 cycles per second downward. At the lower values, up to about 300 cycles per second, they are much more sensitive than the telephone, and approach in sensitivity the best d.c. galvanometers.

Vibration galvanometers are of two kinds, according as the tuned system is a moving magnet or a moving coil ; the two types correspond to the Kelvin suspended needle and the d'Arsonval d.c. galvanometers. A brief discussion of each will now be given.*

36. Moving Magnet Vibration Galvanometers. Moving magnet vibration galvanometers generally resemble in principle of construction the Kelvin galvanometers used with d.c.† The moving part consists of a suspended magnetized system, composed of small permanent magnets or of soft iron polarized by an auxiliary magnetic field. Tuning of the natural period of the suspended part is effected by altering the length and tension of the suspension, or by varying the strength of the auxiliary magnetic field, if any. The moving magnets are deflected by the current to be measured passing round suitable deflecting coils ; the amplitude of the resulting oscillation is determined by observing the breadth of the band of light reflected on to a scale from a mirror mounted on the vibrating part. The principal sources of damping are air friction and elastic hysteresis in the suspension ; the controlling forces are supplied either by torsion of the suspending fibre, by magnetic action of an auxiliary field, or by combinations of both these causes.

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* See O. Werner, *Empfindliche Galvanometer für Gleich-und Wechselstrom*, pp. 1–208, Walter de Gruyter & Co. (1928). J. Krönert, "Nullstrom Indikatoren für Gleich-und Wechselstrom," *Arch. f. tech. Mess.*, J. 850–1 (Feb., 1932). A. Lucchini, "Le moderne tendenze degli indicatori di zero," *L'Elettro.*, Vol. 23, pp. 6–10 (1936). Also *see* the book by Campbell and Childs, previously cited.

† H. Zöllich, "Nadel-Vibrationsgalvanometer," *Arch. f. tech. Mess.*, J. 852–2 (May, 1933).

The earliest vibration galvanometer is the "optical telephone" of Max Wien.* In principle this instrument is the same as an ordinary acoustic telephone, but differs in two important respects: (i) it works in resonance with the applied current and is, therefore, highly sensitive; (ii) the resulting motion of the diaphragm is caused to operate a magnifying lever which carries a mirror, from which a beam of light is reflected to a distant scale, where it is drawn out by the vibration into a luminous band. Fig. 92 (*a*) illustrates two types of optical telephone in diagrammatic outline. The early pattern is practically a telephone with optical magnification. In the later pattern a more effective magnet system is used, in which four deflecting coils, wound so that their relative polarities are as shown by the small letters. With a d.c. resistance of 400 ohms the sensitivity is quoted as about 3 mm. per microampere at 1 metre. Wide range of tuning is effected by changing the diaphragm; fine adjustment by slightly moving the polarizing magnet.

Rubens† has improved Wien's arrangement by replacing the diaphragm by a galvanometer system consisting of 20 small iron needles soldered to a brass wire (*see* Fig. 92 (*b*)). By altering the length and tension of the wire, the instrument can be tuned over a range of 50 to 200 cycles per second. The needles lie in the magnetic field produced by a pair of permanent magnets, upon the poles of which coils are wound in such a direction as to exert a couple on the suspended system. Fine tuning is obtained by altering the positions of the magnets relative to the needles. The sensitivity‡ at 100 cycles per second is 1·5 mm. per microampere at 1 metre.

Schering and Schmidt§ have described a modification of Rubens' type in which the polarizing field is produced by an electromagnet excited by d.c., adjustment of the strength of which tunes the moving system. The general arrangement of the instrument is shown in Fig. 93. The polarizing field i provided by a small U-shaped magnet excited by a direc

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* Max Wien, "Das Telephon als optischer Apparat zur Strommessung," *Ann. der Phys.*, Vol. 42, pp. 593–621, Vol. 44, pp. 681–688 (1891).

† H. Rubens, "Vibrationsgalvanometer," *Ann. der Phys.*, Vol. 56, pp. 27–41 (1895).

‡ F. Wenner, *Bull. Bur. Stds.*, Vol. 6, p. 365 (1910); *see* also E. B. Rosa and F. Grover, *idem*, Vol. 1, p. 291 (1905), for a resonance curve.

§ H. Schering and R. Schmidt, "Ein Vibrationsgalvanometer mit elektromagnetischer Abstimmung für niedrige Frequenzen," *Zeits. f. Inst.*, Vol. 38, pp. 1–11 (1918). Also see H. Schering, *Elekt. Zeits.*, Vol. 39, pp. 410–114 (1918).

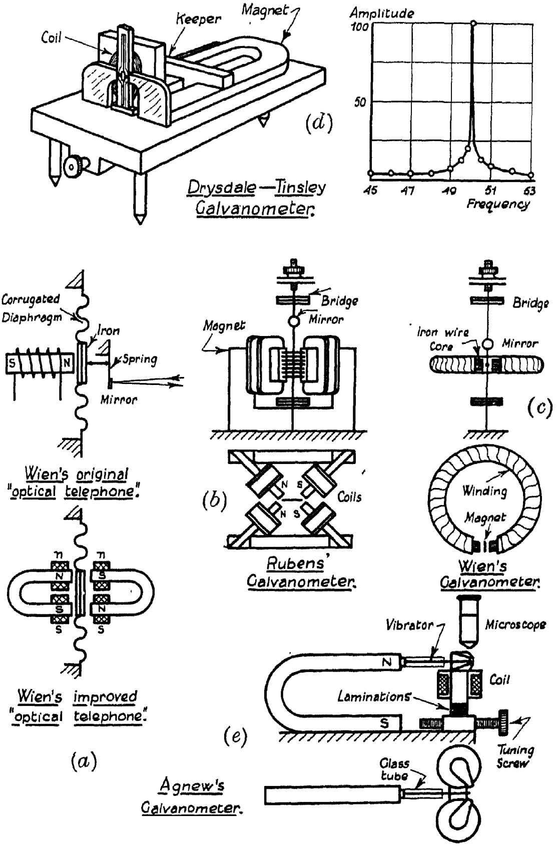
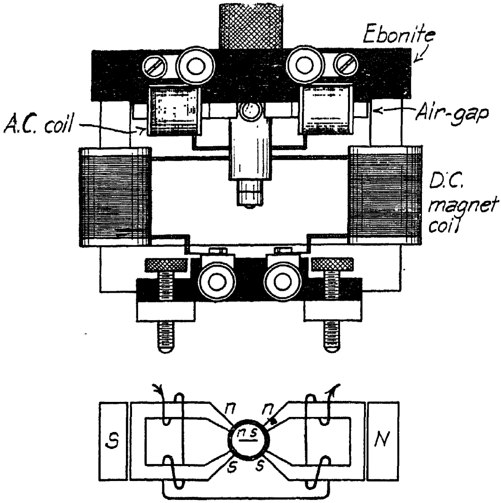


Fig. 92.—Moving Magnet Vibration Galvanometers

current of a few tenths of an ampere flowing in windings mounted on the vertical limbs of the core. Between the poles of this magnet is inserted the a.c. magnet system, consisting of two small laminated U-shaped cores with four magnetizing coils, connected as shown, carrying the alternating current. These coils may be wound for any desired resistance, Schering and Schmidt's original instrument having a total resistance of 92 ohms with 800 turns per coil; a very useful value is from 200 to 500 ohms for use in condenser bridges. The four poles



Diagrammatic plan showing arrangement of A.C. magnets

FIG. 93.—THE SCHERING AND SCHMIDT VIBRATION GALVANOMETER

of the a.c. magnets point radially toward the moving system, which consists of a small piece of thin soft iron suspended by a fine phosphor-bronze strip fixed at both ends and carrying a small plane mirror. In the original galvanometer, needles of 6 mm. × 4 mm. × 0·18 mm. gave a range of 8 to 60 cycles, while 4 mm. × 4 mm. × 0·07 mm. needles were used for a range of 30 to 140 cycles; the suspension was 0·02 cm. diameter phosphor bronze, 4 cm. in length. The moving system is enclosed in a removable ebonite chamber which slides between the magnet poles.

The effect of the a.c. is to produce an alternating transverse distortion of the polarizing field so that the needle is set into oscillation. The vibratory system is tuned by altering the strength of the polarizing current until the magnetic control on the needle is sufficient to throw it into resonance; this control can, of course, be effected at any desired distance from the instrument, which is a great advantage when the instrument is used in high voltage bridges, for which purpose it is now almost always used in preference to other types. The a.c. windings and cores are carried on an ebonite plate and a small air-gap is left between the a.c. magnet cores and the main magnet; thus the a.c. system is very highly insulated from the magnet windings and may safely attain high potentials. The earth capacitance of the instrument is very low, about 8 $\mu\mu$F., since its bulk is slight. The galvanometer is protected from stray magnetic fields of the resonant frequency by encasing it in a laminated silicon steel (or, better, permalloy or mumetal) shield.

The damping is ordinarily very small but can be adjusted with the aid of a movable copper block, which can be placed nearer to or farther from the needle, as desired. As the instrument is specially intended for use at the ordinary supply frequencies some increase in damping may be very desirable, since the frequency of ordinary alternators is not very steady and the fluctuations would result in great loss in sensitiveness in a sharply tuned galvanometer; the increase of damping blunts the resonance peak and provides some measure of frequency-variation tolerance. Schering recommends that the damping be adjusted so that 1 per cent change in frequency results in a reduction of sensitiveness to one-half.

The galvanometer in its original form is constructed by the firm of Hartmann and Braun. A slightly modified design has been prepared at the National Physical Laboratory and is manufactured by the Cambridge Instrument Co. and by H. Tinsley and Co. The instruments have a range, with a single moving system, of about 25 to 120 cycles per second, the sensitivity at 50 cycles per second being of the order of 40 mm. per microampere at 1 metre. Its advantages of remote control tuning, low frequency, high insulation, and small self-capacitance render the instrument of special value in high voltage bridges at commercial frequencies.

Wien* has described a highly sensitive galvanometer shown in Fig. 92 (*c*), in which the current to be detected passes through a coil wound on a ring made of iron wires. The moving system consists of a set of very small magnets fastened to a brass wire so that they are held parallel to the faces of a gap in the ring. Coarse tuning is effected by varying the length and tension of the suspension wire; fine tuning by altering the width of the gap in the electro-magnet. The sensitiveness is inversely proportional to the square of the frequency, and at 100 cycles per second is quoted as 70 mm. per microampere at 1 metre; Wells† has shown that the deflection is proportional to the voltage and shows how the deflection varies with the suspension length when a current of constant frequency flows in the coil.

Drysdale‡ has designed a galvanometer for use at low frequencies (*see* Fig. 92 (*d*)). The moving system consists of a light mirror carrying a soft iron needle, the whole being suspended by a taut silk thread in the field of a powerful permanent horse-shoe magnet. Behind the moving needle the deflecting coil is situated so that the deflecting field is perpendicular to the control field and, hence, sets the needle into vibration. Since the natural frequency of the suspended needle is proportional to the square root of the control field, tuning is effected by altering the field due to the permanent magnet by the expedient of sliding an iron keeper along the magnet limbs. A range of frequencies between about 40 and 160 is thus covered. The deflecting coils are interchangeable and can be chosen to have an impedance suitable for the work in hand. The damping is very slight, resulting in sharp resonance. At 50 cycles, the current sensitivity varies between 1 mm. and 70 mm. per microampere at 1 metre when coils having impedances between 1·26 and 6,000 ohms are used. The corresponding voltage sensitivities are 0·8 to 0·012 mm. per microvolt.

A galvanometer identical in principle with that of Drysdale has been described by Blondel and Carbenay;[*] in it the controlling field is produced by a pair of coils carrying a direct current instead of by a permanent magnet. The instrument is tuned by adjustment of this current, and has a range up to 1,500 cycles per second.

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* M. Wien, "Ueber die Erzeugung und Messung von Sinusströmen," *Ann. der Phys.*, Vol. 4, pp. 425–449 (1901). For modern designs on this principle, see W. Meissner and U. Adelsberger. *Zeits. f. tech. Phys.*, Vol. 11, pp. 102–107, 143–147 (1930); Vol. 13, pp. 475–477 (1932); Vol. 14, pp. 111–118 (1933). The suspended magnets are of cobalt steel and move in two fields with their directions at right angles; one is the a.c. deflecting field and the other a d.c. control field to adjust tuning.

† R. T. Wells, "Note on the vibration galvanometer," *Phys. Rev.*, Vol. 23, pp. 504–506 (1906).

‡ H. Tinsley, "A magnetic shunt vibration galvanometer," *Elecn.*, Vol. 69, pp. 939–941 (1912).

Agnew[†] has designed a galvanometer on a new principle, shown in Fig. 92 (*e*). The moving system consists of a fine steel wire attached to one pole of a permanent magnet by one of its ends; the other end vibrates in the field of an electromagnet, the winding of which carries the alternating current. The vibration of the wire is observed by a microscope mounted above it. With 270 ohm coils and a magnification of 50/100, 0·05 microampere can be detected. The frequency to which the instrument responds is varied by changing the vibrator; fine tuning is secured by screwing an iron rod nearer to or farther from the lower magnet pole. The instrument is sturdy, and very quickly responsive.

A further interesting type is the vibration telescope introduced by Barus.[‡] In this instrument a telescope object lens is mounted on a double bifilar suspension and has attached to it a small piece of silicon iron which is acted upon by a small electromagnet carrying the current to be measured. The ocular of a fixed telescope faces the lens and a distant fiducial mark is focused. When the lens oscillates from side to side, the image is drawn out and can be viewed as a band upon a plate micrometer in the eyepiece. Tuning is effected by altering the tension of the suspensions.

Disadvantages of Moving Magnet Galvanometers. Moving magnet instruments are easily affected by stray magnetic fields of the resonant frequency and must be set up at some distance from the bridge; they should be magnetically shielded. They are also very susceptible to disturbance by mechanical vibrations of the appropriate frequency, and are best arranged on some form of anti-vibration support.

37. Moving Coil Vibration Galvanometers. The moving coil vibration galvanometer is really a d'Arsonval of short periodic time, arranged with a small light coil of few turns mounted on a suspension which can be tuned by alteration of its length and tension.* The coil stands in the field of a permanent magnet and vibrates when the alternating current to be measured is passed through it. The amplitude of the oscillation is observed in the usual way by reflecting a beam of light to a distant scale from a mirror carried by the coil.

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[*] A. Blondel and F. Carbenay, "Analyse harmonique des différences de potential alternatives par la résonance méchanique d'un barreau de fer aimanté," *Annales de Phys.*, Vol. 8, pp. 97–158 (1917).

[†] P. G. Agnew, "A new form of vibration galvanometer," *Bur. Stds. Scientific Papers*, No. 370, pp. 37–44 (1920).

[‡] C. Barus, "An electrodynamometer using the vibration telescope," *Proc. Nat. Acad. Sci.*, Vol. 5, pp. 211–217 (1919).

The development of the m.c. instrument is due to Campbell,† who has designed various types of galvanometer, some being shown in Fig. 94. In the original type the coil is suspended by a silk thread above and bifilar suspensions below, and is tunable over a range from 50 to 800 cycles per second. The later long-range pattern has metal bifilars both above and below, and is tunable over a range from 40 to 1,000 cycles per second; the short range type, on the other hand, is provided with unifilar strip‡ suspensions, originally introduced by Hausrath,§ and covers a range from 10 to 400 cycles per second.

The instrument is capable of very high sensitivity, especially at low frequencies; Campbell‖ has produced a galvanometer giving 400 mm. at 1 metre per microampere at 10 cycles per second. The curve given in Fig. 94 for a typical Campbell bifilar galvanometer show the sensitivity of the instrument when tuned to respond to different frequencies. The sensitiveness falls inversely as the frequency for lower values, but much more rapidly at higher frequencies owing to the enormous increase in the damping with short suspensions. The effective resistance varies between about 500 ohms and 35 ohms over the range of frequency, 50 to 1,000 cycles per second.

The Campbell galvanometers made by the Cambridge Instrument Co. have interchangeable coils covering ranges of 10–100, 20–200, 30–300, 60–600, and 200–1,200 cycles per second respectively. Coarse tuning is effected by moving the bridges under the suspensions, fine tuning by altering the tension. At frequencies of 100, 300, 500, and 600 cycles per second the sensitivities in mm. at 1 metre per microampere are 63, 21, 7·5, and 3·5. Rothwell* has made an interesting application of these instruments by mounting twelve movements side by side in a common magnet; they are tuned at intervals of 104/100 between 128 and 197 cycles per second, and are used in conjunction with a hot wire microphone for sound analysis in signals research.

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* H. Zöllich, "Spulen-Vibrationsgalvanometer," *Arch. f. tech. Mess.*, J· 852–3 (June, 1933). Also *see* the book by Campbell and Childs.

† A. Campbell, "On the measurement of mutual inductance by the aid of a vibration galvanometer," *Proc. Phys. Soc.*, Vol. 20, pp. 626–638 (1907); also *Dictionary of Applied Physics*, Vol. 2, pp. 964–965.

‡ A. Campbell, "On vibration galvanometers with unifilar torsional control." *Proc. Phys. Soc.*, Vol. 25, pp. 203–205 (1913). A. Campbell, "On vibration galvanometers of low effective resistance," *Proc. Phys. Soc.*, Vol. 26, pp. 120–126 (1914). Both types are made by the Cambridge Instrument Co.

§ H. Hausrath, "Die methoden zur Eisenuntersuchung bei Wechselstrom und ein Apparat zur Darstellung dynamischer Hysteresiskurven," *Phys. Zeits.*, Vol. 10, pp. 756–762 (1909). The method had been used earlier by Blondel.

‖ A. Campbell, *Proc. Phys. Soc.*, Vol. 31, pp. 85–86 (1919). For other types of m.c. galv., *see* P. G. Agnew and F. B. Silsbee, *Trans. Amer. I.E.E.*, Vol. 31, pp. 1635–1644 (1912); H. Zöllich, "Ueber ein hochempfindliches Vibrationsgalvanometer für sehr niedrige Frequenzen," *Arch. f. Elekt.*, Vol. 3, pp. 369–383 (1915); A. Blondel, *Annales de Phys.*, Vol. 10, pp. 195–354 (1918).

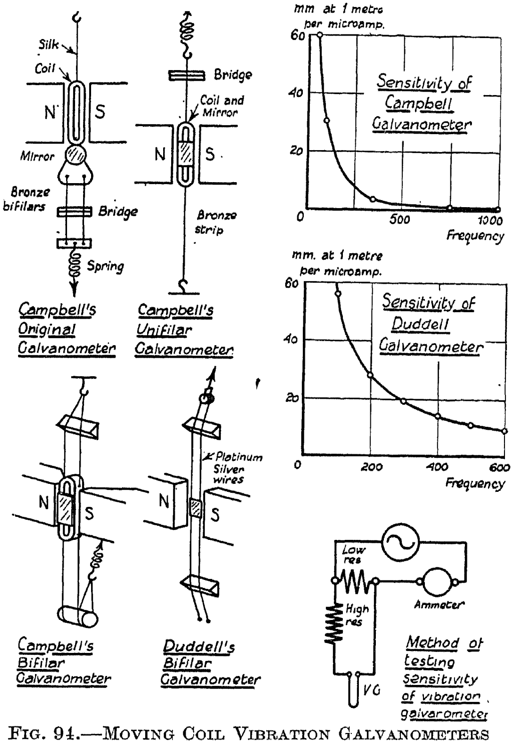
Silk
Coil
N
S
Mirror
Bronze bifilars
Bridge
Spring
Campbell's Original Galvanometer
Bridge
Coil and Mirror
N
S
Bronze strip
Campbell's Unifilar Galvanometer
mm at 1 metre per microamp.
60
40
20
0
500
1000
Frequency
Sensitivity of Campbell Galvanometer
mm. at 1 metre per microamp.
60
40
20
0
200
400
600
Frequency
Sensitivity of Duddell Galvanometer
N
S
Campbell's Bifilar Galvanometer
Platinum Silver wires
N
S
Duddell's Bifilar Galvanometer
Low res
High res
Ammeter
VG
Method of testing sensitivity of vibration galvanometer

FIG. 94.—MOVING COIL VIBRATION GALVANOMETERS

Messrs. H. Tinsley make galvanometers with coils for 5–25, 20–140, 70–300, 250–1,000 cycles per second respectively, interchangeable in the same magnet. The sensitivities in mm. at 1 metre per microampere at the ranges covered by these coils are 600–100, 266–16, 90–13, 15·5–0·5.

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* P. Rothwell, "The mutivibration galvanometer," *Journal Sci. Insts.*, Vol. 2, pp. 251–254 (1925).

A very useful quick-tuning instrument is also made,[*] simultaneous alteration of bridges and the tension being made by a single adjustment, the resonant frequency being marked on a large disc.

The damping is chiefly due to elastic hysteresis in the suspension strips, which are usually of phosphor bronze. The small value of damping is indicated by the sharp resonance curves shown in Fig. 96, p. 276. Owing to the narrow range of high sensitiveness, moving coil galvanometers must be used with a constant frequency source; moreover, in common with other tuned detectors, they show great frequency selectiveness.

Blondel[†] has described a galvanometer consisting of a closed silver or aluminium coil attached to a tunable bifilar suspension. Current is induced in the coil by putting it in a gap in the core of a small transformer excited by the current to be measured.

An ordinary bifilar oscillograph of the Duddell pattern can be used as a vibration galvanometer by removing the oil damping; it is applicable at frequencies above about 2,000.[‡] Duddell,[§] by re-designing the instrument, greatly increased the sensitiveness and extended the range downwards. In his galvanometer the current passes through a loop of fine bronze or platinum-silver wire stretched by means of a spring over two ivory bridges which determine its vibrating length. The wires lie in the strong field of a permanent magnet, and carry a small mirror at the middle of their length. Rough tuning is secured by changing the distance apart of the bridges, which can be moved simultaneously by a right and left-hand screw, so that the mirror is always midway between them. Fine tuning is done by altering the tension of the loop. Instruments of this pattern cover a range from 100 to 1,800 cycles per second and have an effective resistance of about 250 ohms. They are now made by Messrs. Muirhead.

The sensitivity is high, being about 50 mm. at 1 metre per microampere at 100 cycles per second, and falls off inversely as the frequency, provided the loop is not too short. Haworth[||]

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[*] D. C. Gall, "A new quick-tuning vibration galvanometer with calibrated tuning," *Journal Sci. Insts.*, Vol. 5, pp. 134–135 (1928).

[†] A. Blondel, "Galvanomètre de résonance à induction," *Lum. Élect.*, Vol. 18, pp. 72 (1912).

[‡] Mühlenhöver, *Dissertation*, Münster i. W. (1905). E. Giebe and H. Diesselhorst, *Zeits. f. Instk.*, Vol. 26, p. 151 (1906).

[§] W. Duddell, "On a bifilar vibration galvanometer," *Proc. Phys. Soc.*, Vol. 21, pp. 774–787 (1910).

[||] H. F. Haworth, "The maximum sensibility of a Duddell vibration galvanometer," *Proc. Phys. Soc.*, Vol. 24, pp. 230–237 (1912); "Vibration galvanometer design," *idem*, Vol. 25, pp. 264–272 (1913).

has shown how the sensitiveness, especially to applied voltage, may be further increased by the use of an electromagnet and by using the instrument *in vacuo*. In the latter case an increase of 30 per cent is obtainable, since a large part of the damping is due to air friction.

Schering and Schmidt* have constructed a galvanometer of Duddell's pattern having wires as long as 100 cm. and extending the frequency down to 25 cycles. At 50 cycles, 10 mm. per microampere at 1 metre was obtained.

The principle of the Duddell galvanometer has been still further simplified in the single-wire instrument designed by Dr. Moll† and made by Kipp, of Utrecht. The moving system consists of a single-stretched tungsten fibre carrying a small unbalanced mirror, the fibre lying between the poles of an electromagnet ; the transverse vibratory motion of the fibre results in rotational oscillation of the mirror. The range of tuning is 100 to 2,500 cycles per second with very high frequency selectiveness. The sensitivity is 30 mm. at 1 metre per microampere at 260 cycles, falling to 3·5 mm. at 1,000 cycles, and 2 mm. at 2,500 cycles.

It is interesting here to notice an interesting remote-control tuning device designed by Silsbee‡ for use in moving-coil vibration galvanometers. The upper suspension is attached in the usual way ; the lower suspension is anchored to a horizontal flat bronze spring. Attached to this spring is a cylindrical soft iron core which hangs within a coil standing with its axis vertical. A direct current flowing round the coil produces a pull on the iron core and enables a variation in tension of the suspension to be obtained sufficient to give a tuning range of 2 or 3 cycles per second in a 60-cycle instrument. Coarse tuning is effected by the use of sliding bridges. Silsbee's galvanometer has unifilar suspensions, but the tuning device is applicable to all types of suspension. This device gives to moving-coil galvanometers the advantage of remote control of tuning inherent in certain polarized moving-magnet galvanometers, see page 259.

Another arrangement due to Vuylsteke§ uses a double coil system, consisting of two small moving coils rigidly attached one above the other with their planes at right angles, the whole being supported by light bifilar suspensions. The upper coil carries the a.c. to be measured, and in the undeflected position has its plane parallel to the field of a strong permanent or electromagnet. The lower coil lies in a similar magnetic field but has its plane perpendicular to the lines of force; it carries a direct current, produces a couple proportional to the sine of the angle of displacement, and is used to effect tuning. It is shown that by supplying this second coil from a shunt carrying the current which magnetizes the magnet, the current in the coil is proportional to the tuned frequency.

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* A. Schering and R. Schmidt, " Ein empfindliches Vibrationsgalvanometer für niedrige Frequenzen," *Arch. f. Elekt.*, Vol. 1, pp. 254–258 (1913).

† W. J. H. Moll, " A new vibration galvanometer," *Journal Sci. Insts.*, Vol. 2, pp. 361–363 (1925); Vol. 5, pp. 22–23 (1928). S. L. Straneo, "Sul galvanometro a vibrazione di Moll per correnti alternate," *Nuovo Cimento*, Vol. 7, pp. 99–116 (1930). H. W. Koch and O. Brötz, "Veränderung der Resonanzkurve eines Saitengalvanometers durch Parallelkondensators," *Arch. f. Elekt.*, Vol. 29, pp. 688–692 (1935).

‡ A. E. Kennelly, *Electrical Vibration Instruments*, p. 271 (1923).

§ H. A. Vuylsteke, "Nouveau galvanométre à résonance," *Rev. Gén. de l'Él.*, Vol. 37, pp. 537–539 (1935).

38. Vibration Electrometers. Small condensers possess very large impedance when used in a.c. circuits at low frequencies. When measured in a bridge they form branches of high impedance, and to secure sensitiveness the impedance of the detector must also be high. A vibration electrometer is, therefore, useful in such cases and was first employed by Greinacher.* His instrument was developed from the ordinary Wulf electrometer.

Curtis† has designed an instrument by means of which condensers of 10^{-3} μF. can be measured at 50 cycles with ten times the accuracy attainable with a vibration galvanometer. It consists of an aluminium vane mounted with its plane vertical upon a tunable bifilar suspension, the vane corresponding to the "needle" of a quadrant electrometer. Four rectangular plates are placed two in front and two behind the vane, and are connected diagonally. The vane is charged at a high, steady potential; the two pairs of plates are connected to the bridge. The suspended vane is tuned to resonance with the applied p.d. and, to reduce damping, the whole instrument is enclosed in a glass bell-jar, from which the air is exhausted to a certain degree. The sensitivity is of the order of 10,000 mm. at 1 metre per microampere, the deflection being read by reflected light from a mirror carried by the vane. The instrument is not suitable for frequencies over 100.

Krüger‡ has used a piezo-electric quartz resonator as a detector in bridges operated at radio frequencies. It is well known that if a thin slab is cut from a quartz crystal with the electric axis perpendicular to its flat faces, pressure on these faces causes them to become oppositely charged. Conversely, if the faces are charged by putting the slab into a parallel-plate condenser, the slab shortens or lengthens in the direction perpendicular to its surface, according to the sign of the charge. Consequently, an alternating charge will set the crystalline plate into oscillation, and resonance will occur at a definite frequency; the arrangement forms a kind of vibration electrometer. Since the damping is slight, the resonance point is very sharp and can be indicated by an ingenious method due to Giebe and Scheibe.* The crystals are enclosed in a glass envelope containing a mixture of helium and neon at low pressure; when resonance occurs, the charges developed on the surfaces of the crystal slabs are sufficient to set up a glow discharge in the gas. The onset of the discharge is very sudden and is a very precise indicator of exact resonance. Krüger describes resonators covering a range from 35,000 to 10^6 cycles per second.

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* H. Greinacher, "Ueber die Verwendung des Vibrations-Elektrometers in der Wheatstoneschen Brücke," *Phys. Zeits*, Vol. 13, pp. 388–389, 433–434 (1912); "Ueber das Vibrationselektrometer und dessen Verwendung bei Wechselstrommessungen," *Arch. f. Elekt.*, Vol. 1, pp. 471–476 (1913)

† H. L. Curtis, "A vibration electrometer," *Bull. Bur. Stds.*, Vol. 11, pp. 535–552 (1915).

‡ F. Krüger, "Über die Verwendung von Leuchtquarz resonatoren als Vibrationselektrometer bei Messungen in der Wheatstoneschen Brücke, insbesondere zur Messung des Skineffektes an Drähten aus Eisen und Mumetall," *Ann. der Phys.*, Vol. 26, pp. 167–176 (1936).

39. Method of Using Vibration Galvanometer. Before endeavouring to balance the bridge in which a vibration galvanometer is to be used, the instrument must be tuned. To do this a small current is passed through it, frequently by connecting the galvanometer into the unbalanced bridge and heavily shunting it.† The length and tension of the suspension, or other tuning adjustment, is then varied until the band of light on the scale attains its maximum breadth; the moving system is then in resonance with the applied current. The bridge may now be balanced, the galvanometer shunt being gradually removed to give increased sensitiveness as the deflection is reduced to zero.

Great care should be taken to see that the spot of light reflected from the mirror is quite sharp when no current is applied to the instrument. The mirror should be a good one, and should give a good image of the lamp filament or the cross-wire used to show the deflection. The simplest optical system is a plane mirror in front of which is a convex lens of 1 m. focal length, which brings the spot of light to a focus on the scale. A "Point'o'lite" lamp forms an excellent source of illumination, especially for instruments in which a very small mirror is used. The instrument should be set up so that it is free from mechanical vibration and is unaffected by stray alternating magnetic fields of the resonance frequency.

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* E. Giebe and A. Scheibe, "Sichtbarmachung von hochfrequenter Longitudinalschwingungen piezoelektrischer Kristallstäbe," *Zeits. f. Phys.*, Vol. 33, pp. 335–344 (1925).

† For a shunt specially designed for use with vibration galvanometers, see H. Schering and G. Reichardt, "Ein Empfindlichkeitsregler für Nullinstrumente," *Arch. f. Elekt.*, Vol. 12, pp. 493–497 (1923).

Freedom from the effects of mechanical vibration can usually be adequately obtained by standing the galvanometer on a heavy slate or stone base which is supported on rubber sponges resting upon a sturdy shelf or pillar; such sponges are a highly efficient means of insulating the galvanometer from mechanical disturbances. In cases where the disturbances are particularly severe it may be necessary to use one of the more complicated anti-vibration pendulum mountings, such as the well-known Julius suspension. Moving-coil galvanometers are naturally free from stray field troubles ; moving-magnet galvanometers must, however, usually be put into an efficient laminated iron shield, especially in bridges where large currents with consequent considerable stray fluxes may occur.

It is useful to notice here that the vibration galvanometer may be successfully used not only in the laboratory but also on the works test-bed. For this purpose, Messrs. H. Tinsley and Co. make a portable vibration galvanometer set specially designed for works conditions, comprising a moving-coil galvanometer together with lamp and scale fitted up in a suitable box. The galvanometer has a nominal frequency of 50 cycles per second with a tuning range of 2 or 3 per cent, the sensitivity being 9 mm. per microampere at the short-scale distance used.

The vibration galvanometer has one disadvantage in comparison with some other types of detector in that it does not show whether a particular setting is too high or too low. Saunders* has described two ways of overcoming this defect by the use of stroboscopic observation. In one method a stroboscopic disc driven synchronously with the supply is used ; when the circuit is unbalanced the spot of light viewed through the disc will be stationary on one side or the other of zero, depending on whether the bridge setting is too high or too low. In another method the illumination of the galvanometer is provided by a neon lamp energized from the same source as the bridge supply, thus providing fixed synchronization.

For an additional adjustment, possible in some instruments, for greatest voltage sensitivity, see page 280.

40. Theory of the Vibration Galvanometer. Owing to the importance of the vibration galvanometer as a sensitive detector in bridge work and in other classes of measurements, e.g. testing of instrument transformers, the analysis of alternating wave forms, etc., it is of interest to examine the theory of the instrument to find the best conditions under which to use it and also to determine data for design. The theory of apparatus working in mechanical resonance with a periodic current has been known for a long time,* but the application to the theory of vibration galvanometers is relatively new.

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* J. B. Saunders, *Journal Opt. Soc. Amer.*, Vol. 17, pp. 326–327 (1928).

For the purposes of theory vibration galvanometers may be divided into two classes, according to the possible number of degrees of freedom possessed by the moving system. The first class includes instruments of the moving coil or the suspended magnet type where a bifilar or a flat strip suspension is used; the system has only one degree of freedom, namely, rotation about the axis of suspension, provided that the moving system is symmetrically mounted on the axis and that the latter possesses lateral rigidity. In the second class, represented by the Duddell string type of galvanometer, there is an infinite number of degrees of freedom.

The theory of galvanometers with one degree of freedom has been given by Wenner.† His terms and theory will be adopted here, but his analytical methods will be superseded by the symbolic vector method as worked out by Butterworth‡ and extended by him to cover the case of instruments with an infinite number of degrees of freedom.

41. Vibration Galvanometers with One Degree of Freedom. General. In order to fix ideas, let the theory of the widely-used Campbell galvanometer be considered. In this instrument a moving coil vibrates in a magnetic field and is controlled by a bifilar suspension, the length and tension of which can be varied. With obvious modification of detail, the theory will also apply to moving magnet instruments and also to the acoustic telephone.

When at rest the coil of the galvanometer lies with its plane in the direction of the magnetic field. Suppose the field to have strength H, and that the plane of the coil has area A. If the coil has N turns and carries a current whose instantaneous value is i, the couple deflecting the coil from its initial position will be $HANi \cos \theta$, where θ is the angle between the field and the plane of the coil. When θ is small, the deflecting couple is, very nearly, $HANi = Gi$; G is called the *constant of displacement.*

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* H. L. F. von Helmholtz, *On the Sensations of Tone*, 2nd edn., Appendix 8, pp. 398–400 (1885).

† F. Wenner, "Theoretical and experimental study of the vibration galvanometer," *Bull. Bur. Stds.*, Vol. 6, pp. 347–378 (1910).

‡ S. Butterworth, "On the vibration galvanometer and its application to inductance bridges," *Proc. Phys. Soc.*, Vol. 24, pp. 75–94 (1912). *See also* H. Zöllich, "Mechanische Resonanz-Schwingungen in der Messtechnik," *Wiss. Veröff. Siemens Konz.*, Vol. 2, pp. 378–400 (1922). "Vibrationsgalvanometer. Allgemeine Eigenschaften," *Arch. f. tech. Mess.*, J. 852–1 (Nov., 1932). "Nullstrom-Indikatoren. Steigerung der Empfindlichkeit elektrischer Messgeräte durch mechanische Resonanz," *Arch. f. tech. Mess.*, J. 850–2 (Apr., 1933).

The deflecting couple is opposed by three retarding couples—

(i) A restoring couple, due to the elasticity of the suspension, proportional to the displacement. If c is the *constant of restoration,* the couple is $c\theta$.

(ii) A damping couple due to air-friction, imperfect elasticity of the suspension, etc. It is usual to take this couple as proportional to the angular velocity of the coil. If b be the *constant of damping*, the couple is $b \cdot d\theta/dt$. Blondel and Carbenay* have described a moving magnet galvanometer in which it was necessary to assume the damping proportional to the square of the velocity; other workers have also made this correction to the theory of oscillating bodies, especially in cases where air-damping at high velocities is involved.

(iii) A kinetic reaction couple proportional to the angular acceleration of the coil. If a be the *constant of inertia* of the coil, the couple is $a \cdot d^2\theta/dt^2$.

The equation of motion is obtained by equating the total resisting couple to the deflecting couple, i.e.

$$a\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} + c\theta = Gi.$$

The quantities a, b, c, G are expressed in absolute units, and are called by Wenner the *intrinsic constants* of the instrument.

Two distinct problems now arise: (i) the conditions of motion of a tuned galvanometer carrying a current $i = i_1 \cos\omega t$; (ii) the behaviour of a galvanometer when connected in a circuit, such as a bridge network, which contains an alternating e.m.f. The first problem is concerned with the current sensitivity of the galvanometer, and is similar to the theory of the oscillograph worked out so thoroughly by Blondel.† The second problem involves the consideration of the effect of the voltage induced in the galvanometer coil as it swings in the magnetic field, since this back e.m.f. will have an important influence on the voltage sensitivity of the instrument.

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* A. Blondel and F. Carbenay, "Analyse harmonique des différences de potentiel alternatives par la résonance méchanique d'un barreau de fer aimanté," *Annales de Phys.*, Vol. 8, pp. 97–158 (1917); see also B. O. Pierce, *Proc. Amer. Acad.*, Vol. 44, pp. 63–88 (1909); and R. Grammel, *Phys. Zeits.*, Vol. 14, pp. 20–21 (1913). See also Section 44.

† A. Blondel, "Théorie des oscillographes," *Écl. Élec.*, Vol. 33, pp. 115–125 (1902), and Vol. 36, pp. 326–346 (1903).

42. Vibration Galvanometers with One Degree of Freedom. Current Sensitivity. Let the galvanometer carry a current $i = i_1 \cos \omega t$ represented by a vector **i**. Then the complete solution of the equation of motion consists of the sum of two parts: (i) a transient portion, representing the natural free motion of the coil, involving the initial conditions of the moving system at the time of switching on the current; (ii) a periodic forced oscillation corresponding to the steady vibratory motion of the coil after the initial disturbances have subsided. It is with (ii) that we are chiefly concerned, but a few words are first necessary regarding (i).

The *transient part of the solution*, θ_T, is obtained by solving the equation

$$a\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} + c\theta = 0.$$

By the ordinary theory of differential equations if the roots of the auxiliary equation

$$ax^2 + bx + c = 0$$

are $\quad m_1,\ m_2 = \{-b \pm \sqrt{b^2 - 4ac}\}/2a = -\alpha \pm \beta,$

the solution is

$$\theta_T = A\varepsilon^{m_1 t} + B\varepsilon^{m_2 t} = \varepsilon^{-\alpha t}\{A\varepsilon^{\beta t} + B\varepsilon^{-\beta t}\}$$

provided $\quad b^2 \lesseqgtr 4ac;$

or $\quad \theta_T = \varepsilon^{-\alpha t}\{A + Bt\}$

when $b^2 = 4ac$; A and B are found from the initial conditions.

Hence owing to the factor $\varepsilon^{-\alpha t}$ the transient part of the solution is gradually damped out. In vibration galvanometers b is usually small, so that $4ac$ is greater than b^2; in this case

$$\theta_T = \varepsilon^{-\alpha t}\left\{C\cos\left(\frac{\sqrt{4ac - b^2}}{2a}t + \phi\right)\right\};$$

C and ϕ are again determined by the initial conditions. The free galvanometer system thus performs *damped natural oscillations* of frequency $(\sqrt{4ac - b^2})/4\pi a$. If the system had no damping the *undamped natural frequency* would be $f_o = \frac{1}{2\pi}\sqrt{\frac{c}{a}}$; or $\omega_o = \sqrt{c/a}$.

When the damping is such that the system just ceases to be capable of free oscillations $\beta = 0$ and the galvanometer is *critically damped*. The appropriate value of b is $b_c = 2\sqrt{ac}$.

This degree of damping is approximately that aimed at in oscillographs, but is not nearly approached in vibration galvanometers; it is useful, however, to express the actual damping in terms of the critical value, i.e. b/b_c is the *degree of damping*.

The effect of the transient oscillations of the coil is to produce initially when the galvanometer is switched into circuit a complex motion, due to the superposition of the free and forced oscillations. The transient terms gradually die out until the instrument is in a steady state of forced oscillation corresponding to the current i, as the records of the motion of a moving coil galvanometer obtained by Zöllich* clearly show.

Hence for the present purpose the transient part of the motion may be neglected, attention being confined to the forced periodic motion of the coil.

The *periodic forced oscillation* is obtained by solving

$$a\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} + c\theta = Gi_1 \cos \omega t = \sqrt{2}GI \cos \omega t.$$

Clearly θ must be a periodic function of ωt; let it be represented by the harmonic vector $\boldsymbol{\theta}$. Then by the methods shown on pages 25–37

$$\{(c - a\omega^2) + j\omega b\}\boldsymbol{\theta} = G\mathbf{i},$$

$$\boldsymbol{\theta} = \frac{G\mathbf{i}}{(c - a\omega^2) + j\omega b} = \frac{G}{\sqrt{(c - a\omega^2)^2 + \omega^2 b^2}} \cdot \underline{/-\text{arctan}} \; \frac{\omega b}{c - a\omega^2} \cdot \mathbf{i}$$

from page 36.

Hence the amplitude of the motion of the coil is

$$\theta_1 = \frac{Gi_1}{\sqrt{(c - a\omega^2)^2 + \omega^2 b^2}} = \frac{\sqrt{2}\,GI}{\sqrt{a^2(\omega_o^2 - \omega^2)^2 + \omega^2 b^2}}$$

the motion lagging in phase by an angle

$$\psi = \arctan \frac{\omega b}{c - a\omega^2};$$

i.e. $$\theta = \theta_1 \cos(\omega t - \psi).$$

For a given value of the current I it is required to make the amplitude of the motion as large as possible. Clearly, θ_1 is proportional to $G = HAN$; since the area and turns of the coil are fixed, the amplitude increases as the field in which the coil lies is made stronger. Hence *the magnet should be powerful* and can be an electromagnet (see, however, p. 280).

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* H. Zöllich, "Über ein hochempfindliches Vibrationsgalvanometer für sehr niedrige Frequenzen," *Arch. f. Elekt.*, Vol. 3, pp. 369–383 (1915).

Further increase of amplitude is secured by "tuning" the instrument. As Max Wien has pointed out, there are two ways of tuning a galvanometer to resonance with a source of supply: (i) by adjusting the frequency of the source; (ii) by adjusting the constants of the galvanometer until resonance is attained.

FREQUENCY TUNING. Let the galvanometer be such that a, b, c, and G are fixed, and let ω be continuously varied; then θ_1 will be a maximum when ω reaches the value given by

$$\omega_f{}^2 = \frac{c}{a} - \frac{1}{2}\left(\frac{b}{a}\right)^2 = \omega\,^2 - \frac{1}{\ }\Big(\frac{b}{\ }$$

the amplitude then being

$$\frac{2\sqrt{2}GaI}{b\sqrt{(4ac-b^2)}} = \frac{\sqrt{2}GI}{b\omega_n},$$

since the natural damped frequency of the galvanometer is obtained from

$$\omega_n{}^2 = \frac{4ac-b^2}{4a^2} = \frac{c}{a} - \frac{1}{4}\left(\frac{b}{a}\right)^2 = \omega_o{}^2 - \frac{1}{4}\left(\frac{b}{a}\right)^2.$$

This shows that for the greatest sensitiveness, the *damping* b and the *natural frequency of the instrument must be low*; resonance then occurs when the frequency of the source $\omega_f \fallingdotseq \omega_n \fallingdotseq \omega_o$, since b is negligible.

Frequency tuning is easily studied in a general way. Introducing the undamped natural frequency and the critical damping gives $\omega_o = \sqrt{c/a}$, $b_c = 2\sqrt{ca}$; the amplitude then becomes

$$\theta_1 = \frac{\sqrt{2}\,GI}{c\sqrt{\left(1-\frac{\omega^2}{\omega_o{}^2}\right)^2 + 4\frac{b^2}{b_c{}^2}\cdot\frac{\omega^2}{\omega_o{}^2}}}.$$

When the frequency is very low, i.e. $\omega \rightarrow 0$, the amplitude is $\sqrt{2}\,GI/c$; then if

$$\gamma = \frac{\omega}{\omega_o} = \frac{\textit{applied frequency}}{\textit{undamped natural frequency}}$$

and $$\delta = \frac{b}{b_c} = \frac{\textit{damping constant}}{\textit{critical damping constant}} : \textit{degree of damping},$$

$$\frac{\textit{Amplitude}}{\textit{Amplitude for zero frequency}} = \frac{1}{\sqrt{(1-\gamma^2)^2 + 4\gamma^2\delta^2}}$$

$$\tan\psi = 2\gamma\delta/(1-\gamma^2).$$

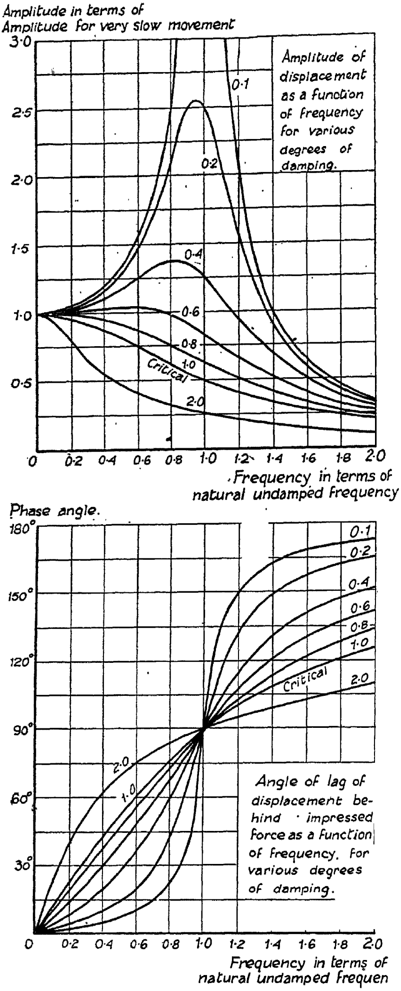

FIG. 95.—FREQUENCY TUNING OF A VIBRATING SYSTEM

Fig. 95 shows these expressions plotted as a function of γ for various degrees of damping. When the damping is equal to or in excess of the critical value, gradually raising the applied frequency continually reduces the amplitude of motion. When the damping is less than critical, the amplitude attains a maximum for some value of frequency less than the undamped frequency of the moving system, diminishing again as the frequency is still futher raised. The maximum amplitude is greater as the damping is reduced, and occurs more and more nearly at the value of applied frequency equal to the undamped frequency of the system. Hence the great importance in vibration galvanometers of reducing the damping as much as possible. At the same time these curves show the selective sensitiveness of the galvanometer for currents of the resonating frequency and its insensitiveness to all other applied frequencies, e.g. to harmonics in the deflecting current.

Referring to the curves illustrating the phase of the motion, it is clear that, whatever be the degree of damping, the deflecting couple and the resulting motion will be in phase when the frequency is very low. As the frequency is increased, the motion lags behind the deflecting couple until, when the applied frequency and the undamped natural frequency are equal, they are in quadrature. Thereafter the phase-angle increases until, whatever be the damping, the couple and the motion are in opposition of phase when the frequency is infinite. A careful study of these curves, which apply to all cases of forced oscillations in mechanical systems, is recommended to the student.*

CONTROL TUNING. In most bridges, tests are to be carried out at a definite frequency, so that ω is fixed. Hence the tuning is more usually done on the galvanometer itself, thereby enabling one instrument to cover a range of frequencies. It is not easy to produce a continuous variation of the moment of inertia a; the damping b is seldom sufficiently adjustable to serve as a means of tuning. Thus, tuning is effected by alteration of c by variation of the length and tension of the suspension in coil galvanometers, or of the polarizing field in moving magnet instruments.

With ω, G, a and b fixed, θ_1 will be a maximum when $c = a\omega^2$; that is, when $\omega_0 = \omega$, the undamped natural frequency of the moving system being equal to the fixed applied frequency. The amplitude has the maximum value

$$\sqrt{2}\,GI/\omega b$$

and the motion is $\pi/2$ later than the applied current. For the greatest sensitivity the magnetic field should be strong, the damping slight, and the applied frequency low.

To illustrate the control tuning of a vibration galvanometer, curves are plotted in Fig. 96 for a Campbell† instrument in which $a = 6{\cdot}9 \times 10^{-6}$, $b = 23{\cdot}2 \times 10^{-6}$, $G = 585$, giving the amplitude in arbitrary units in terms of the control constant c when applied frequencies of 50 and 100 are used. Resonance occurs when the applied and undamped frequencies are equal, and the extreme sharpness of tuning is to be noted, showing the selective frequency property of the galvanometer due to its slight damping. For a frequency of 100, $c = 2{\cdot}72$, so that $b_c = 2\sqrt{ac} = 8{\cdot}68 \times 10^{-3}$; hence $b/b_c = 0{\cdot}0027$.

To show how much more sensitive the galvanometer is to currents of the frequency to which it is tuned, consider the response of an instrument to the nth harmonic when tuned to the fundamental. The amplitude due to the fundamental is $\sqrt{2}\,GI/\omega b$, since ω_0 is tuned to equal ω. With ω_0 at this value, let ω be changed to $n\omega$ in the general equation for the amplitude, p. 272; then the amplitude due to a current of n times fundamental frequency is, for the same current strength, $\sqrt{2}\,GI/\{\sqrt{a^2(\omega^2 - n^2\omega^2)^2 + n^2b^2\omega^2}\}$. Neglecting the damping term, this is $\sqrt{2}GI/\omega^2a(n^2 - 1)$; hence

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* C. C. Hawkins, *The Dynamo*, pp. 116–126, 6th edition. A. Blondel, *loc. cit.*

† *Dictionary of Applied Physics*, Vol. 2, p. 974.

$$\textit{Sensitivity to fundamental/sensitivity to nth harmonic}$$
$$\doteqdot \omega a(n^2 - 1)/b.$$

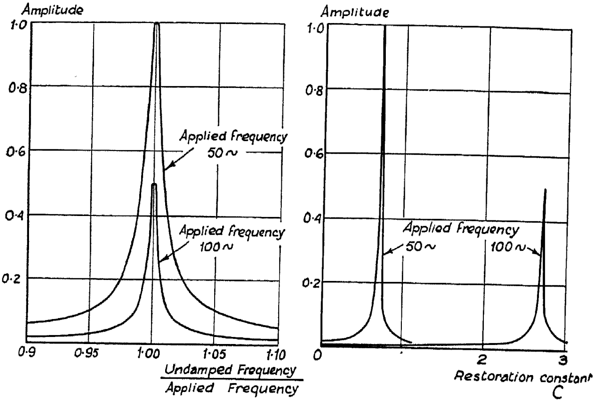

FIG. 96.—CONTROL TUNING OF A VIBRATION GALVANOMETER

In the case of the Campbell instrument treated above, when tuned to resonate at 100 cycles per second the ratio works out to be 1,495 when $n = 3$; i.e. the galvanometer is 1,495 times more sensitive to the fundamental than it is to the third harmonic, and the ratio is considerably greater for higher harmonics. Hence it is not necessary to supply absolutely pure wave forms of current to bridges in which a vibration galvanometer is used, since the sensitiveness of the instrument is so much greater for current of fundamental frequency than it is for the harmonics.

43. Vibration Galvanometers with One Degree of Freedom. Voltage Sensitivity. In its usual application to a.c. bridges, the vibration galvanometer is used to detect small differences of potential between the branch points of the bridge. Its voltage sensitivity then becomes important, and will now be investigated.*

When the coil of the galvanometer is vibrating, it cuts the field issuing from the magnet, and therefore has an e.m.f. induced in it. The current passing through the coil is thus determined in part by the back e.m.f. appearing in the coil due to its vibratory motion in the field, this e.m.f. opposing the alternating p.d. applied to the circuit in which the galvanometer is placed.

If the plane of the moving coil makes an angle θ with the direction of the field H, the flux through it will be $HA \sin \theta$, and the linkages will be $HAN \sin \theta = G \sin \theta$. When θ is small the linkages are $G\theta$, so that the back e.m.f. is

$$e_b = -G\frac{d\theta}{dt}.$$

Since θ is a periodic function of ωt, e_b can be represented by a uniformly rotating vector $\mathbf{e}_b = -j\omega G\boldsymbol{\theta}$.

Suppose the galvanometer to be inserted in a circuit to which an electromotive force $e = e_1 \cos \omega t$ is applied. If the entire circuit, inclusive of the galvanometer, has resistance R and reactance X, its impedance operator will be $R + jX$. If $\mathbf{e}$ be the vector of applied e.m.f., the current is given by

$$\mathbf{i} = (\mathbf{e} + \mathbf{e}_b)/(R + jX) = (\mathbf{e}/z) - (j\omega G\boldsymbol{\theta}/z) \quad . \quad (a)$$

From page 272, the symbolic equation of motion of the galvanometer is

$$\{(c - a\omega^2) + j\omega b\}\boldsymbol{\theta} = G\mathbf{i}\,; \quad . \quad . \quad .. \quad . \quad (b)$$

substituting for the current from above expresses $\boldsymbol{\theta}$ in terms of $\mathbf{e}$,

$$\left\{(c - a\omega^2) + j\omega\left(b + \frac{G^2}{z}\right)\right\}\boldsymbol{\theta} = \frac{G}{z}$$

Inserting $z = R + jX$, and simplifying, gives

$$\{[R(c - a\omega^2) - \omega bX] + j[X(c - a\omega^2) + \omega(bR + G^2)]\}\boldsymbol{\theta} = G\mathbf{e};$$

so that

$$\theta = \theta_1 \cos(\omega t - \psi), \quad . \quad . \quad . \quad . \quad . \quad . \quad (c)$$

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* F. Wenner, *loc. cit.*; S. Butterworth, *loc. cit.*; H. Zöllich, *loc. cit.* *See* also for a very full discussion, A. Blondel, "Sur l'analyse harmonique directe de l'onde des courants alternatifs par résonance méchanique ou électrique," *Annales de Phys.*, Vol. 10, pp. 195–334 (1918).

where the amplitude

$$\theta_1 = \frac{\sqrt{2}GE}{\sqrt{[R(c-a\omega^2)-\omega bX]^2+[X(c-a\omega^2)+\omega(bR+G^2)]^2}},$$

and the phase

$$\tan\psi = \frac{X(c-a\omega^2)+\omega(bR+G^2)}{R(c-a\omega^2)-\omega bX},$$

E being the effective value of $e = e_1 \cos \omega t$.

Alternatively, elimination of θ gives the relation between $\mathbf{e}$ and $\mathbf{i}$, thus,

$$z\mathbf{i} = \mathbf{e} - jw \cdot \frac{G^2\mathbf{i}}{(c-a\omega^2)+j\omega b}$$

$$\text{or} \quad \mathbf{e} = \left[R + jX + j\omega G^2\left\{\frac{(c-a\omega^2)-j\omega b}{(c-a\omega^2)^2+\omega^2b^2}\right\}\right]\mathbf{i},$$

$$= \left[R + \frac{\omega^2G^2b}{(c-a\omega^2)^2+\omega^2b^2} + j\left\{X + \frac{\omega G^2(c-a\omega^2)}{(c-a\omega^2)^2+\omega^2b^2}\right\}\right]\mathbf{i},$$

$$(R' + jX')\mathbf{i} \quad . \qquad . \qquad (d)$$

so that

$$i = \frac{e_1}{\sqrt{R'^2+X'^2}}\cos(\omega t - \phi), \text{ where } \tan\phi = X'/R'.$$

R' is called the effective resistance of the galvanometer circuit and X' the effective reactance.

Control Tuning. The conditions for maximum sensitivity of a vibration galvanometer when used as a voltmeter are quite different from those applying to its current sensitivity. Referring to Equation (c) above, let a, b, and ω be fixed and, for the present, let G, R, and X be fixed also. Then let c be adjusted until the amplitude θ_1 has a maximum value. This will be found to occur when

$$(c - a\omega^2) = -\frac{\omega XG^2}{X^2+R^2};$$

the value of the amplitude when the instrument is tuned is

$$\frac{\sqrt{2}GE}{\omega[b(R^2+X^2)^{\frac{1}{2}} + G^2R(R^2+X^2)^{-\frac{1}{2}}]} \quad . \qquad . \qquad (e)$$

The phase is then given by $\tan\psi = R/-X$ or $\psi = \pi - \arctan\frac{R}{X}$. When the circuit has zero reactance, e.g. as is very nearly the case when the galvanometer alone is in circuit, then the amplitude is $\sqrt{2}GE/\omega(bR+G^2)$.

Optimum Sensitivity. The maximum amplitude obtained by tuning the galvanometer in the above manner does not represent the greatest sensitiveness which is possible. The expression (e) can itself be made to attain a maximum value by adjustment of either of the quantities G or X. In both cases the optimum sensitiveness of the instrument as a voltmeter is secured when

$$b/R = G^2/(R^2+X^2).$$

The optimum amplitude is then

$$\sqrt{2}E/2\omega\sqrt{bR}$$

When the galvanometer has been control tuned and has then been adjusted to the optimum condition by alteration of its magnet field, or by varying the circuit inductance, or by the use of an interbridge transformer, it is of interest to examine the resulting current and the back e.m.f. in the coil. Referring to Equation (d), let the conditions $(c-a\omega^2)/\omega X = -G^2/(R^2+X^2) = -b/R$ be inserted; then the effective resistance $R' = 2R$ and the effective reactance $X' = 0$. Hence, $i = (e_1/2R)\cos\omega t$, or, in r.m.s. values, $I = E/2R$. Thus, when a vibration galvanometer is used as a voltmeter and is adjusted to optimum sensitiveness, the effective resistance of the galvanometer circuit is twice its d.c. resistance, and the current through it is in phase with the applied voltage.

When the galvanometer is so adjusted, it is easy to show that the back e.m.f. in the coil is given by $\mathbf{e}_b/\mathbf{e} = -\frac{1}{2}\left(1 - j\frac{X}{R}\right)$. If the reactance is negligible, as, for example, when the galvanometer alone is in circuit, then $\mathbf{e}_b/\mathbf{e} = -\frac{1}{2}$. Thus, when the instrument is used in a bridge and is adjusted to the optimum conditions, the back e.m.f. in the coil is half the applied voltage and is in opposition of phase therewith. Under these conditions, half the power supplied to the instrument is dissipated in heat and half in mechanical work, the latter maintaining the motion of the moving system as a synchronous motor. This condition is well known, and can be realized in practice by adjusting the strength of the galvanometer field by the use of an electromagnet, as has been shown by Haworth* and later by Zöllich. After control tuning has been effected, the optimum is attained by adjusting the electromagnet; the effective resistance of the instrument is then $2R$, as is shown by the curves of Fig. 97, taken from the paper by Zöllich previously cited.

An electromagnet has the disadvantage that it and its associated battery may produce large capacitance effects when the instrument is in an a.c. bridge. The battery supplying the magnet should be small and well insulated. The trouble can be completely overcome by using a permanent magnet of adequate strength provided with a magnetic shunt.

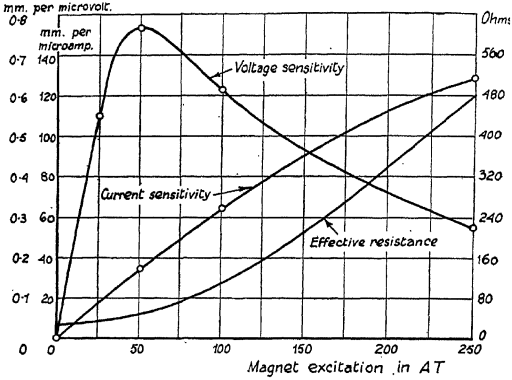

FIG. 97.—VOLTAGE SENSITIVITY, CURRENT SENSITIVITY, AND EFFECTIVE RESISTANCE OF A MOVING COIL VIBRATION GALVANOMETER PLOTTED AS FUNCTIONS OF THE STRENGTH OF THE MAGNET

Another way of avoiding the troubles due to an electromagnet is to adapt the instrument impedance to that of the network by connecting it to the bridge through a transformer of suitable ratio. This method was described by Wenner and provides a simple means of attaining optimum sensitivity.

MOTIONAL RESISTANCE AND REACTANCE. Referring to Equation (d), suppose the galvanometer only to exist in the circuit, R being its resistance and X its reactance. Then the effective resistance R' is composed of the d.c. resistance of the instrument and a term, called its *motional resistance*, arising from the movement of the galvanometer coil. Similarly, the effective reactance X' consists of X together with the *motional reactance* of the coil. The quantities $R' - R$ and $X' - X$ have been investigated in a number of cases by Kennelly,* and are shown to have interesting physical properties.

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* H. F. Haworth, "The maximum sensibility of a Duddell vibration galvanometer," *Proc. Phys. Soc.*, Vol. 24, pp. 230–237 (1912).

In most galvanometers the inductance is very small, so that X is negligible ; the effective reactance of the instrument is then equal to $\omega G^2(c - a\omega^2)/[(c - a\omega^2)^2 + \omega^2 b^2]$ its motional reactance. If the galvanometer is tuned so that $c/a = \omega_o^2 = \omega^2$, the reactance is zero and the galvanometer behaves as if it were a non-inductive resistance $R + (G^2/b)$. If the frequency be lower than this value, i.e. $\omega < \omega_o$, the instrument has positive reactance and behaves as if it were inductive. On the other hand, when $\omega > \omega_o$, the reactance is negative and the instrument acts as a condenser.†

44. Vibration Galvanometers with an Infinite Number of Degrees of Freedom. Asymmetric Systems. Galvanometers of the bifilar loop or Duddell pattern do not behave according to the theory just laid down for a moving coil instrument. The stretched wires carrying at their mid-point a load in the shape of a mirror, perform oscillations in the same way as the vibrating string of a monochord when centrally loaded. They have, therefore, an infinite number of normal modes of vibration, the relations between which are determined by the relative masses of the mirror and the wires.

Butterworth‡ has worked out the theory of such instruments and has shown how to find the infinite number of resonance frequencies. He points out that if the mirror be fairly heavy in comparison with the vibrating loop, the damping of the higher harmonics is large and the instrument acts as if it had only one degree of freedom. With too light a mirror, the frequency selectivity of the galvanometer is spoilt. The wires can then vibrate in some complex manner, and the spot of light, instead of moving in a straight line across the scale, will describe closed curves resembling Lissajou's figures. Similar effects will occur if the two sides of the loop do not come into resonance together, due to inequality of tension or to unsymmetrical mounting of the mirror upon them.*

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* A. E. Kennelly and G. W. Pierce, *Proc. Amer. Acad.*, Vol. 48, p. 113 (1912); A. E. Kennelly and H. O. Taylor, *Proc. Amer. Phil. Soc.*, Vol. 55, p. 415 (1916). A. E. Kennelly, *Electrical Vibration Instruments*, Ch. xxii (1923).

† Zöllich, *loc. cit.*; also *see* S. Butterworth, " On a null method of testing vibration galvanometers," *Proc. Phys. Soc.*, Vol. 26, pp. 264–273 (1914).

‡ S. Butterworth, *Proc. Phys. Soc.*, Vol. 24, pp. 75–94 (1912).

It has also been shown† that multiple resonance is not confined exclusively to instruments of the Duddell pattern. A vibration galvanometer of the moving magnet or moving coil type may exhibit more than one point of resonance, since the moving system may possess more than one degree of freedom arising either from imperfect lateral rigidity of the suspension or from the fact that the axis of suspension does not pass through the centre of mass of the moving system. Rosa and Grover‡ some years ago showed a resonance curve for an instrument of the Rubens type possessing two degrees of freedom, the curve having two well-defined resonance peaks separated by a region of low sensitiveness. This curve is shown in Fig. 98(*a*). An even more complex example, taken from a moving coil instrument, is illustrated by Zöllich; this shows a peak of very high sensitivity followed by a region of instability, after which there is a second, but lower, resonance peak. These effects are due to the fact that the moving system, though apparently simple, has inherent out-of-balance which make it in effect equivalent to two individual systems coupled together. Assuming the moving part to have two degrees of freedom, e.g. one of rotation and one of lateral translation in consequence of unsymmetrical mounting and imperfect rigidity, Ll. Jones has satisfactorily worked out by Lagrange's method the theory of such double resonances, obtaining results in agreement with experiment. The theory is mathematically analogous to that of the electrical resonances in a pair of coupled circuits.

Another class of dissymmetry in the resonance curve is not infrequently exhibited by vibration galvanometers, particularly in those of the moving magnet type. Referring to Fig. 98(*b*), several resonance curves are shown, each drawn for a constant value of current. With small currents, i.e. low amplitudes, the resonance curve is of the normal symmetrical type. With greater currents the curve increases in amplitude and becomes distorted, the resonance peak bending over towards the left. Finally, with large currents the curve leans over so far that within a region, shown by the chain dotted lines for the upper curve, there are two possible values of amplitude for a given frequency together with an unstable value, shown dotted. One or other of these possible amplitudes is obtained according as the trial is made with increasing or with decreasing frequency. For example, if the frequency is increased the portion *afb* of the curve will be traced; further increase then causes the amplitude suddenly to change from *b* to *c*, thereafter continuing by the curve *cd*. If the frequency be reduced, the amplitude follows the curve *dce*, suddenly falling from *e* to *f* and thence steadily to *a*.

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* L. M. Chatterjee, "Über das anomale Verhalten eines Vibrationsgalvanometer," *Zeits. f. Phys.*, Vol. 96, pp. 720–725 (1935).

† R. Ll. Jones, "Vibration galvanometers with asymmetric moving systems," *Proc. Phys. Soc.*, Vol. 35, pp. 67–80 (1923). Similar effects have been studied in connection with the vibrations of telephone diaphragms by Kennelly and Taylor in the paper cited on the preceding page.

‡ See *Bull. Bur. Stds.*, Vol. 1, p. 298 (1905).

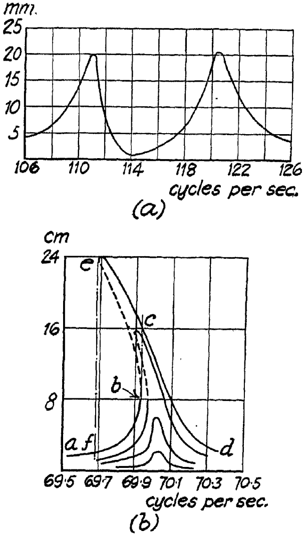

Fig. 98.—Multiple and Asymmetric Resonance in Vibration Galvanometers

These effects have been described by Waibel* in connection with a moving magnet galvanometer used in the range 400 to 800 cycles per second; their explanation and detailed investigation is due to Appleton.† The curves shown in Fig. 98(*b*) are taken from Appleton's paper and refer to a galvanometer of the Drysdale-Tinsley type.

The shape of the resonance curves indicates the existence of some non-linear property of the galvanometer system either in the damping forces or in the restoring forces. The fact that the resonant frequency falls as the amplitude increases suggests that an important cause of the asymmetry and instability is restoring forces which are not proportional to the displacement. In the particular instrument concerned, such non-linear restoring forces may be due to (i) decrease in the pole-strength of the moving iron with increasing angular displacement; (ii) proportionality of restoring couple to sin θ and not to θ; (iii) inhomogeniety of the polarizing field. Appleton has developed the mathematical theory, taking these factors into account, and has obtained theoretical resonance curves which show the general characteristics of those obtained experimentally.

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* F. Waibel, *Ann. der Phys.*, 4th ser., Vol. 72, pp. 161–192 (1923).

† E. V. Appleton, "On the anomalous behaviour of a vibration galvanometer," *Phil. Mag.*, 6th ser., Vol. 47, pp. 609–619 (1924).

Mallett* has investigated such distorted resonance curves, particularly in connection with tuning forks vibrating with large amplitude, and has shown that the complete explanation requires a modification in the damping terms also. Thus, instead of the equation of motion being

$$a\ddot{\theta} + b\dot{\theta} + c\theta = Gi$$

a close approximation is

$$a\ddot{\theta} + b(1 + \lambda\theta^2)\dot{\theta} + c(1 - \nu\theta^2)\theta = Gi$$

Mallett also points out that in addition to the amplitude effect, with its resulting lowering of resonant frequency, distortion of the curve, and introduction of instability, there may exist simultaneously the " coupled circuit " effect arising from the presence of more than one degree of freedom in the suspended system. The general motion is then extremely complex.

45. Measurement of the Sensitivity of a Vibration Galvanometer. The vibration galvanometer possesses several characteristic properties which are used to compare its sensitiveness with other detecting instruments. Moreover, these quantities can be used to determine the value of the intrinsic constants in the equation of motion for use in design.

(i) *Alternating Current Sensitivity.* Referring to Fig. 94, let the galvanometer be connected in series with a high resistance, so that the back e.m.f. in the instrument can be neglected, and tapped across a known low resistance carrying alternating current which is accurately measured by a dynamometer. The instrument is tuned to be in resonance with the supply, which should be taken from a source of constant frequency. From the value of the current and the resistances, the current through the instrument is calculated ; the total width of the resulting band of light (proportional to twice the amplitude) on the scale is observed; then a.c. sensitivity = width of band in mm. at 1 metre scale distance per microampere.

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* E. Mallett, "Distortion of resonance curves of electrically driven tuning forks," *Proc. Phys. Soc.*, Vol. 39, pp. 334–358 (1927). "Some notes on a fork frequency meter," *World Power*, Vol. 8, pp. 133–138 (1927).

(ii) *Alternating Voltage Sensitivity.* The high resistance is cut out, the current from the source being much reduced. The p.d. across the instrument is then known, and the a.v. sensitivity = width of band in mm. at 1 metre scale distance per microvolt.

(iii) *Direct Current Sensitivity.* Reverting to the connections in the figure, let the alternator be replaced by a battery, and let the direct current through the instrument be calculated, the single deflection being observed. Then, in the usual way, d.c. sensitivity = single deflection in mm. at 1 metre per microampère. The ratio a.c. sensitivity/d.c. sensitivity is called the *resonance magnification* and should be as large as possible.

(iv) *Resonance Frequency.* The frequency of the source to which the instrument is tuned is measured by some suitable means.

(v) *Resistance.* The galvanometer resistance should be obtained in a Wheatstone bridge.

For the manner of deducing the intrinsic constants from these five quantities, and for typical values, see *Dictionary of Applied Physics*, Vol. 2, pp. 972–974.*

A quantity of considerable practical interest is the *Resonant Range*, which is arbitrarily defined as the percentage change in resonant frequency which will reduce the resonant deflection to one-half, either for (*a*) constant current, or (*b*) constant voltage. This quantity is a measure of the sharpness of resonance, and will be small when the damping is slight. If the source supplying the bridge is of very steady frequency the resonant range may be safely kept low. If, however, the frequency changes are likely to be fairly considerable, as in bridges supplied from ordinary commercial alternators, it may be necessary to increase the resonant range by artificial means. This can readily be done with moving coil instruments by shunting them; a resonant range of 1 per cent is then easily obtained without too serious a loss in absolute sensitivity. In unshunted instruments the range may be as low as 1 part in 1,000 or even less.

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* The reader may with advantage consult A. E. Kennelly, *Electrical Vibration Instruments*, Macmillan Co. (1923); S. Jimbo, "On the resonant sharpness of a vibration galvanometer," *Journal I.E.E., Japan*, No. 467, pp. 611–620 (1927); and particularly Chapter IV of the book by Campbell and Childs.

Meissner and Adelsberger* have shown that by adding a condenser in the galvanometer circuit, giving with the inductance of the galvanometer a tuned circuit of frequency f_1, it is possible to get a resonance curve which is nearly flat over the frequency range from f_1 to the natural mechanical frequency of the moving system. The constructional features of the galvanometer and the details of the necessary electric circuits are fully discussed (see p. 260). The sensitivity of a galvanometer working at 62 cycles per second changes by less than 0·5 per cent for a frequency variation of 23 per cent. Such flattened resonance is valuable when working with sources, such as large alternators in works testing, where close regulation of frequency is difficult to secure. Obtained in this way there is not the loss of sensitivity occurring when flattened resonance curves are produced by shunting the galvanometer with resistance; the range of flatness is, moreover, considerably wider.

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* W. Meissner and U. Adelsberger, "Vibrationsgalvanometer mit weitgehender Frequenzunabhängigkeit," *Zeits. f. tech. Phys.*, Vol. 11, pp. 102–107, 143–147 (1930). "Vibrationsgalvanometer mit frequenzunabhängigem Ausschlagsbereich," *Zeits. f. tech. Phys.*, Vol. 13, pp. 475–477 (1932), Vol. 14, pp. 111–118 (1933).

CHAPTER IV

THE CLASSIFICATION OF BRIDGE NETWORKS

1. Introduction. The reader approaching the subject of alternating current bridge measurements for the first time will be impressed by the very large number of networks which have been proposed by various writers for use in practice; and he may find some difficulty in choosing from the profusion of methods at his disposal the one best suited to the measurement of a given quantity. It is the object of this chapter to present a concise description and classification of bridge networks, together with a summary of the literature concerning them.

Since a given network may be of service for the measurement of more than one quantity, it is clear that confusion may arise by attempting to classify bridge methods according to the quantities which they are designed to measure. It is simpler, for the primary classification developed in this chapter, to consider the networks according to the arrangements of resistance, inductance, and capacitance which compose the balancing branches of the networks, and which enter into the balance conditions. It is proposed, therefore, to examine the construction of each bridge network and to classify it in the manner just suggested. The balance conditions can then be worked out, the vector diagram drawn, and the conditions for sensitivity deduced. The purposes to which each network can be put will be referred to, and, so far as possible, experimental results of the use of each bridge will be given.

The bridge networks having been classified in this chapter with respect to the way in which they are built up, it will be easy to show in Chapter V how to choose the network most suitable for a given measurement, and to point out the practical precautions which it is necessary to observe in order that reliable results may be secured.

In Chapter I the reader has been shown that modern alternating current bridge methods for the measurement of induction coefficients are the direct result of applying an interrupted current and a telephone to the old ballistic bridges. It will be of service in the present classification to make a few observations upon these old methods, from which so many of the present-day methods are derived.

In the use of a ballistic method a suitable network is arranged and adjusted, so that the galvanometer in the bridge conductor remains undeflected when steady currents flow in the branches, i.e. the ordinary Wheatstone bridge relation among the branch resistances is first obtained. Then, without interfering with this condition, such adjustments are made in the constants of the branches that the galvanometer remains undisturbed when the source of current is applied to or removed from the network, i.e. the second condition for balance is that no ballistic effect is exerted on the galvanometer.

Now a ballistic galvanometer can remain undeflected under either of two conditions: (i) when the *aggregate quantity* of electricity passed through the instrument during the continuance of the transient condition in the network is zero; or (ii) when the *current* in the galvanometer is zero at every instant. Obviously, if condition (ii) be satisfied, the first condition is automatically fulfilled; the converse is not necessarily true. Assuming steady current balance to be first secured, balance for the transient state can be obtained by imposing in addition one or other of these conditions (i) and (ii). Ballistic bridges are thereby divided into three definite classes* which have *aggregate balance*, *continuous balance*, or *conditional continuous balance*; according as (i), (ii), or (ii) with further conditions is satisfied.

For example, Rimington's method (*see* p. 377) for comparison of a condenser with a self-inductance essentially has aggregate balance under condition (i), and cannot be balanced if (ii) be imposed. On the other hand, Maxwell's method (p. 309) of comparing two self-inductances can be balanced by means of condition (ii) and has continuous balance. Again, such a bridge as Carey Foster's method (p. 457) for the comparison of a condenser and a mutual inductance is primarily balanced under condition (i), but, by imposing a further condition, balance can be obtained under (ii), giving conditional continuous balance.

Now if such bridge networks be supplied with alternating current, the detecting instrument is essentially some form of current-measuring or detecting device, such as a telephone or a vibration galvanometer; hence, as has been shown in Chapter II, the condition for balance is that there shall be no current in the detector at any instant. It follows, therefore, that any ballistic bridge which has continuous balance can be used without modification with alternating current, provided that the battery be replaced by an alternator and that a telephone or vibration galvanometer be substituted for the ballistic galvanometer. By this means a large and important class of alternating current bridges is obtained.

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* In this connection *see* a paper by A. O. Allen, "On measurements of inductance," *Phil. Mag.*, 6th series, Vol. 25, pp. 520–534 (1913); L. H. Harris and H. Williams, "An improved form of Maxwell's d.c. inductance bridge and a method of measuring the time-constant of the core of a magnet," *Journal P.O.E.E.*, Vol. 23, pp. 36–41 (1930). Complete discussions of the theory of ballistic bridges, using Heaviside calculus, are given by A. Cabras, "Contribution à la théorie mathématique de la mesure des inductances," *Rev. Gén. de l'Él.*, vol. 26, pp. 959–963 (1929) and by A. T. Starr, "Ballistic and perfect balances in bridges treated by the operational calculus," *Phil. Mag.*, Vol. 12, pp. 265–280 (1931).

It will be obvious that a ballistic bridge which has aggregate balance cannot be balanced by the use of an alternating current. If, however, the steady current or Wheatstone bridge condition be abandoned, it becomes possible to use such bridges with alternating current. Balance is usually obtainable only at one frequency, and the use of such networks is limited, owing to their practical inconvenience.

Ballistic bridges of the class which may be made to have conditional continuous balance can be adapted to alternating current at once by adopting the slight modification necessary to make the balance continuous.

In addition to these alternating current adaptations of old ballistic methods, there is a large class of methods specially developed as the result of alternating current practice. These methods are either modifications of the old ballistic bridges devised to simplify the practical procedure of securing balance; or they are entirely new methods developed in the course of research work for the purpose of some special measurement. One of the two balance conditions is not necessarily the ordinary Wheatstone bridge resistance relation, and a knowledge of the frequency of the alternating current may enter explicitly into the balance conditions.

As will be shown later in this chapter, the two conditions of balance deduced on page 52 for an alternating current bridge may be functions of the frequency of the current applied to the network. When these conditions are independent of frequency, either a telephone or a tuned detector may be employed, since the bridge is balanced for currents of all frequencies simultaneously. It is, moreover, not important that the wave-form of the applied potential difference be sinusoidal, since the adjustment balancing the fundamental of the wave balances simultaneously the harmonics. On the other hand, when the balance conditions involve the frequency of supply, this frequency must be independently measured, and the knowledge of it becomes, as it were, a third condition of quantitative measurement. Moreover, if in such a case the applied wave of potential difference be impure, the use of a telephone will be inadmissible, since, although the condition of balance appropriate to the fundamental of the wave be satisfied, balance for the harmonics is not simultaneously secured, and silence in the telephone will be impossible. In such cases the use of a tuned detector, the response of which is small to harmonics of the frequency to which it is tuned, is essential. This is usually a simpler expedient than that of purifying the wave-form in order that a telephone may be used.

The first classified collection of a.c. bridge networks was published by Max Wien,[*] in 1891, a number of old ballistic bridges being adapted for use with alternating current and certain new networks introduced. H. Rowland,[*] in 1898, collected together the circuit diagrams and balance conditions of some 27 bridges, and added others in later papers. In 1908 A. Campbell[†] published a collection of bridges in which mutual inductance occurs, the theory of these having been given by Heaviside many years previously. Important summaries of bridge methods have also been published by C. E. Hay,[‡] in 1912, and D. I. Cone, in 1920.[§] The latter paper gives very useful and concise diagrams for a number of bridges used in modern practice.[‖]

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[*] Max Wien, "Messung der Inductionsconstanten mit dem 'optische Telephon,'" *Ann. der Phys.*, Vol. 44, pp. 689–702 (1891). For a collection of the old ballistic methods, *see* W. E. Sumpner, *Journal S.T.E.*, Vol. 16, pp. 344–379 (1888).

In the succeeding sections of this chapter, a large number of bridge networks are passed in review and classified according to the way in which they are constructed. It is found that the networks fall under the following headings—

(i) Containing Resistance and Self-inductance.
(ii) Containing Resistance and Capacitance.
(iii) Containing Resistance, Self-inductance, and Capacitance.
(iv) Containing Resistance, Self-inductance, and Mutual Inductance.
(v) Containing Resistance, Self-inductance, Mutual Inductance, and Capacitance.

It will be found that the bridge networks examined in the following classification fall into three main groups. The first group contains

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[*] H. Rowland, "Electrical measurements by alternating currents," *Amer. J. Sc.*, 4th series, Vol. 4, pp. 429–448 (1897); also *Phil. Mag.*, 5th series, Vol. 45, pp. 66–85 (1898).

H. Rowland and T. D. Penniman, "Electrical Measurements," *Amer. J. Sc.*, 4th series, Vol. 8, pp. 35–57 (1899).

[†] A. Campbell, "On the use of variable mutual inductances," *Phil. Mag.*, 6th series, Vol. 15, pp. 155–171 (1908); *Proc. Phys. Soc.*, Vol. 21, pp. 69–87 (1910). Also, "Inductance measurements," *Elecn.*, Vol. 50, pp. 626–627 (1908).

[‡] C. E. Hay, "Alternate current measurements, with special reference to cables, loading coils, and the construction of non-reactive resistances," *Journal P.O.E.E.*, Vol. 5, pp. 451–454 (1913); also *Professional Papers*, No. 53.

[§] D. I. Cone, "Bridge methods for alternating current measurements," *Journal Amer. I.E.E.*, Vol. 39, pp. 640–647 (1920). See also L. Cahen and J. Carvallo, "Les mesures des grandeurs électriques sous courant alternatif de fréquence musicale," *J. de Phys.*, Vol. 5, pp. 113–125 (1924); and W. F. Shackleton and J. G. Ferguson, "High frequency measurement of communication apparatus," *Trans. Amer. I.E.E.*, Vol. 46, pp. 519–527 (1927).

[‖] For a more recent classification see the following papers by J. Krönert: "Messbrücken Berechnung von Wechselstrombrücken," *Arch. f. tech. Mess.*, J. 920–1 (Dec., 1931); "Wechselstrombrücken vom Anderson-Typ," *ibid.*, J. 921–1 (Aug. 1931); "Wechselstrombrücken vom Wheatstone-Typ," *ibid.*, J. 921–2 (June, 1932); "Wechselstrombrücken mit einer gegenseitigen Induktion," *ibid.*, J. 921–4 (Sept., 1932).

Wheatstone impedance networks without mutual inductance, including in addition the Anderson network and the Kelvin double bridge network, both of which may be readily transformed into Wheatstone form, as has been shown in Chapter II. Included in the second group are Wheatstone networks in which there is mutual inductance between appropriate pairs of branches (usually between one pair only), including many networks which can be put into Wheatstone form with one or more branches of zero or of infinite impedance. Finally, the third group comprises networks which are not of Wheatstone form and in which there is usually mutual inductance. The particular properties of networks in each group will be gathered from the discussion given in this chapter, but there are several general matters which are worth preliminary examination at this stage.

✓ **1a. The Classification of Wheatstone Networks.** Of the large number of circuits available for a.c. bridge measurements a considerable proportion are Wheatstone or four-branch networks of simple impedance elements without mutual inductance, specific examples being considered in later Sections of this chapter. It is the purpose of this Section to analyse these four-branch networks into their various classes, following the principles established by Ferguson,* in order to find out the most useful practical forms.

In Fig. 20, p. 51, let z_1 be the impedance to be measured; then when the bridge is balanced

$$z_1 = z_2 z_4 / z_3 ;$$

or

$$R_1 + jX_1 = (R_2 + jX_2)(R_4 + jX_4)/(R_3 + jX_3),$$

in terms of the resistances and reactances of the branches. Balance may be secured by adjusting any or all of the six parameters on the right-hand side of this equation but, since there are only two quantities R_1 and X_1 to measure, it is necessary and sufficient to adjust only two of the six terms. There are fifteen possibilities, but many of these are inconvenient. If, for example, the two varied quantities are in different branches, then the cost of the bridge set-up will usually be greater than when the adjustments are both made in a single branch, since variable impedance standards are more costly than fixed standards. Eliminating two-branch adjustments, the number of possibilities reduces to three, according as either (R_2, X_2) or (R_4, X_4) or (R_3, X_3) is used. Again, for rapid balancing, it is desirable that the resistance adjustment should be independent of the change made in the corresponding reactance; this means that the above equation when reduced to its simplest form can be written

$$R_1 + jX_1 = A + jB$$

where A and B are real, one of the balancing adjustments appearing only in A and the other only in B. Two cases arise—

In the first, balance is made either by z_2 or z_4; as these both appear in the numerator these are equivalent adjustments and we may suppose z_4 to be variable, with z_2 and z_3 fixed. The balance equation is

$$z_1 = R_1 + jX_1 = \frac{(R_2 + jX_2)}{(R_3 + jX_3)}(R_4 + jX_4) = \frac{z_2}{z_3} z_4 \equiv A + jB.$$

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* J. G. Ferguson, "Classification of bridge methods of measuring impedances," *Trans. Amer. I.E.E.*, Vol. 52, pp. 861–868 (1933).

For A to involve only one of the adjustments (R_4 or X_4) while B contains only the other, the ratio z_2/z_3 must be either real or imaginary but not complex. The adjustments will then be independent and such bridges are known as *ratio bridges.*

In the second case, balance is secured by changing z_3 with z_2 and z_4 fixed. The equation is now

$$z_1 = R_1 + jX_1 = (R_2 + jX_2)(R_4 + jX_4)(G_3 + jB_3) = z_2 z_4 y_3 \equiv A + jB,$$

where $y_3 = 1/z_3$, $G_3 = R_3/(R_3^2 + X_3^2)$ and $B_3 = -X_3/(R_3^2 + X_3^2)$. For independent adjustment the product $z_2 z_4$ must be either real or imaginary, but not complex. Such bridges are known as *product bridges*. It is seen, therefore, that there are only two practical methods of balancing a bridge, namely, either by adjusting a branch adjacent to the unknown (ratio bridges) or opposite to it (product bridges).

RATIO BRIDGES. In these bridges z_2/z_3 must be either real or imaginary. Taking the former, *when z_2/z_3 is real* we may write

$$z_2/z_3 = (R_2 + jX_2)/(R_3 + jX_3) = K,$$

where K is a real quantity. Separating components this gives

$$R_2/R_3 = X_2/X_3 = K$$

and since R_2 and R_3 are essentially positive K must be positive also; hence X_2 and X_3 must be reactances of the same kind, e.g. two inductances or two condensers. If Z_2, ϕ_2 and Z_3, ϕ_3 are the magnitudes and phase-angles of the operators the above relation is equivalent to (see p. 52)—

$$(Z_2/\underline{\phi_2})/(Z_3/\underline{\phi_3}) = (Z_2/Z_3)/\underline{\phi_2 - \phi_3} = K \equiv (Z_1/Z_4)/\underline{\phi_1 - \phi_4},$$

so that $\phi_2 - \phi_3 = \phi_1 - \phi_4$ must be zero to maintain K positive. In other words, $X_2/R_2 = X_3/R_3$ or $\tan\phi_2 = \tan\phi_3$, i.e. $\phi_2 - \phi_3 = 0$; thus z_2 and z_3 may be any pair of equal-angled impedances standing in the desired ratio K. Substituting in the balance condition,

$$R_1 + jX_1 = K(R_4 + jX_4),$$

i.e. $$R_1 = KR_4 = R_4R_2/R_3 \text{ and } X_1 = KX_4 = X_4X_2/X_3,$$

which is equivalent to Equation (8*e*) on p. 53. Thus X_4 must be a reactance of the same kind as X_1. In real-ratio bridges, therefore, resistance in the unknown is balanced by resistance in the adjustable branch and reactance by reactance.

When z_2/z_3 is imaginary then writing

$$z_2/z_3 = (R_2 + jX_2)/(R_3 + jX_3) = jK$$

and separating components gives

$$-R_2/X_3 = X_2/R_3 = K,$$

so that X_2 and X_3 must be reactances of opposite kinds, e.g. one a coil and the other a condenser. This may be written alternatively as

$$X_2/R_2 = -R_3/X_3 \text{ or } \tan\phi_2 = -\cot\phi_3 = \tan[\phi_3 \pm (\pi/2)],$$

so that $$\phi_2 - \phi_3 = \phi_1 - \phi_4 = \pm\pi/2,$$

and $$(Z_2/Z_3)/\underline{\phi_2 - \phi_3} = (Z_1/Z_4)/\phi_1 - \phi_4 = jK,$$

where K may be positive or negative. The balance condition becomes

$$R_1 + jX_1 = jK(R_4 + jX_4)$$

that is $$R_1 = -KX_4 = X_4R_2/X_3 \text{ and } X_1 = KR_4 = R_4X_2/R_3$$

Thus X_4 must be a reactance of the same kind as X_3 and X_1 of the same kind as X_2; but X_2 and X_3 are of opposite kinds, so that X_1 and X_4 must also be opposite. In imaginary-ratio bridges, therefore, resistance in the unknown is balanced by reactance in the adjustable branch and reactance by resistance, the two reactances being of opposite kinds. In many of the most important practical bridges z_2 and z_3 each consist of a single element, and since they have $\pm\pi/2$ phase difference it follows that either R_2 and X_3 or X_2 and R_3 must be simultaneously zero. Hence one of these two branches is a pure resistance and the other either a loss-free condenser or a resistanceless coil. Of the remaining branches z_1 and z_4 must have opposite reactances and differ by $\pm\pi/2$ in phase-angle, as proved; if R_2 and X_3 vanish X_1 resembles X_2, while if X_2 and R_3 vanish X_4 resembles X_3.

PRODUCT BRIDGES. In this class z_2z_4 must be either real or imaginary. In the case *when z_2z_4 is real*, writing

$$z_2z_4 = (R_2 + jX_2)(R_4 + jX_4) = K,$$

where K is a real quantity, and separating components gives

$$R_2R_4 - X_2X_4 = K, \text{ and } R_2X_4 + R_4X_2 = 0.$$

The second relation states that

$$X_2/R_2 = -X_4/R_4 \text{ or } \tan\phi_2 = -\tan\phi_4, \text{ i.e. } \phi_2 + \phi_4 = 0;$$

hence, since R_2 and R_4 are positive, X_2 and X_4 must be reactances of opposite kinds. If now $G_4 = R_4/(R_4^2 + X_4^2)$ and $B_4 = -X_4/(R_4^2 + X_4^2)$ we can write

$$z_2z_4 = z_2/y_4 = (R_2 + jX_2)/(G_4 + jB_4) = K$$

which separates into

$$R_2/G_4 = X_2/B_4 = K$$

Substituting in the balance condition,

$$R_1 + jX_1 = K(G_3 + jB_3)$$

i.e. $$R_1 = KG_3 = G_3R_2/G_4 = G_3X_2/B_4,$$

and $$X_1 = KB_3 = B_3R_2/G_4 = B_3X_2/B_4$$

From the second condition it follows that X_1 has the same sign as B_3, i.e. X_1 is a reactance of opposite kind to X_3. In real-product bridges resistance in the unknown is balanced by conductance in the adjustable branch, and the unknown reactance is balanced by susceptance of the same sign, i.e. by reactance of opposite sign. Of particular practical interest is the bridge with z_2 and z_4 pure resistances, i.e. X_2 and X_4 zero giving $R_2 = K/R_4$, $G_4 = 1/R_4$ and $B_4 = 0$. Then $R_1 = R_2R_4G_3$ and $X_1 = R_2R_4B_3$.

In the cases *where z_2z_4 is imaginary* write

$$z_2z_4 = (R_2 + jX_2)(R_4 + jX_4) = (R_2 + jX_2)/(G_4 + jB_4) = jK$$

and separating the components gives

$$-R_2/B_4 = X_2/G_4 = K;$$

that is $X_2/R_2 = -G_4/B_4 = R_4/X_4$,

or $\tan\phi_2 = \cot\phi_4 = -\tan[\phi_4 \pm (\pi/2)]$

i.e. $\phi_2 + \phi_4 = \phi_1 + \phi_3 = \pm\pi/2$

The reactances X_2 and X_4 must now be of the same kind. Using the balance condition,

$$R_1 + jX_1 = jK(G_3 + jB_3)$$

i.e. $R_1 = -KB_3 = B_3R_2/B_4 = -B_3X_2/G_4$,

and $X_1 = KG_3 = -G_3R_2/B_4 = G_3X_2/G_4$.

From the first condition B_3 and B_4, i.e. X_3 and X_4 must be of the same kind; while the second condition shows that X_1 and X_2 must be of the same kind. Hence all the reactances are of the same kind. In imaginary-product bridges the unknown resistance is balanced by susceptance and the unknown reactance by conductance in the adjustable branch.

General Considerations. It is desirable that the fixed branches should be as simple as possible and satisfy accurately the required phase-angle condition. For these reasons single-element branches are preferred in practice, and consist either of resistors or condensers. Resistors are easily constructed with practically zero phase-angle, and air condensers constitute accurate quadrature standards. Condensers with solid dielectrics, such as mica, have low losses and a very small defect from quadrature. In any case, residual imperfections, whether in resistors or condensers, are easily allowed for by small correction terms. This is not so with inductors, where the appreciable resistance considerably affects the phase-angle and requires quite large corrections; this limitation will be referred to later.

In Figs. 99 and 100 the eleven possible forms of bridges with fixed branches containing a single pure element, resistance, or reactance as required, are shown. In Fig. 99 (*c*) the series resistances of the inductors will be allowed for by making them in the same relation as the inductances. In Figs. 99 (*f*) and (*g*) any resistance in the inductor must be compensated by capacitance in series with the resistor and, as this series capacitance is usually large the correction is unsatisfactory. These bridges are, therefore, unpractical. In Fig. 100 (*b*) the series resistance of an actual inductor must be balanced by resistance shunting the condenser and, as Fig. 100 (*a*) produces the desired angle relation by simpler means, Fig. 100 (*b*) must be regarded as unpractical also. Finally, in Fig. 100 (*c*) the compensation can only be secured by pure inductance shunting the fixed resistance, and this is impossible. There are, therefore, only seven forms of bridges with single-element fixed branches, namely, Figs. 99 (*a*), (*b*), (*c*), (*d*), and (*e*), and Figs. 100 (*a*) and (*d*).

Of these seven standard types, Figs. 99 (*a*), (*b*), and (*c*) are capable of measuring impedances of any kind, inductive or capacitive, by the use

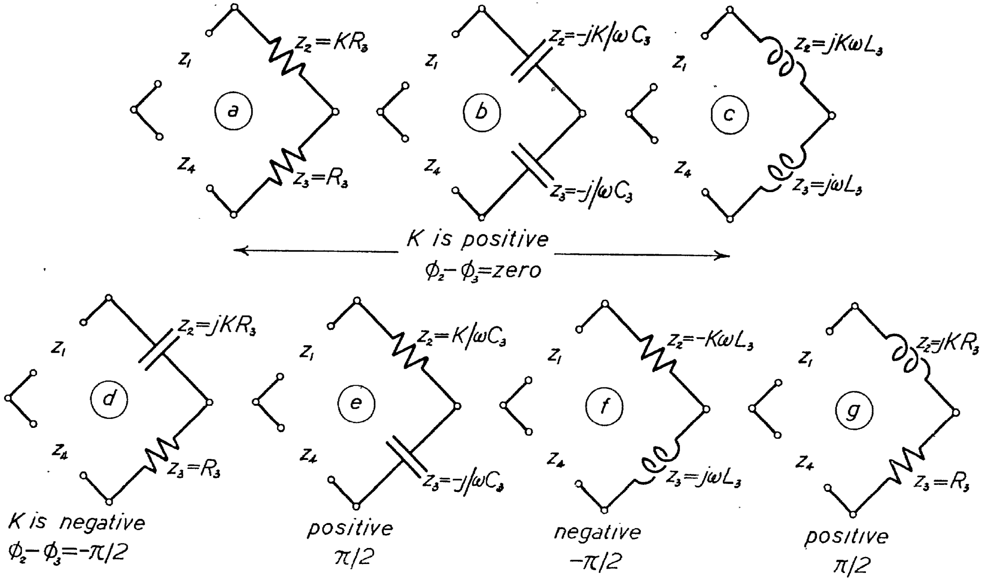

FIG. 99.—RATIO BRIDGES

of an appropriate standard of similar kind. Fig. 100 (*a*) measures any kind of impedance in terms of a standard of opposite kind. Figs. 99 (*d*) and (*e*) are complementary and between them cover a complete range of impedances; (*d*) deals with capacitive impedances in terms of an inductance standard, while (*e*) measures inductive impedances in terms of a capacitance standard. The bridge, Fig. 100 (*d*), measures capacitive impedance in terms of a similar standard, the most important representative of this class being the Schering bridge of p. 352. Sometimes if a given kind of standard is adopted, e.g. a condenser-resistance combination, "universal" bridges may be devised; for example Fig. 99 (*a*) may be used for measuring capacitive impedances and Fig. 100 (*a*) for inductive impedances. Numerous examples have been described in the literature and in maker's catalogues, and will be referred to in Chapter V. The following table gives typical examples of bridges in each of the seven classes.*

Ratio Bridges

Fig. 99 (*a*) .	Maxwell's (Fig. 110) and Wien's (Fig. 113) for inductance comparison. Series resistance (Fig. 114), Parallel resistance (Fig. 115), and Wien's (Fig. 115) for comparison of capacitances. Series tuned arm (Fig. 137) and Parallel tuned arm (Fig. 138).
Fig. 99 (*b*) .	Fleming and Dykes' (Fig. 129) for capacitance comparison.
Fig. 99 (*c*) .	Grover's [Fig. 140 (*a*)] for capacitance comparison.
Fig. 99 (*d*) .	Owen's [Fig. 140 (*b*)] for capacitance measurement.
Fig. 99 (*e*) .	Owen's [Fig. 140 (*b*)] for inductance measurement.

Product Bridges

Fig. 100 (*a*) .	Maxwell's (Fig. 130) and Hay's (Fig. 133) for comparison of inductance with capacitance.
Fig. 100 (*d*) .	Schering's (Fig. 133) for capacitance comparison.

Of all types the simple comparison bridge with resistance ratios is most common, Fig. 99 (*a*), especially the equal-ratio bridge with $K = 1$, though Fig. 99 (*b*) is often used when high-impedance arms are required, as in testing small condensers at low frequencies. Capacitance ratio branches are also useful when the measurement of an inductive impedance with superposed direct current is made, since the direct current is excluded from the ratio branches. Inductive ratio arms are advantageous when heavy currents are to be dealt with, since they are easily designed to have low losses. When modified by the inclusion of mutual inductance between them, they have other practical advantages that will be discussed in the following Section. ✗

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* A note may be added about notation for branch elements. The use of subscripts to denote the various branches is best for general theory discussed in this and the following subsection. Typographically it is, however, rather inconvenient for general use and is avoided in describing actual bridges, where there are seldom more than two reactances, by using P, Q, S, R for the resistances in branches 1, 2, 3 and 4. Reactances of the same kind are distinguished by subscripts, irrespective of the branches they are in.

1b. Use of Inductive Ratio Arms. It is usual in practice to employ bridges in which the ratio arms are fixed non-reactive resistances in a definite numerical ratio, most usually unity. In high frequency measurements, where the ratio arms often possess high ohmic values, difficulties are encountered on account of earth capacitance effects, necessitating more or less complex systems of shielding and exact balance of the ratios. Walsh* has recently shown in a comprehensive paper that many of the difficulties arising with non-reactive ratios may be readily overcome by using inductive impedances, with mutual inductance between them, in their place; the principle involved is a radical departure from standard practice and is likely to find considerable application in various classes of bridges.

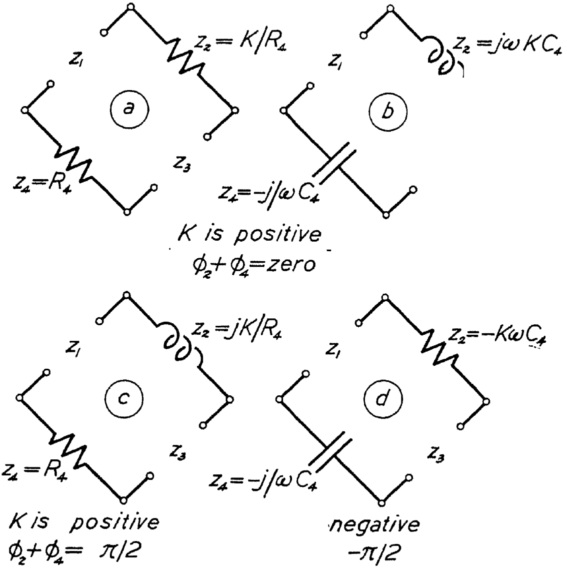

FIG. 100.—PRODUCT BRIDGES

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* R. Walsh, "Inductive ratio arms in alternating current bridge circuits," *Phil. Mag.*, 7th series, Vol. 10, pp. 49–70 (1930). The original idea is due to A. D. Blumlein. There is an error in Walsh's paper that has been corrected by A. T. Starr, "A note on impedance measurement," *W. Eng. and Exp. W.*, Vol. 9, pp. 615–617 (1932).

Referring to Fig. 101, A, B, C, and D are the usual four branch-points of a bridge in which z_1 and z_4 are operators of unknown and standard impedances. The impedances z_2 and z_3 are inductive coils with mutual inductance operator $m = j\omega M$; these coils have a small resistance in comparison with their reactance and a close magnetic coupling. They commonly consist of toroidal windings upon a silicon steel, or still better a perm-alloy core. Using the transformation on page 70, or the equivalent expression from page 75 with $m = m_{23}$, the network is balanced if

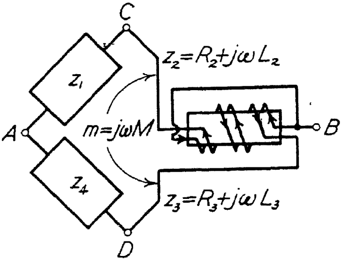

FIG. 101.—THE USE OF INDUCTIVE RATIO ARMS

$$\frac{z_1}{z_4} = \frac{z_2 + m}{z_3 + m} = \frac{R_2 + j\omega\,(L_2 + M)}{R_3 + j\omega\,(L_3 + M)}.$$

For the arrangement to be practically useful the right-hand expression should reduce to a real integer $n : 1$, the most important case being the equal ratio bridge with $n = 1$.

Suppose the two windings to be very closely coupled, then neglecting leakage we may take $M^2 = L_2L_3$; if we now arrange $L_2 = n^2L_3$, then $M = nL_3$, so that $R_2 + j\omega\,(n^2L_3 + nL_3) = R_2 + j\omega n\,(1 + n)L_3$; and $R_3 + j\omega\,(L_3 + nL_3) = R_3 + j\omega\,(1 + n)L_3$. If we arrange further to make $R_2 = nR_3$, then the ratio of the two operators $(z_2 + m)/(z_3 + m)$ is exactly $n : 1$. Thus to secure a bridge of ratio $n : 1$ it is necessary to provide two closely-coupled coils with self-inductances in the ratio $n^2 : 1$ and resistances in the ratio $n : 1$. The conditions are obviously fulfilled for a ratio of unity by using a toroidal bifilar winding on an iron core; Walsh shows that they are also exactly fulfilled for $n \neq 1$, where there is obviously magnetic leakage, if the arrangement consists of $n + 1$ windings between any pair of which the mutual inductance is KL_3 where K is the leakage factor and L_3 the inductance of any one section. A multifilar arrangement of this kind is described giving a ratio of 2 : 1.

The important case of unity ratio deserves further comment. Very closely balanced ratio-coils are easily constructed by winding twisted-pair conductors upon an iron core, and connecting them in the network so that they are series-assisting in the sense CBD, as shown in Fig. 101. Currents entering the coils at B pass round them in opposite directions, so that the mutual inductance between them is negative, i.e. opposing their self-inductances. When the bridge is balanced, equal currents enter the coils at B. As the coupling is close $M = -L_2 = -L_3$ very nearly, and the total reactive drops down BC and BD are practically zero. Since the resistances $R_2 = R_3$ are very small, when the bridge is balanced B, C, and D are at nearly the same potential; this greatly simplifies any shielding or earthing devices intended to remove earth capacitance effects from C and D, indeed B may be directly earthed.

When the bridge is slightly unbalanced, the mutual and self inductance drops are additive round the mesh *CBD* and thus considerable reactance is presented to out-of-balance current in that mesh; in this respect, therefore, inductive ratio arms behave like high-impedance ratios. When the bridge is balanced, the current at *B* is determined by z_1 and z_4 in parallel, since the drop down the ratios is nearly zero, i.e. by $z_1/2$; if resistance ratios were used, the impedance across *AB* would be $(R_2 + z_1)/2$. The inductive ratio bridge, therefore, takes more current and is, for the same voltage, more sensitive.

Walsh studies in detail the effects of earth and inter-capacitances in bridges with coupled inductive ratios, and shows that the electric shielding of the circuit is usually simpler than when resistance ratios are used; the ratio arms are compact and easily fit into any shielding scheme. Their capacitance effects are so small that impedances for use at frequencies of 5,000 to 20,000 cycles per second are made without serious difficulties. Bridges with inductive ratios are readily used with superposed direct current, the low resistance of the ratio arms reducing the d.c. power expended to about $\frac{1}{100}$ of that necessary when 1,000 ohm resistance ratios are employed; this application appears to have been suggested by Shackelton and Ferguson, *loc. cit.* on page 320. A unity ratio bridge on Walsh's principle to measure inductances from 10 μH. to 100 H. is made by Messrs. Muirhead, provision being made for superposed d.c. if desired.

1c. The Convergence of Bridge Balances. The process of balancing any type of bridge consists in making successive adjustments of two parameters until the detecting instrument has no current in it. In Section 1*a* it has been shown that for practical convenience the adjustments should be made in a single branch of the network and that they should be quite independent; on this basis a classification of Wheatstone networks was made. In practice it is also important that balance shall be attained rapidly, i.e. the successive adjustments must quickly cause the detector current to converge towards zero. It is the purpose of this Section to work out the conditions which favour this result.

On p. 77 it is shown that the current in the detector of a Wheatstone network—a general type covering most actual or transformable bridges used in practice—is given by Equation (13*c*),

$$\dot{\mathbf{i}} = \frac{z_1 z_3 - z_2 z_4}{\Delta} e_1 \mathbf{1}.$$

It is further shown that the locus of the current vector is a circle when a single resistance or reactance in any of the branch operators is varied. Balance occurs when any such locus passes through the origin, thus making the vector zero; this occurs for $z_1 z_3 = z_2 z_4$. On p. 80 an actual case is worked out for an unbalanced bridge, the circular loci being drawn to show how balance is gradually converged upon. While this process is exceedingly graphic it is, in general, rather laborious. Since interest is mainly confined to the conditions near balance, a much simpler process, due to Küpfmüller,* can be used. When near balance the denominator Δ changes very little with the varied parameter but the numerator alters considerably; hence, we can take the vector **i** when the bridge is nearly balanced to be proportional to the numerator, i.e. to

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* K. Küpfmüller, "Ueber die Konvergenz der Brückenmessverfahren," *E.u.M.*, Vol. 51, pp. 204–208 (1933).

$$z_1z_3 - z_2z_4 = A + jB = (R_1 + jX_1)(R_3 + jX_3) - (R_2 + jX_2)(R_4 + jX_4)$$

Since this is linear in any one of the quantities R_2, X_2, R_3, X_3, R_4, X_4 that may be varied, R_1 and X_1 belonging to the unknown impedance, it may be put into the form $(\alpha_0 + \alpha_1\xi)$ where α_0 and α_1 are constant complex operators and ξ is a variable. On p. 42 it is proved that such an expression leads to a linear locus, the line passing through the end of $\alpha_0\mathbf{1}$ and being inclined to the horizontal or A-axis by the phase-angle of α_1. A radius vector $\mathbf{D} = (A + jB)\mathbf{1}$ from the origin to this line is proportional to the detector current near balance, and this current is a minimum when the vector is perpendicular to the locus; see Fig. 102 (*a*). In practice two defects may prevent this minimum being actually reached—

(i) The variation of the parameter may be made in steps, as with a decade resistor, in such a way that the minimum lies between two steps.

(ii) Even though the parameter be provided with a fine adjustment which enables the minimum to be passed through, the detector will be insufficiently sensitive to be affected by changes of current smaller than a certain amount, depending on the type of detector. These defects result in any position of the vector between *a* and *b* being indicated as the minimum point, the difference between *Oa* or *Ob* and the minimum being the least detectable current. The range of uncertainty from the minimum corresponds in practice with a current-change of 5 to 10 per cent in a telephone and 0·1 to 10 per cent in deflectional instruments.* If this change is a fraction α of the current to be measured, the angle ε is 18° to 20° with a telephone and 3° to 25° in other detectors. Since $Ob = (\text{Minimum})/\cos\varepsilon$ and $\alpha = (Ob - \text{minimum})/(\text{minimum})$ it folows that $\cos\varepsilon = 1/(1 + \alpha)$.

By operating one adjustable parameter, the vector **D** may be moved along Locus 1 in Fig. 102 (*b*) to any point in the range *ab* which includes the minimum. If now a second adjustable parameter is varied, the first being fixed, **D** moves along a new line, Locus 2, to a similar uncertainty range *cd* about a new and smaller minimum. By returning to the first adjustment, a third, still smaller, minimum may be reached, and so on. The origin, denoting balance, is thus approached in a step-wise convergence as shown by Fig. 102 (*c*). The rapidity of the convergence depends upon the angle γ between the two loci, being slow if the angle is small, most rapid when they cut at right angles, and zero if the two loci are parallel. Hence, if a rapid convergence is to be obtained the angle between the loci for the chosen pair of parameters should be as nearly 90° as possible. Figs. 103 (*a*) and (*b*) show respectively a slow and an ideally rapid convergence, ε being 5° in both cases.

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* It will be understood that only detectors which measure current independently of its phase are considered here. The special case of phase-selective devices like mechanical rectifiers, dynamometers, etc., are examined in a paper by H. Poleck, "Mechanisiertes Abgleichverfahren für Wechselstrom-Messbrücken bei Verwendung phasenabhängiger Nullindikatoren," *Arch. f. Elekt.*, Vol. 28, pp. 492–506 (1934).

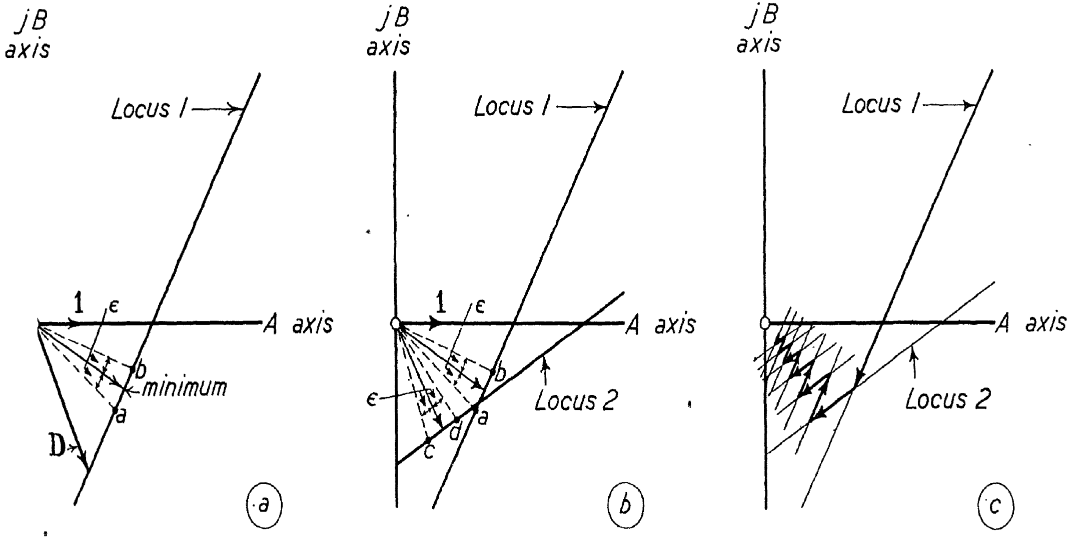

FIG. 102.—THEORY OF CONVERGENCE OF BRIDGE BALANCE

In general, any pair of the six parameters in z_2, z_3, and z_4 may be chosen, giving a total of fifteen possibilities. It is possible, however, that one or more branches may be non-reactive or non-resistive; if the number of available parameters be n ($n \lesseqgtr 6$) the number of methods for attempting a balance will be $n(n-1)/2$. Each method will have different balance convergence; some may not balance at all.

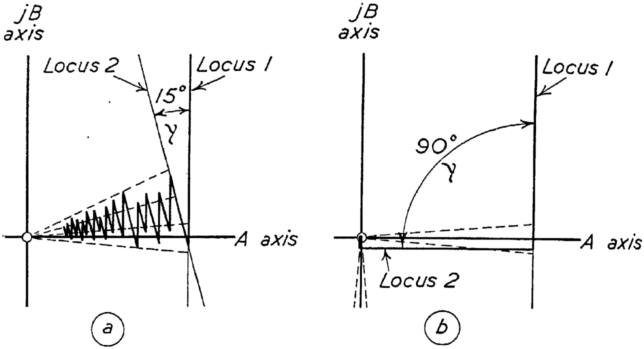

FIG. 103.—SLOW AND RAPID CONVERGENCE

As a simple example consider the resistance-ratio bridge of Fig. 99 (*a*) in which there are six possible ways of adjusting for balance. Since in this case

$$A + jB = R_3(R_1 + jX_1) - R_2(R_4 + jX_4)$$

it is easy to see that the four loci are as follows—

R_4 varied; a line parallel to the A axis.
X_4 varied; a line parallel to the jB axis.
R_2 varied; a line inclined at $\phi_4 = \arctan (X_4/R_4)$ to the A axis.
R_3 varied; a line inclined at $\phi_1 = \arctan (X_1/R_1)$ to the A axis.

Since at balance $\phi_1 - \phi_4 = 0$, near balance ϕ_1 and ϕ_4 must be nearly equal and the last pair of loci nearly parallel; hence the pair (R_2, R_3) cannot result in a balance. Most rapid balance is obtained by the pair (R_4. X_4) since their loci cut normally. When ϕ_1 and ϕ_4 are near 90°, i.e. when X_1/R_1 and X_4/R_4 are $\gg 1$ (very reactive arms), the pairs (R_2, R_4) and (R_3, R_4) converge rapidly, and the pairs (R_2, X_4) and (R_3, X_4) very slowly. The reverse is the case when X_1/R_1 and $X_4/R_4 \ll 1$, the angles ϕ_1, ϕ_4 approaching zero (slightly reactive arms). For intermediate angles these pairs have only fair convergence. The loci are illustrated in Fig. 104 (*a*); a numerical example follows on p. 305.

A further simple case is the resistance-product bridge of Fig. 100 (*a*) with six possibilities, the conditional equation being

$$A + jB = (R_1 + jX_1)(R_3 + jX_3) - R_2R_4$$

from which it is seen that the loci are as follows—

R_2 or R_4 varied; lines parallel to the A axis.
R_3 varied; a line inclined at $\phi_1 = \arctan(X_1/R_1)$ to the A axis.
X_3 varied; a line inclined at $\phi_1 + (\pi/2)$ to the A axis.

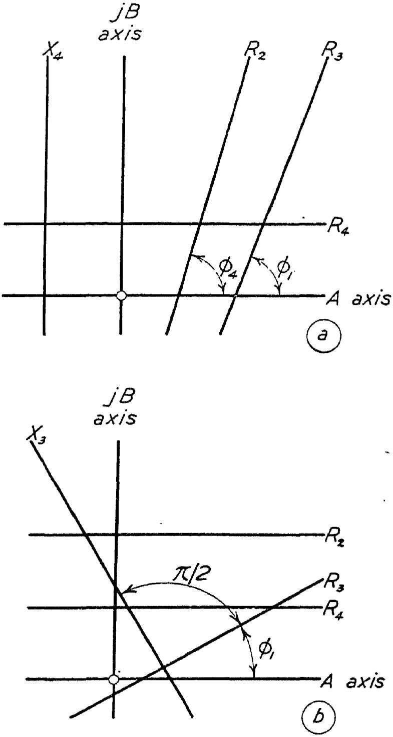

Fig. 104.—Convergence Loci for Resistance Ratio and Product Bridges

The pair (R_2, R_4) never achieves balance. Ideally rapid balance is given by the pair (R_3, X_3), the loci for which cut normally. The pairs (R_2, R_3) and (R_3, R_4) give good convergence if $X_1/R_1 \gg 1$ and poor convergence if $X_1/R_1 \ll 1$; the pairs (R_2, X_3) and (R_4, X_3) converge badly under the former condition and well under the latter. The loci are shown in Fig. 104 (b).

The numerator of the detector current operator for the Anderson bridge of Fig. 22 is given in Equation (9) on p. 56, namely,

$$z_7(z_1z_3 - z_2z_4) - z_2\{z_6(z_3 + z_4) + z_3z_4\}$$

and this also can give linear loci near balance. In the commonest practical form of the bridge $z_1 = P + j\omega L$, $z_2 = Q$, $z_3 = S$, $z_4 = R$, $z_6 = r$ and $z_7 = 1/j\omega C$ as shown on p. 380. Insertion of these in the above expression will include terms involving $1/C$ which for variation of C will not give linear loci. Since $1/C$ occurs also in terms of the current denominator suppose both numerator and denominator cleared of $1/\omega C$ by multiplication with ωC. Then we can write for the usual bridge.

$$(SP - QR) + j\omega\{SL - CQ[r(S + R) + SR]\} = A + jB.$$

Inspection shows at once that the locus with P varied is parallel to the A axis, and that the loci with C or r varied are parallel to the jB axis. Thus the pairs (P, C) and (P, r) give ideal convergence, and both involve inclusion of a variable resistance in series with the coil under test, the second is preferred in practice (see p. 382). The pair (C, r) does not converge. All other loci, varying Q, S, or R cut the A axis at easily calculable angles, and in special cases may give useful convergences.

Bridges with mutual inductance can be dealt with by using the expression on p. 71, or any of its special forms on pp. 75–76. The most interesting case is the Carey Foster bridge of p. 457, for which the numerator in the detector current operator is

$${}_3 - z_2z_4 - \qquad +$$

where, $z_1 = 0$, $z_2 = Q$, $z_3 = S - (j/\omega C)$, $z_4 = R + j\omega L$ and $m_{46} = j\omega M$. Inserting these values and clearing of inverse terms to leave linear variations only,

$$(M + CQR) + j\omega C[M(Q + S) + QL] = A + jB.$$

The loci with either S or L varied are parallel to the jB axis, so the pair (S, L) never balances. The locus with R varied is parallel to the A axis, so that the pairs (R, S) and (R, L) give ideally rapid convergence. The locus with M varied cuts the A axis at an angle arctan $\omega C(Q + S)$ which is generally small; hence convergence with the pairs (M, L) and (M, S) is good, the second being preferred in the usual procedure of condenser measurement. The loci with C or Q varied cut the axis at calculable angles but are not generally so useful as balancing elements.

It is important to have a numerical measure of convergence. In Fig. 105 the possible deviation from the minimum OM_1 on Locus 1 is OE, $(OE - OM_1)/OM_1$ being the least detectable fractional change in the detector current, α, defined on p. 300. From this extreme point suppose adjustment is made along Locus 2, OF being the possible detectable deviation from the new minimum OM_2. The detector current has been reduced in the ratio OF/OE; hence the larger OE/OF the more rapid the convergence. To limit the range of values expressing the rapidity of convergence it is convenient to write for the convergence coefficient,

$$k = \log(OE/OF) = \log[\cos\varepsilon/\cos(\gamma - \varepsilon)]$$

the logarithm being to base 10. The number of adjustments, m, required to reduce the current to $1/n$ of its value at the beginning of the balancing process is then $m = (\log n)/k$. From the diagram it will be seen that the expression for k is valid only if $\gamma > 2\varepsilon$; if $\gamma < 2\varepsilon$ balance is not possible and $k = 0$. Thus it is essential that ε be less than $\gamma/2$ for convergence to occur at all. Küpfmüller gives the following range of values for k to define convergence of various kinds—

Value of k	> 0·6	0·3 to 0·6	0·15 to 0·3	< 0·15
Convergence	Very good	good	slow	bad

As a numerical example take the case of a simple resistance-ratio bridge using a telephone with $\varepsilon = 25°$ and a vibration galvanometer with $\varepsilon = 8°$. The following table gives the values of k for various ratios of X_1/R_1 in each of the possible adjustments. In all cases the convergence is better with the galvanometer than with the telephone.

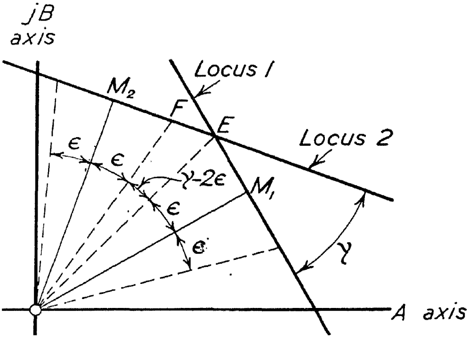

FIG. 105.—THE CONVERGENCE COEFFICIENT

Pair	$X_1/R_1 = 10$			$X_1/R_1 = 1$			$X_1/R_1 = 0{\cdot}1$		
	γ	Value of k		γ	Value of k		γ	Value of k	
		Tel.	Galv.		Tel.	Galv.		Tel.	Galv.
R_4, X_4	90°	0·33	0·85	90°	0·33	0·85	90°	0·33	0·85
R_3, R_4	84° 20′	0·25	0·62	45°	0	0·1	5° 40′	0	0
R_2, R_4	84° 20′	0·25	0·62	45°	0	0·1	5° 40′	0	0
R_3, X_4	5° 40′	0	0	45°	0	0·1	84° 20′	0·25	0·62
R_2, X_4	5° 40′	0	0	45°	0	0·1	84° 20′	0·25	0·62

1d. Properties of Shunted Coils and Condensers. The branches of bridge networks are, in general, comprised of suitable combinations of resistances, inductances, and capacitances. The properties of the series combination are simple and self-evident; thus for R, L, and C in series the impedance operator will be $R + j\left(\omega L - \frac{1}{\omega C}\right)$, which can be written as $R + j\omega\left(L - \frac{1}{\omega^2 C}\right)$. Hence a condenser may be regarded as possessing a negative inductance $-1/\omega^2 C$ and a branch will have inductive, capacitive, or zero reactance according as ωL is greater than, less than, or equal to $1/\omega C$. The properties of parallel combinations are not nearly so simple, and are of considerable practical importance; some of these will now be briefly examined.

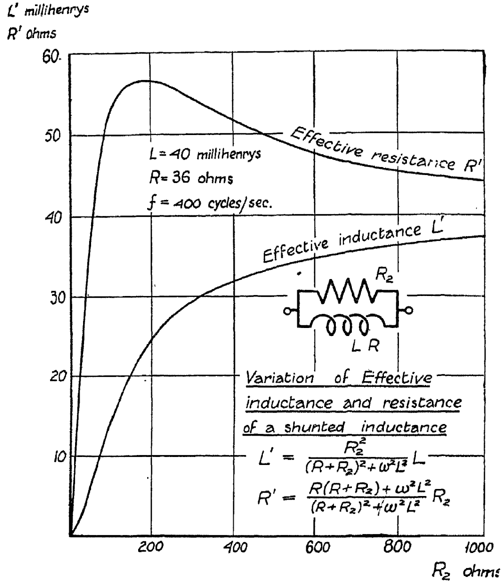

FIG. 106.—VARIATION OF EFFECTIVE INDUCTANCE AND RESISTANCE OF A SHUNTED INDUCTANCE

Consider first a coil of resistance R and inductance L shunted by a resistance R_2. Then, as shown in Fig. 10, the effective resistance and inductance of the combination may be written as

$$R' = R + \frac{R_2(\omega^2L^2 - R^2) - R(\omega^2L^2 + R^2)}{(R + R_2)^2 + \omega^2L^2}$$

$$L' \qquad \frac{R_2{}^2}{(R + R_2)^2 + \omega^2L^2}$$

Assuming constant frequency, it is of interest to examine the variation of R' and L' as R_2 is varied from 0 to ∞; the following results are easily deduced and are illustrated in Fig. 106 for a coil in which $R = 36$ ohms and $L = 40$ mH. the frequency being 400 cycles per second.

(i) L' is always less than L. Hence the apparent inductance of a coil can be reduced to any desired extent by shunting it with a suitable resistance, a fact which was pointed out by Niven* so long ago as 1887.

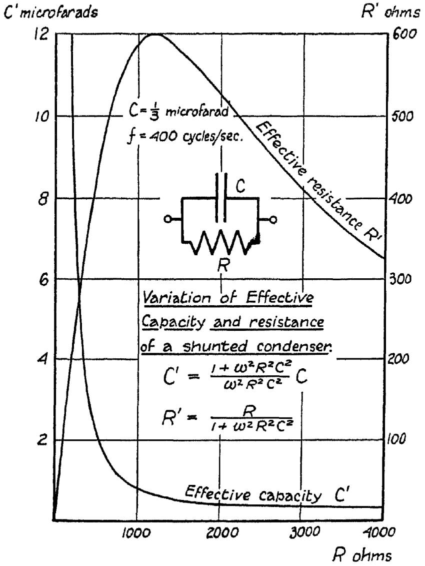

FIG. 107.—VARIATION OF EFFECTIVE CAPACITANCE AND RESISTANCE OF A SHUNTED CONDENSER

L' and L are equal when R_2 is infinite; the rate at which L' approaches L as R_2 is increased is slower for higher frequencies and larger values of L.

(ii) The effective resistance of the combination, R', is equal to the resistance of the coil when $R_2 = R(\omega^2L^2 + R^2)/(\omega^2L^2 - R^2)$. For values of R_2 less than this $R' < R$. When R_2 is greater than this the effective resistance of the combination *always* exceeds R, rising to a maximum and then gradually falling towards R as R_2 tends towards infinity.

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* C. Niven, "On some methods of determining and comparing coefficients of mutual induction," *Phil. Mag.*, 5th series, Vol. 24, pp. 225–238 (1887); H. E. Harrison, "Effects produced by using an inductionless shunt with a choking coil," *Elecn.*, Vol. 31, pp. 119–121 (1893).

Consider next the case of a condenser of capacitance C shunted by resistance R; then from Fig. 10 the effective resistance and capacitance of the combination will be

$$R' = \frac{R}{1 + \omega^2 C^2 R^2}$$

$$C' \quad \frac{1 + \omega^2 C^2 R^2}{\omega^2 C^2 R^2} \quad C$$

These expressions are plotted in Fig. 107 for a condenser in which $C = 1/3\ \mu$F., R being varied, while the frequency is constant at 400 cycles per second. From the graphs it is easy to verify the following properties of the combination—

(i) C' is always greater than C, so that the effective capacitance of a condenser may be increased to any desired extent by shunting it with a sufficiently low resistance.

(ii) The effective resistance of the combination, R', increases with R until a maximum value is attained when $R = 1/\omega C$; thereafter R' continually decreases towards zero as R tends toward infinity.

The two shunted elements just considered are of frequent occurrence in bridge networks, where they serve one of two purposes; either to adapt a given standard of inductance or capacitance to a wider range of measurements, or to modify a given inductive or capacitive impedance so that it may be measured by a bridge with given standards. Examples of both procedures will be found later in the present chapter.

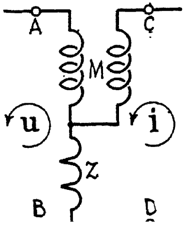

FIG. 108.—THE MUTUAL INDUCTANCE-IMPEDANCE PAIR

1e. The MZ Pair. The general principle of a bridge measurement is, as has been pointed out, the balancing of an unknown impedance by a standard impedance in which the resistance and reactance components can be separately controlled. In Wheatstone networks the standard impedance may be composed of suitable arrangements of self-inductances, mutual inductances, and condensers with resistances. The use of inductive or capacitive reactance does not call for special comment and the influence of direct mutual inductance between the branches of a Wheatstone network has already been fully treated on page 70. There is one detail, however, of which important use is made, chiefly in bridges which are not of the Wheatstone type, namely, the mutual inductance-impedance combination introduced by Campbell[*] in 1910, and applied by him to a variety of measurements in 1917. It has since been used by many other workers both in bridge measurements and in a.c. potentiometry. In Fig. 108, M is a mutual inductance, in series with the primary of which is an impedance $z = R + jX$, where AB are current terminals and CD are potential terminals. When a cyclic current **u** flows in the primary and a cyclic current $\mathbf{i}$ in the secondary, the e.m.f. applied externally between C and D will be

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[*] A. Campbell, *Proc. Phys. Soc.*, Vol. 22, p. 497 (1910); Vol. 29, p. 345 (1917).

$$z_2\mathbf{i} + z(\mathbf{i} - \mathbf{u}) + j\omega M\mathbf{u},$$

where z_2 is the impedance operator for the secondary.

If $\mathbf{i}$ be zero then the e.m.f. is $-[R + j(X - \omega M)]\mathbf{u}$. When $X = 0$ we have the MR pair, which is the balancing equivalent of an impure mutual inductance, see page 154. Again, if $R = 0$ and $X = -1/\omega C$ we have the well-known Campbell frequency bridge. Other values for z yield various methods for dealing with four-terminal impedances, some of which are noticed later.

1f. Tuned Branches. In some bridges three of the branches are resistances and the fourth contains a combination of resistance, inductance, and capacitance adjusted to be non-reactive. Such an arrangement is a tuned or resonant branch.

In the simplest case R, L, and C in the tuned branch are in series. Then

$$z = R + j[\omega L - (1/\omega C)],$$

and the reactance will vanish if

$$\omega L = 1/\omega C$$

or

$$f = 1/[2\pi\sqrt{(LC)}]$$

The impedance is then least, equal to the resistance, and the current is greatest and in phase with the voltage. In setting up such a branch it is a great convenience to have the relation of f to LC in graphical form; this is given in Fig. 109 in double logarithmic co-ordinates for a range from 10 to 10,000 cycles per second. This graph is also useful in other similar, though not necessarily resonant, instances, e.g. the relation between MC and f in the Campbell frequency bridge of Fig. 169 (*a*).

The parallel resonant branches are rather more complex. For example, if a condenser C is put in parallel with R, L, Fig. 10 shows that

$$z = \frac{R + j\omega[L(1 - \omega^2 LC) - CR^2]}{(1 - \omega^2 LC)^2 + \omega^2 C^2 R^2}$$

The reactance will vanish if

$$\omega^2 = (1/LC) - (R^2/L^2)$$

and the operator is then

$$z = L/CR.$$

Applications and further developments will be found on p. 402.

NETWORKS CONTAINING RESISTANCE AND SELF-INDUCTANCE

2. Maxwell's Method. In the second volume of his *Treatise on Electricity and Magnetism*,* Maxwell describes a simple method for comparing the self-inductances of two coils by the use of a ballistic galvanometer and a battery. Max Wien,†

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* 1st Edn., p. 357 (1873). *See* also M. Brillouin, "Comparison des coefficients d'induction," *Ann. de l'Ecole Normale*, Vol. 11, pp. 339–424 (1882).

† Max Wien, "Messung der Inductionsconstanten mit dem 'optische Telephon,'" *Ann. der Phys.*, Vol. 44, pp. 689–712 (1891).

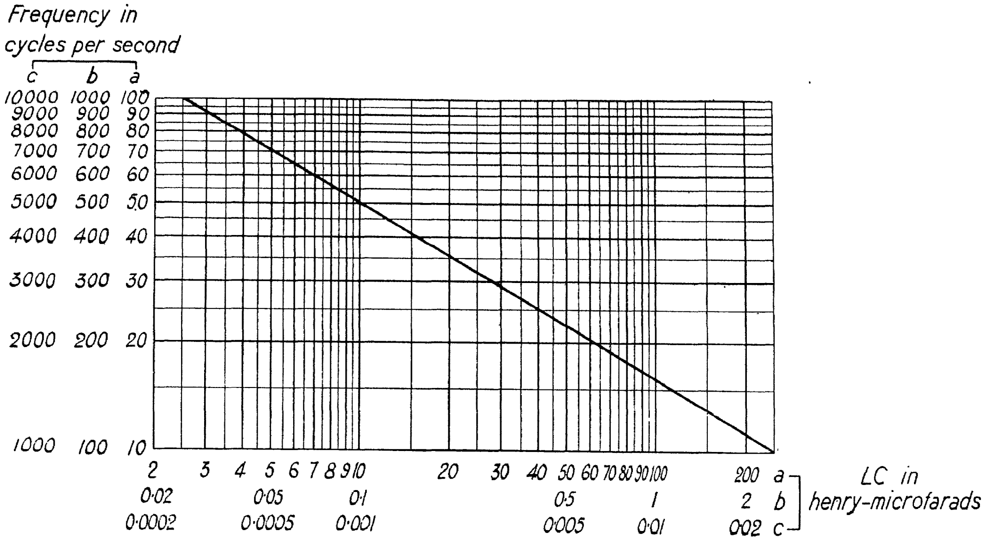

FIG. 109.—RESONANCE IN TUNED BRANCHES

in 1891, appreciating that the method possesses continuous balance, adapted it to alternating current.

In Fig. 110 (*a*), let L_1 and L_2 be the two self-inductances, the branches of the network in which they are connected having resistances P and R respectively. A resistance r is arranged so that it may be included at will in P or in R. The remaining branches are composed of resistances Q and S, frequently coils in a ratio box. The branch impedance operators are

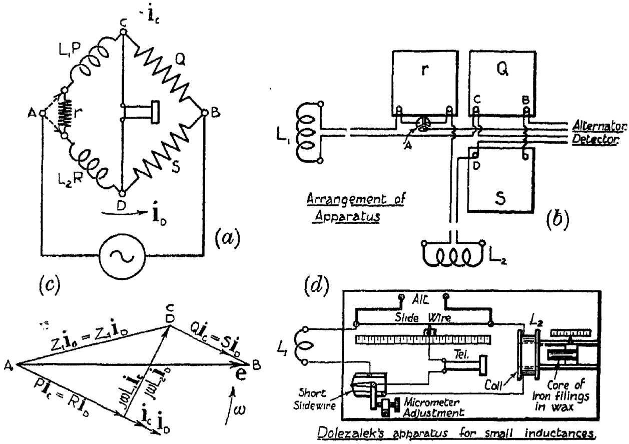

FIG. 110.—MAXWELL'S METHOD FOR COMPARING TWO SELF-INDUCTANCES

$$z_1 = P + j\omega L_1,\ z_2 = Q,\ z_3 = S,\ z_4 = R + j\omega L_2,$$

where P or R includes r according to the needs of the experiment. Then, for a four-branch network, the balance condition is (p. 52)—

$$z_1 z_3 = z_2 z_4,$$

or

$$S(P + j\omega L_1) = Q(R + j\omega L_2)\,;$$

whence, by separating the two components,

$$\frac{L_1}{L_2} = \frac{P}{R} = \frac{Q}{S}$$

are the two conditions which must be satisfied if no current flows in the detector. Since the conditions do not involve ω, the bridge can be balanced even when the wave form of applied potential difference is impure, and a telephone can be satisfactorily employed to indicate balance.*

In practical working, the following procedure is the most convenient. Adjustment of the bridge will be most easily attained *if the standard inductor, L_2, be of the type in which the inductance can be continuously varied* without alteration of resistance.† As a preliminary, the resistance of the unknown inductance L_1 should be found by means of a Wheatstone bridge, the value obtained acting as a guide in deciding where to connect the resistance r. A suitable arrangement of the apparatus is shown diagrammatically in Fig. 110 (*b*). The inductances L_1 and L_2 are arranged at some distance from one another, so that the mutual inductance between them may be negligible; they are connected to the remainder of the network by bifilar leads, so that errors due to inductance of the leads may be a minimum. The branches Q and S are shown as separate resistance boxes, but it is often convenient to combine them in some form of ratio box. The connections from the source of current and to the detector are preferably of twisted wire; in general, care should be taken to avoid loops of any considerable area in the connections used in the bridge by arranging that the leads are grouped in pairs. By such means, stray inductance errors are minimized as far as possible. A three-point plug serves to connect the alternator terminal A to one or other end of the resistance r; this resistance may be composed of a dial or plug box in combination with a fine-adjustment rheostat‡ or slide wire for the purpose of securing accurate balance.

Assuming the value of L_1 to be entirely unknown, balance may be found by a process of methodical trial and error. Start by making $Q/S = 1$, setting the resistance r in AC or in AD, so that P/R is about 1 also. Then, by successive adjustment of L_2 and r, endeavour to obtain balance. Clearly, if $L_1 =$ or $< L_2$, balance may be at once secured, but if it be not within the range of the standard inductor, a second trial is necessary. Observe at which end of the range of L_2 the indication of the detector is least; then alter the ratio Q/S to bring the balance point within the values obtainable in L_2. Re-adjust r to make P/R about equal to the new ratio, and then obtain balance by alteration of r and L_2 in successive steps.

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* For an adaptation of the method to phase-selective detectors, such as the Sumpner galvanometer or rectifying commutators, *see* T. Parnell, "An alternating-current bridge method of comparing two fixed inductances at commercial frequencies," *Proc. Phys. Soc.*, Vol. 29, pp. 259–268 (1917).

† For example, the Ayrton-Perry Inductance or other forms shown on pp. 141–147. *See* p. 302 for the possible ways of balancing.

‡ Of constant or calculable inductance, pp. 124–127.

Should it be found that the alteration of L_2 over its whole range makes no appreciable difference to the indication of the detector when $Q = S$, a considerable alteration in the value of Q/S is requisite, say to 10 or to 1/10. An attempt to balance should then be made by the procedure just described. It is always best, wherever the available standards render it possible, to work with $Q = S$, and to extend the range of a small variable inductor by means of additional fixed inductance standards connected in series with it. Such extra coils should be put at some distance from the variable inductor and from the coil under test, so that there shall be no errors due to mutual actions between them; or they should be toroidally wound. By using $Q = S$, any residual errors and slight inequalities which they may possess can be eliminated by re-balancing the bridge when Q and S are interchanged and averaging the two results.*

If only fixed inductance standards are available, balance must be secured by successive adjustment of Q/S and r. After a preliminary trial has indicated the order of magnitude of L_1, a standard L_2 of about equal magnitude should be chosen. The resistance boxes Q and S are then preferably connected by a slide wire, the point B being the sliding contact thereon; the final precise adjustment of Q/S can then be obtained.

It is worth while to notice at this stage two artifices which may be of service in adapting the bridge to work with available standards. Properly, these artifices constitute separate methods and will be considered in detail in their proper place. Suppose that L_1 *is much larger than any available standard* L_2. Connect in the branch AC a condenser of capacitance C; then the apparent inductance of the branch is $L_1 - (1/\omega^2 C)$, which by suitable choice of C may be brought within the range of the standard L_2. It should be noticed that the frequency must be constant and known. Again, suppose that L_1 *is much smaller than the standard* L_2. Connect a resistance R_2 in parallel with the branch AD. Then, referring to the discussion given on page 306, the apparent inductance of the branch is

$$\frac{L_2 R_2^2}{(R + R_2)^2 + \omega^2 L_2^2}$$

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* *See* Chap. V for further precautions in precise work.

which can be made as small as is desired by suitable choice of R_2. This in effect is the Wien-Dolezalek method described on p. 324. In the use of both these artifices such magnitudes of C or of R_2 should be chosen as will convert the bridge into one with equal ratios, in order that residual errors may be reduced to a minimum.

Experimental Examples. The following typical examples will serve to illustrate the balancing procedure described above.

(i) The various branches were made up as follows. AC, a coil L_1 of nominally 40 millihenrys inductance and 5 ohms resistance in series with a decade resistance box and constant inductance rheostat (r, in Fig. 110 (a)). AD, an Ayrton-Perry variable self-inductance L_2 of 43 millihenrys maximum value, and $R = 10{\cdot}69$ ohms. Q and S were equal coils in a ratio box, and were set at 10 ohms each. The source was a triode oscillator working at a frequency of 459·3; the detector a tuned Duddell vibration galvanometer. Balance was secured by successive alterations of r and L_2, the balancing values being 5·50 ohms and $40{\cdot}72_0$ millihenrys respectively. Hence $L_1 = 40{\cdot}72_0$ millihenrys and the effective resistance of L_1 is $10{\cdot}69 - 5{\cdot}50 = 5{\cdot}19$ ohms.

(ii) In a second test made on a coil of about 0·6 henry, Q was set at 1,500 ohms and S at 100 ohms, these now being coils in separate decade boxes. r was included in the branch with the unknown coil. The settings of r and L_2 at balance were 65·15 ohms and $40{\cdot}40_6$ millihenrys respectively, so that $L_1 = 15 \times 0{\cdot}04040_6 = 0{\cdot}6060_9$ henry, and its effective resistance is $15 \times 10{\cdot}69 - 65{\cdot}15 = 95{\cdot}20$ ohms. The frequency in this test was 407·1.

3. The Vector Diagram for Maxwell's Method. The vector diagram for the balanced bridge is drawn in Fig. 110 (c). Since the points C and D are always at the same potential, and the branches CB and DB are pure resistances, the currents passing the branch points C and D must be in phase. Then if AB be the vector of potential difference, **e**, applied between the terminals A, B, the remaining vectors are easily constructed as shown. Relative to A, the common potentials of C and D are represented by coincident points C, D, i.e. $z_1\mathbf{i}_C = z_4\mathbf{i}_D$. To these must be added $Q\mathbf{i}_C = S\mathbf{i}_D$ to give the vector **e**. From the geometry of the figure it is easily seen that $P\mathbf{i}_C = R\mathbf{i}_D$, $L_1\mathbf{i}_C = L_2\mathbf{i}_D$, and $Q\mathbf{i}_C = S\mathbf{i}_D$; hence $L_1/L_2 = P/R = Q/S$.

4. Sources of Error in Maxwell's Method. The method described above in its simplest form is, in practice, subject to various sources of error, which will now be examined.

Eddy Current Effects. It is to be noted that although the balance conditions given in the above do not involve the frequency, it does not follow that balance obtained at one frequency will be retained at any other. This is due to the fact that the quantities involved are effective inductances and resistances, the values of which are different with alternating current from the values with direct current, owing to the effects of eddy currents in the resistances and the standard coils. With properly constructed resistance boxes and well stranded inductances, the effect of eddy currents can be reduced to a very small amount, the alternating and direct current values being practically identical. Thus, if three branches be made up of standard apparatus, as free from eddy current effects as possible, the bridge serves to find the inductance and the effective resistance of a coil at a given frequency. It has been applied by Dolezalek* in this connection to measure the additional resistance in alternating current apparatus due to the effect of eddy current losses; this investigator has also described self-contained bridges capable of measuring an inductance of 10^{-7} henry with an accuracy of 1 or 2 per cent (Fig. 110 (*d*)).

Mutual Inductance. In arranging the apparatus for the simple bridge just described, the two coils have been placed so that the mutual inductance between them is very small. This can be secured by putting them at some distance apart, with their planes perpendicular. However, in precise work, the mutual inductance effect, though small, may not be negligible, and it becomes necessary to examine the way in which it modifies the balance conditions.

Let M be the mutual inductance between the coils in branches 1 and 4. Then, in the expression given on page 71, put all the mutual operators zero except $m_{14} = j\omega M$, giving $\alpha = 0$, $\beta = 0$, $\gamma = j\omega M$, $\delta = 2j\omega M$. Then

$$(SP - QR) + j\omega\{S(L_1 + M) - Q(L_2 + M)\} = 0\,;$$

whence balance occurs if

$$\frac{L_1 + M}{L_2 + M} = \frac{P}{R} = \frac{Q}{S};$$

it should be noted that M may be positive or negative.

If the coils to be compared are equal, i.e. if $L_1 = L_2$, then the above equation is independent of M. Then $P = R$ and $Q = S$. Thus, *in an equal ratio bridge balance is unaffected by the mutual inductance between the coils,* so that it is not necessary in this case to go to any trouble to reduce mutual action to a minimum.

Residual Errors in the Ratio Branches. It has been assumed that the resistances Q and S are perfect, i.e. they contain no residual inductance or capacitance. Now, although this is very nearly true in the case of resistances specially constructed for alternating current work, the small residuals may have an important effect, especially in bridges where the ratio is such that Q and S are very unequal. If these are of low value the residual inductance may preponderate; if of high value, the residual self capacitance may be important. In any case, the effect can be represented by writing $Q + j\omega\lambda$ for Q and $S + j\omega\mu$ for S where λ and μ are the residuals.

The balance condition for the bridge is then

$$(P + j\omega L_1)\ (S + j\omega\mu) = (R + j\omega L_2)\ (Q + j\omega\lambda),$$

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* F. Dolezalek, "Messeinrichtung zur Bestimmung der Induktionskonstanten und des Energieverlustes von Wechselstromapparaten," *Zeits. f. Inst.*, Vol. 23, pp. 240–248 (1903).

which, on separation of the two components, gives

$$\frac{L_1}{L_2} = \frac{Q}{S} - \left(\frac{P\mu - R\lambda}{L_2 S}\right) = \frac{Q}{S} - k_1$$

and

$$\frac{P}{R} = \frac{Q}{S} + \omega^2 \left(\frac{L_1\mu - L_2\lambda}{RS}\right) = \frac{Q}{S} +$$

as found by Giebe.*

To show the importance even of small residuals, consider a bridge† in which $L_1 = 0{\cdot}1$ henry, $P = 20$ ohms, $L_2 = 0{\cdot}01$ henry, $R = 2$ ohms, $S = 5$ ohms. Then if Q and S are free from residual effects, Q must be 50 ohms for balance. Now let $\lambda = 0$ and $\mu = 0{\cdot}25$ microhenry, then the second equation gives $Q = 45{\cdot}57$ ohms when the frequency is 3,000 cycles per second. Hence, the value of Q requisite for balance is reduced by 4·43 ohms or 8·86 per cent. If the residual inductance had been neglected and L_1/L_2 calculated from the observed Q/S, the result would have been in error by 8·86 per cent, which is a considerable amount.

Error due to these residual effects can be avoided in three ways: (*a*) By making the bridge symmetrical, so that $L_1 = L_2$, $Q = S$, and $P = R$. Any slight difference between Q and S is eliminated in the mean of the values obtained for balance when their positions are interchanged. Or by means of a small auxiliary inductance in Q or S, make $Q/S = \lambda/\mu$, so that the residuals of these branches are proportional to their resistances. Then $L_1/L_2 = Q/S = P/R$ as if residuals were not present. With an equal ratio bridge, let balance be secured in the ordinary way. Then alter the small auxiliary inductance and the other adjustments of the network until, on reversing the positions of Q and S, balance is not disturbed. (*b*) By a substitution method. The bridge is balanced with the unknown coil in position, and then re-balanced when the coil is replaced by one of the same resistance and calculable inductance. The adjustable rheostats in the branches should be of constant inductance, and the substituted coil should be as nearly equal to the unknown coil as possible. This procedure (since the balanced bridge is only very little disturbed from its initial state) entirely eliminates residual errors, and determines with high precision the difference between the unknown and the standard. Grover and Curtis‡ have employed it to measure the inductance of low value resistance coils. (*c*) By the use of a specially constructed bifilar bridge in which the residuals are calculable.

This third method has been developed by Giebe§ for the precise comparison of two inductances on a bridge of large ratio. His arrangement is shown diagrammatically in Fig. 111 (*a*) for a bridge in which the ratio is 10, and forms an excellent illustration of the care which must be exercised when precision is desired. The two coils to be compared were wound with carefully stranded wire, their inductances and resistances being $L_2 = 10^7$ cm., $R = 2{\cdot}84$ ohm, $L_1 = 10^8$ cm., $P_1 = 18{\cdot}77$ ohm. They are connected to the rest of the bridge by bifilar leads about 1 metre long, so that mutual inductance between the coils and the bridge may be neglected.

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* E. Giebe, "Messung induktiver Widerstände mit hochfrequenten Wechselströmen. Methode zur Messung kleiner Selbstinduktionskoefficienten," *Ann. der Phys.*, Vol. 24, pp. 941–959 (1907).

† From an example given by E. Orlich, *Kapazität und Induktivität*, p. 236.

‡ F. W Grover and H. L. Curtis, "The measurement of the inductances of resistance coils," *Bull. Bur. Stds.*, Vol. 8, p. 463 (1913). For a self-contained bridge with full allowance for residual effects (General Radio Co.), *see* R. F. Field, "The measurement of a small inductance," *G.R. Exp.*, Vol. 8, pp. 1–5 (Mar., 1934). The range is 1 μH. to 1 H. up to 10,000 cycles per second.

§ E. Giebe, *loc. cit.*; also "Präzisionsmessungen an Selbsinduktionsnormalen," *Zeits. f. Inst.*, Vol. 31, pp. 6–20, 33–52 (1911).

The four branch points *A*, *B*, *C*, *D* are arranged to be as close together as possible, the telephone or galvanometer being connected to *CD* and the alternator to *AB*, each by a pair of bifilar leads. The points *A*, *C*, *D* are represented by pins fixed in a piece of ebonite, soldered connection being made at them to the other branches of the network. The point *B* is represented by a knife-edge contact moving on a manganin wire about 2 cm. long and 0·36 mm. diameter for fine adjustment of the ratios.*

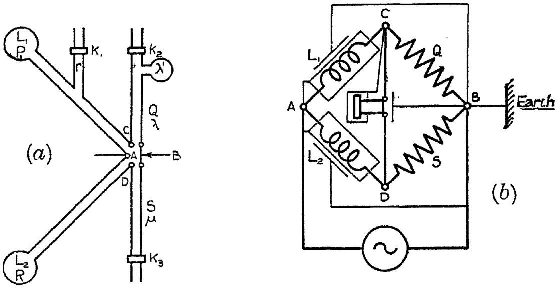

FIG. 111.—GIEBE'S BIFILAR BRIDGE AND SHIELDING ARRANGEMENTS

The ratio branches *Q* and *S* are resistances designed so that their residuals may be readily calculated.† Each is composed of a pair of parallel, bare manganin wires stretched out on a suitable board, and bridged at the ends remote from the branch points by the short-circuiting pieces K_2, K_3. If l be the length of the wires, a the distance between their axes, r their radius, the self-inductance of such a parallel wire resistance is $L = \left[4 \log \left(\frac{a}{r}\right) + 1\right] l.$ cm. The dimensions and approximate constants of these resistances are

	Q	*S*
r mm.	0·025	0·075
a mm.	1·3	1·3
l cm.	102·4	87·1
Resistance	390 ohms	39 ohms
Inductance	1,720 cm. (λ_1)	1,080 cm. (μ)

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* For detailed drawings of a bifilar bridge, *see* W. Hüter, "Kapazitätsmessungen an Spulen," *Ann. der Phys.*, Vol. 39, pp. 1350–1380 (1912).

† See p. 118.

Now, referring to the balance conditions, if Q and S are such that $Q/S = \lambda/\mu$, then the equations reduce to $L_1/L_2 = Q/S = P/R$ just as if there were no residuals present. Hence, to ensure that this condition is approximately fulfilled, a coil of inductance $\lambda' = 9{,}870$ cm. is connected in series with Q. Thus, $\lambda = \lambda_1 + \lambda' = 1{,}720 + 9{,}870 = 11{,}590$ cm., so that $\mu/\lambda = 1{,}080/11{,}590 \fallingdotseq 1/10$. The resistance of λ' is negligible, it being a copper coil. It should be noticed that the correction term k_2 can be kept small by keeping the radius of the wire of which S is composed as small as possible ; as the radius is reduced, S increases at a greater rate than μ, so that k_2 is diminished.

A bifilar resistance r is included in series with L_1 for purpose of adjustment. Balance is secured by successive alterations of r and of the slider B. The movement of the latter allows of a variation of ratio of about 2 per cent ; further adjustment can be obtained by movement of the bridges K_2, K_3. The following table shows how accurately, with such precautions, a bridge of ratio 10 can be balanced at various frequencies when the residuals are allowed for.

$$k_1 = 0{\cdot}000006$$

Frequency	Q/S		P_1	P	R	k_2	Observed P/R	Calculated $P/R = \frac{Q}{S} + k_2$
50	10·000	$9{\cdot}6_3$	18·77	$28{\cdot}4_0$	2·84	0	10·00	10·00
2,040	10·026	$9{\cdot}6_9$	18·77	$28{\cdot}4_6$	2·84	$-0{\cdot}01_2$	10·02	$10{\cdot}01_4$
3,035	10·053	$9{\cdot}5_0$	18·77	$28{\cdot}2_7$	2·84	$-0{\cdot}02_6$	9·97	$10{\cdot}02_7$

It will be seen that the values of P/R directly observed and those calculated from the corrected value of Q/S are in very close agreement. The discrepancies are discussed by Giebe.

Kreielsheimer,* following a suggestion of Jaeger and Meissner, has used a shielded bifilar bridge for the measurement of the effective resistance of iron wires at frequencies of the order of 10^6 cycles per second. Details of the circuit are fully described, the accuracy being tested by measuring the a.c. resistance of a copper wire, the error being less than ± 4 per cent. From tests on the iron wires the high-frequency permeability is deduced with an error of ± 1·7 per cent. A somewhat similar method has been used by Krüger† to test iron and mumetal wires at radio frequencies, glow discharge quartz resonators being used as tuned detectors (see p. 266).

Self capacitance of L_1. Self capacitance‡ in the coil under test, especially if it has a great number of turns, will introduce an error at higher audio frequencies. It has been shown on page 135 that the effect can be represented by a condenser C in parallel with the coil, the

* K. Kreielsheimer, "Zur Messung der magnetischen Permeabilität von Eisendrahten bei Hochfrequenz in der Wheatstoneschen Brücke," *Zeits. f. Phys.*, Vol. 55, pp. 753–770 (1929).

† F. Krüger, "Über die Verwendung von Leuchtquarz-resonatoren als Vibrationselektrometer bei Messung in der Wheatstoneschen Brücke, insbesondere zur Messung des Skineffekt an Drähten aus Eisen und Mumetall," *Ann. der Phys.*, Vol. 26, pp. 167–176 (1936).

‡ See E. Giebe, *loc. cit.* ; also Max Wien, "Messung der Inductionsconstanten mit dem 'optische Telephon,'" *Ann. der Phys.*, Vol. 44, p. 711 (1891), and F. Dolezalek, "Ueber Prazisionsnormale der Selbstinduktion, *Ann. der Phys.*, Vol. 12, pp. 1142–1152 (1903).

combined impedance operator being $(P_1 + j\omega L_1)/\{(1-\omega^2 C L_1) + j\omega C P_1,\}$ so that the effect of self capacitance is to increase the resistance of the coil to

$$P'_1 \fallingdotseq (1 + 2\omega^2 L_1 C)P_1,$$

and its inductance to

$$L'_1 \fallingdotseq (1 + \omega^2 L_1 C)L_1.$$

These values must replace P_1 and L_1 in all the preceding expressions.

The value of C can be found by measuring L'_1 at two frequencies, and its effect can be allowed for. For example, in Giebe's bifilar bridge described above, the value of L_1 is 10^8 cm., and C is $0{\cdot}0001_8$ microfarad. The values of P'_1 and L'_1 at two frequencies are

Frequency	2,040 cycles/second	3,035 cycles/second
P'_1	18·86 ohm	18·97 ohm.
L'_1	$1{\cdot}0025 \times 10^8$ cm.	$1{\cdot}0055 \times 10^8$ cm.

from which an idea of the magnitude of the effect can be obtained, remembering that $P_1 = 18{\cdot}77$ ohm at low frequency.

Earth Capacitances. Considerable error may arise in measurements at high frequencies when the earth capacitances are neglected, and without making some compensation for them, balance may be difficult or impossible to obtain. The general question of earth capacitances will be discussed on page 525, but it will be well to mention at this stage the general nature of the effects involved.

Every branch of the network has capacitance with respect (*a*) to surrounding earthed objects, and (*b*) to every other branch of the network. To a first approximation the earth capacitances (*a*) may be represented by condensers connected between the branches and earth; and in a similar way the intercapacitances (*b*) may be considered as a set of condensers joining the several branches. These condensers constitute, as it were, branches additional to those forming the original network, and by their presence the balance conditions may be profoundly affected. Naturally, the effects will be more pronounced at high frequencies than at low, since the reactances of the capacitance paths connecting the branches to one another and to earth become less as the frequency is raised.

In order to make the effect of earth and intercapacitances as small and as definite as possible, it is necessary to shield the several branches from one another by enclosing them in metal boxes or shields, the potentials of the latter being maintained at definite values with respect to earth and to the branch points of the network.

Fig. 111 (*b*) shows diagrammatically the shielding arrangements adopted by Giebe for the bifilar bridge. The inductive coils L_1, L_2 are enclosed in metal boxes maintained at the potential of A. The bifilar ratio resistances Q, S, are earthed at the branch-point B. Shields connected to B make the earth capacitances of the boxes definite in value; they merely shunt the source.

A further important capacitance effect when a telephone is used as a detector is the capacitance between the observer and the telephone. Though at balance the potentials of the points CD are identical, it does not follow that their common potential will be the same as that of the observer. A capacitance current will flow from the branch points through the telephone to earth, *via* the observer, and silence can not be secured unless the capacitance effect be eliminated. Several devices for securing this result are described in the next chapter. Giebe's arrangement is to enclose the telephone in a metal case connected to the branch-point *C*, an earthed screen making the capacitance definite. The observer applies his ear to a tube of insulating material projecting through the walls of the metal case.

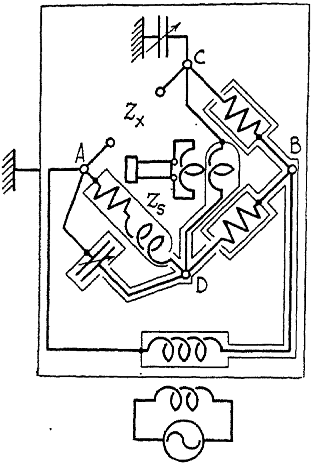

FIG. 112.—SHACKLETON'S SHIELDED IMPEDANCE BRIDGE

A very thorough study of parasitic capacitance effects in the Maxwell bridge has been made by Shackleton,* resulting in the design of a completely shielded, self-contained bridge for the measurement of inductive impedances ranging from 100 to 10,000 ohms, with time constants not less than 10 at frequencies up to 50,000 cycles per second, such as occur in research on telephonic apparatus. The bridge has two equal ratio branches of 1,000 ohms each, carefully wound so as to have very low residuals; these are contained in shielded cases connected to the point *B*, Fig. 112. The adjustable branch Z_s contains a variable non-reactive resistance providing a range of 1,000 ohms in steps of 0·01 ohm, and a three-decade variable inductance. This inductance is of toroidal form with "Permalloy" cores, thus considerably reducing the bulk and the quantity of wire necessary for large standard inductometers. A suitable system of reversing switches, omitted from the diagrams enables reversal of the ratio arms and of Z_s and Z_x to be effected, so that inequalities in the branch impedances or their residuals can be detected and allowed for. The ratio coils and the standard impedance are enclosed in individual shields; a second system of shields makes definite the capacitance effects between the source and the detector and the branches of the network. The shields are arranged so that all earth and intercapacitance effects are rendered definite and, so far as is possible, are balanced. The residual unbalanced capacitances are reduced to a shunt across the unknown impedance Z_x and the capacitances from A and C to earth. To eliminate the effect of these on the balance two 500 $\mu\mu$F. air condensers are connected as shown, one across AD to balance the capacitive residual across AC, and the other from the point C to earth.

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* W J. Shackleton, "A shielded bridge for inductive impedance measurements at speech and carrier frequencies," *Journal Amer. I.E.E.*, Vol. 46, pp. 159–166 (1927); *Bell Syst. Tech. J.*, Vol. 6, pp. 142–171 (1927). W. J. Shackleton and J. G. Ferguson, "High frequency measurement of communication apparatus," *Trans. Amer. I.E.E.*, Vol. 46, pp. 519–527 (1927).

In practice, Z_x and Z_s are first removed and the out-of-balance capacitance residuals balanced out by adjustment of the condensers, which are then left untouched. The impedances Z_x, Z_s are now inserted and the bridge rebalanced by adjustment of the resistance and inductance of the latter; the balance values are equal to the effective resistance and inductance of Z_x. The original papers should be consulted for full details of construction and of procedure in high precision testing.

Webb and Brookes-Smith* have described a Maxwell bridge used by the Standard Telephones and Cables Ltd. for measurements of inductance and effective resistance on small samples (about 2 yards) of continuously loaded telephone cable. A range of 1 to 300 μH., 0·0001 to 10 ohms and 100 to 70,000 cycles per second is covered by a simple substitution method. The ratio branches Q and S are 20 ohms each. The total resistance R is 20 ohms, consisting of fixed resistors together with a variable 1 ohm resistor and a small inductor for zero adjustment. The branch AC contains the cable sample, three variable inductors with maximum values of 350, 20 and 0·1 μH. respectively, and a five-decade resistor with ten steps each of 1, 0·1, 0·01, 0·001 and 0·0001 ohms each. The inductors have stranded astatic coils; the resistors are specially constructed to have low contact resistance. After an initial balance with the specimen short-circuited, the specimen is inserted and balance restored by adjustment of the inductors and resistors in the same arm; the changes give the inductance and resistance of the sample. Provision is made for substitution of d.c. so that the increase of resistance due to eddy currents and loading losses can be determined. The detector leads, ratio arms, and variable resistor are contained in shields joined to point A, which is earthed. The detector is an amplifier with a gain of 80 db. and a telephone, up to 4,000 cycles per second; above this frequency a heterodyne arrangement is used.

5. Giebe's Modification of Maxwell's Method for Small Inductances. Giebe† has shown that Maxwell's method can, by means of a simple modification, be used for the measurement of small inductances which have a large resistance; as, for example, resistance coils. Briefly, the principle is to use as the ratio branches coils of high inductance and low resistance; it is then easy to show that residual errors in the bridge, when used to compare two small inductances of comparatively high resistance, are very small and can be allowed for experimentally. By this means Giebe was able to measure inductances of the order of 500 cm. with time constants as low as 10^{-8} second.

Referring to Figs. 110 and 111, the following procedure is adopted. L_1 and L_2 are known large inductances of relatively low resistances. In Giebe's arrangement, L_1 is nominally 0·1 henry and L_2 0·01 henry; self capacitance is important in the case of L_1, altering its inductance to

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* J. K. Webb and C. Brookes-Smith, "The measurement of small values of inductance and effective resistance," *Elect. Comm.*, Vol. 13, pp. 9–13 (1934).

† *Loc. cit.*

$L'_1 = (1 + \omega^2 L_1 C)L_1$ and its resistance to $P'_1 = (1 + 2\omega^2 L_1 C)P_1$ at frequency $\omega/2\pi$. S is a high resistance of which the small inductance, μ, is to be measured; Q is a parallel wire resistance of which the residual inductance may be calculated. Let a bifilar bridge be set up, as in Fig. 111, balance being secured by adjustment of the sliders K_1, K_2, and B. Then, from the balance equations on p. 316, balance is attained when

$$\frac{P'_1 + r}{R} = \frac{Q}{S} + \omega^2 \frac{(L'_1\mu - L_2\lambda)}{RS}.$$

Now re-balance the bridge with direct current by adjustment of r only, the balance value being r'; then

$$(P_1 + r')/R = Q/S,$$

assuming Q, R, and S do not vary appreciably with frequency. Subtracting the two equations and solving for μ, gives

$$\mu = \frac{S[(P'_1 - P_1) + (r - r')]}{\omega^2 L'_1} + \frac{L_2}{L'_1}\lambda.$$

Substituting for P'_1 gives,

$$\mu = \frac{S[r - r' + 2\omega^2 L_1 C P_1]}{\omega^2 L'_1} + \frac{L_2}{L'_1}\lambda,$$

all the quantities on the right-hand side being known.

To show the degree of precision attainable, Giebe compares two parallel wire resistances of calculable inductances. The observed values for the two balances were as follows, $L_1 = 0{\cdot}1$ henry, $L_2 = 0{\cdot}01$ henry, $C \doteqdot 0{\cdot}00015$ microfarad, $L'_1 = 0{\cdot}10017$ henry, $P_1 = 18{\cdot}77$ ohm, $= 11{\cdot}95$ ohm, $r' = 9{\cdot}59$ ohm, $\omega = 2\pi \times 2{,}016$, $S = 2{\cdot}01$ ohms. The first term on the right-hand side is then equal to 303×10^{-9} henry. Hence,

$$\text{Measured value of } \mu - \frac{L_2}{L'_1}\lambda = 303 \text{ cm.}$$

The calculated values of μ and λ were 380 cm. and 744 cm. respectively; so that the calculated value of the above quantity is $380 - \frac{0{\cdot}01}{0{\cdot}10017} \times 744$ i.e. 306 cm; agreeing with the observed value to within 1 per cent.

6. Wien's Methods. Max Wien* has described a modification of Maxwell's method by means of which a self-inductance can be found in terms of resistance and frequency. Two arrangements are shown in Fig. 113. The first is that described by Wien in 1891, L_1 and L_2 being unknown inductances, P_2, P_3, and r rheostats, and Q, S the ratio branches joined by a slide wire. The second gives Wien's† bridge of 1896, as used by him for the measurement of standard inductances of 0·001, 0·01, and 0·1 henry respectively. This arrangement differs from the former in the omission of the rheostat P_3, and in this form has been used by Dolezalek* to investigate the influence of stranding on the errors in standard inductances. A. Campbell,† in 1905, has described the use of the method to measure inductances of the order of 100 microhenrys.

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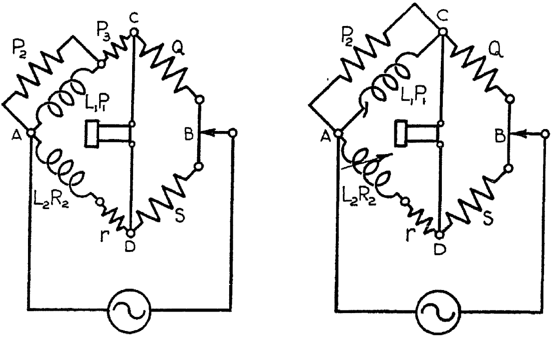
* Max Wien, "Messung der Inductionsconstanten mit dem 'optische Telephon,'" *Ann. der Phys.*, Vol. 44, pp. 689–712 (1891).

† Max Wien, "Einheitsrollen der Selbstinduction," *Ann. der Phys.*, Vol. 58, pp. 553–563 (1896).

In practical working, L_1 is the unknown inductance ; L_2 is an adjustable inductance which need not be known. Balance is attained by first fixing P_2 and L_2, and then adjusting successively the ratio Q/S and the rheostats P_3 and r. In using the second method of connection, it may be necessary to include a rheostat in P_1.



Wien

Wien - Dolezalek

FIG. 113.—WIEN'S METHODS FOR THE DETERMINATION OF SELF-INDUCTANCE

Referring to the more general arrangement, put $R_2 + r = R$; then the impedance operators for the branches are

$$z_1 = P_3 + \frac{P_2(P_1 + j\omega L_1)}{P_1 + P_2 + j\omega L_1},\ z_2 = Q,\ z_3 = S,\ z_4 = R + j\omega L_2.$$

Substituting in the balance equation

$$z_1 z_3 = z_2 z_4$$

and collecting terms

$$S\{P_3(P_1 + P_2) + P_1P_2\} + Q\{\omega^2 L_1 L_2 - R(P_1 + P_2)\}$$
$$+ j\omega[L_1\{S(P_2 + P_3) - QR\} - L_2Q(P_1 + P_2)] = 0.$$

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* F. Dolezalek, "Ueber Präzisionsnormale der Selbstinduktion," *Ann. der hys.*, Vol. 12, pp. 1142–1152 (1903).

† A. Campbell, *Proc. Phys. Soc.*, Vol. 19, pp. 171–172 (1905).

Now the total resistance of the branch AC is $P = P_3 + \frac{P_1 P_2}{P_1 + P_2}$; substituting in the above equation and putting the two components separately equal to zero, gives

$$\omega^2 L_1 L_2 = (P_1 + P_2)(QR - SP)/Q,$$

and

$$\frac{L_1}{L_2} = \frac{Q(P_1 + P_2)}{S(P_2 + P_3) - QR}$$

for the balance conditions, from which L_1 or L_2 can be calculated independently.

In the case of the more commonly used Wien-Dolezalek bridge, shown in the right-hand diagram, these conditions reduce to a simple form. Put $P_3 = 0$ in the above equations, then the balance conditions reduce to*

$$\omega^2 L_1 L_2 = R(P_1 + P_2) - P_1 P_2 \frac{S}{Q}$$

and

$$\frac{L_1}{L_2} = \frac{Q(P_1 + P_2)}{SP_2 - QR} \qquad (a)$$

Solving these equations for the value of L_1, gives

$$\omega^2 L_1^2 = (P_1 + P_2)^2 \frac{R - \frac{P_1 P_2 S}{(P_1 + P_2)Q}}{P_2 \frac{S}{Q} - R} \qquad (b)$$

from which L_1 can be found in terms of resistance and frequency. Wien has shown how, in practice, the knowledge of all these separate resistances can be avoided by making two additional measurements with direct current. After balance has been secured with alternating current, apply direct current and a reflecting galvanometer to the bridge. Then the resistance r must be adjusted so that the branch AD falls in resistance by an amount

$$r' = R - \frac{P_1 P_2}{(P_1 + P_2)} \cdot \frac{S}{Q}.$$

Then remove the inductive coil L_1; the resistance r must then be adjusted so that the resistance of AD increases by

$$= P_2 \frac{S}{Q} - R.$$

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* The first of these conditions is incorrectly given in the paper by Dolezalek quoted above, the factor $P_1 + P_2$ being omitted.

Putting these observed alterations of r in the above equation makes

$$L_1 = \frac{S}{Q} \cdot \frac{P_2^2}{\omega} \sqrt{\frac{r'}{r''}} \cdot \frac{1}{r' + r''} \qquad (c)$$

It is now necessary to determine the best arrangement of the network. Referring to Equation (b), it will be seen that the numerator and denominator both involve the difference between terms. Accuracy can only be expected, therefore, if P_2 be chosen so that these differences have reasonably large values. Following the usual practice, put $Q/S = 1$ in Equation (a) and find L_2, R in terms of L_1, P_1 and P_2; the resulting expressions are

$$R = P_2 \frac{\omega^2 L_1^2 + P_1(P_1 + P_2}{\omega^2 L_1^2 + (P_1 + P_2)^2}$$

$$\text{and} \qquad L_2 = L_1 \frac{P_2^2}{\omega^2 L_1^2 + (P_1 + P_2)^2} \qquad (d)$$

Now put these values in the numerator of Equation (b); then if P_1 is small compared with ωL_1, it is easy to show that the numerator is a maximum if

$$P_2 = \omega L_1.$$

In a similar manner, insertion of these values in the denominator will show that the latter increases with P_2, attaining neither a maximum nor a minimum; it has, however, a sufficiently large value if $P_2 = \omega L_1$.

In practice, therefore, the following procedure may be conveniently adopted—

(i) Measure approximately the values of L_1 and P_1.

(ii) Set up the bridge, making $Q \doteq S$ and P_2 about equal to ωL_1.

(iii) Calculate from Equations (d) the values of R and L_2, setting the rheostat r and the variable inductance in AD to these values.

(iv) Attain balance by adjustments of the slider B and L_2, noting their positions.

(v) Measure the frequency, $f = \omega/2\pi$.

(vi) Make the two measurements with direct current corresponding to Equation (c).

When the above procedure is adopted, accuracy of about 1 part in 1,000 is obtainable.

The bridge should be used only at relatively low frequencies, say 400 to 500 cycles per second, since, on account of the skin-effect at high frequencies, the two direct current balances will not bear any simple relation to the values which the resistances will have with alternating current. Moreover, owing to these eddy effects the bridge current will no longer be sinusoidal, and the use of the telephone becomes difficult. It then becomes necessary to use the vibration galvanometer, the sensitivity of which is greater at lower frequencies than at high.

Experimental Example. The following figures were obtained in a test at 459·3 cycles per second on a coil, L_1, of about 40 millihenrys. Using the above procedure, P_2 was set at 100 ohms, which is approximately the value of ωL_1. To enable the d.c. balances to be made, a resistance of 20 ohms was included in series with L_1, thus making P_1 about 25 ohms. Using these values in Equations (d) makes $R \doteqdot 51$ ohms and $L_2 \doteqdot 15{\cdot}6$ millihenrys; the latter was an Ayrton-Perry variable inductance of resistance $R_2 = 10{\cdot}69$ ohms. The resistance r was a decade box in series with a constant inductance rheostat, and in the a.c. tests was set at 40·0 ohms to make R of about the required value. Q and S were decade resistances, the former being in series with a low reading rheostat. The a.c. balances were obtained by adjustments of Q and L_2 for various fixed values of S, using a Duddell vibration galvanometer and a triode oscillator. The d.c. balances were made by reading the changes in the setting of r, using a cell and reflecting moving coil galvanometer.

S ohms.	Q ohms.	P_2 ohms.	r' ohms.	r'' ohms.	L_1 mH.
30	34·03	100	32·86	37·55	40·59
60	68·10	100	32·86	37·55	40·56
90	102·10	100	32·86	37·55	40·58
				Average	**40·58** mH.

The Wien-Dolezalek bridge lends itself not only to the measurement of self-inductance in terms of frequency and resistance* but can also be used as a frequency bridge, as Kurokawa and Hoashi† have shown. Referring to Equations (a), the conditions of balance can be written in the form—

$$P_2 = R\,\frac{1 + \dfrac{P_1}{R}\cdot\dfrac{L_2}{L_1}}{\dfrac{S}{Q}\cdot\dfrac{L_2}{L_1}}$$

$$f = \frac{R}{2\pi\left[L_1L_2\left(\dfrac{S}{Q}-\dfrac{L_2}{L_1}\right)\right]^{\frac{1}{2}}}\left[1-\left(\frac{P_1}{R}\right)^2\frac{S}{Q}\cdot\frac{L_2}{L_1}\right]^{\frac{1}{2}}$$

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* For a process differing from that just described, see J. G. Ferguson and B. W. Bartlett, *Bell. Syst. Tech. J.*, Vol. 7, pp. 420–437 (1928).

† K. Kurokawa and T. Hoashi, "A linear frequency bridge," *Journal I.E.E., Japan*, No. 437, pp. 1132–1138 (1924). P. Kaspareck, "Instruments de mesure de courants alternatifs employés en téléphonie," *Annales des Postes, Télég. et Téléph.*, Vol. 14, pp. 461–499 (1925).

where f is the frequency in cycles per second. The following values are fixed: $L_1 = 1{,}000\ \mu\text{H.}$, $L_2 = 100\ \mu\text{H.}$, $S/Q = 0{\cdot}3533$ $P_1 = 150$ ohms; balance is secured by adjustment of R and P_2. With these values it is easy to show that

$$f = R\left(1 - \frac{795}{R^2}\right)^{\frac{1}{2}}$$

which at high frequencies is very nearly $f = R$.

The authors point out that if the frequency be always calculated from $f = R$ the error will be 0·64, 0·16, 0·039 and 0·0099 per cent at 250, 500, 1,000, and 2,000 cycles per second. Hence the arrangement is an approximately linear frequency bridge.

NETWORKS CONTAINING RESISTANCE AND CAPACITANCE

7. De Sauty's Method. The simplest way to compare two condensers is to make use of the method introduced by Mr. De Sauty,* of the Eastern Telegraph Co., and first applied to alternating current measurement by Max Wien,† in 1891.

Referring to Fig. 114, C_1 is the condenser to be tested, C_2 a suitable standard condenser; Q and S are non-reactive resistances. The branch impedance operators are $z_1 = 1/j\omega C_1$, $z_2 = Q$, $z_3 = S$, $z_4 = 1/j\omega C_2$; so that balance occurs when

$$S/j\omega C_1 = Q/j\omega C_2,$$

or

$$C_1/C_2 = S/Q.$$

The usual two balance conditions thus coalesce and become one. In practice, Q is fixed and S is adjusted until the detector indicates zero.

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* Latimer Clark and Robert Sabine, *Electrical Tables and Formulae*, p. 62 (1871). For the ballistic bridge, *see* W. E. Sumpner, *loc. cit.* (1888).

† Max Wien, "Messung der Inductionsconstanten mit dem 'optische Telephon,'" *Ann. der Phys.*, Vol. 44, p. 697 (1891). J. Hanauer, "Ueber die Abhängigkeit der Capacität eines Condensators von der Frequenz der benutzen Wechselströme," *Ann. der Phys.*, Vol. 65, pp. 789–814 (1898). The following investigations were undertaken with the aid of an electrodynamometer as detector, W. Stroud and J. H. Oates. "On the application of alternating current to the calibration of capacity boxes, and to the comparison of capacities and inductances," *Phil. Mag.*, 6th series, Vol. 6, pp. 707–720 (1903); E. M. Terry, "On the variation of a capacity with temperature," *Phys. Rev.* Vol. 21, pp. 193–197 (1905).

The vector diagram for the balanced bridge is easily drawn, and is practically self-explanatory.

It is interesting to note the effect of residual inductance in the resistances Q and S upon the accuracy of the measurements. If λ and μ be the residuals, the operators for Q and S become $Q + j\omega\lambda$ and $S + j\omega\mu$ respectively. The conditions for balance are then

$$C_1/C_2 = S/Q = \mu/\lambda.$$

Hence not only the resistances but the residuals also must be in the ratio C_1/C_2. It may be necessary in this case to include in series with S or Q a small variable self-inductance in order that the second condition may be fulfilled. The bridge then becomes a variant of Grover's series inductance method discussed on page 403.

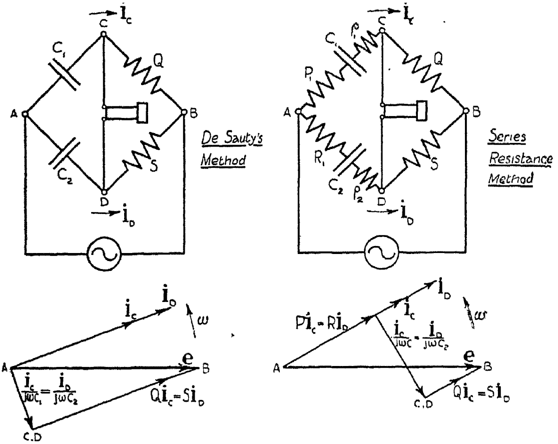

FIG. 114.—DE SAUTY'S METHOD AND THE SERIES RESISTANCE METHOD FOR COMPARING TWO CONDENSERS

It is pointed out on page 83 that De Sauty's bridge has the greatest sensitivity when $C_1 = C_2$ and, therefore, $Q = S$. With a vibration galvanometer as detector, the frequency should be low for great sensitiveness ; hence, if C_1 and C_2 are small, the impedances of the branches in which they lie will be very large. Hence, when condensers are tested at low frequencies (say, less than 0·1 microfarad at 100 cycles per second or lower), the ratio branches should have a high resistance, and both source and detector should be of high impedance. (It is often convenient in such a case to connect the source and detector to the bridge through step-up transformers.) This rule is in agreement with Glazebrook's* analysis of the bridge when used ballistically.

Since high-value resistors for Q and S may have considerable capacitive residuals they may with advantage be replaced by air condensers. De Sauty's bridge for small condensers is then a four-condenser arrangement (see also p. 372). Griffiths† has described a bridge on this principle in which C_2 is a fixed air standard; Q and S are replaced by two "ganged" variable air condensers, balance being secured by alteration of their ratio by a single adjustment. By this means a range from 0·00003 to 1 μF. can be read on a single scale, and by suitable design of the condensers approximately constant percentage accuracy can be secured at all parts of the scale. The apparatus is made by Messrs. H. Sullivan & Co.

Experimental Example. In a test at 1,500 cycles per second, C_1 and C_2 were variable air condensers, the maximum values of which were to be compared. Q was fixed at 10,000 ohms, S being adjusted for balance. To allow for the effects of earth capacitance, each condenser was provided with a shield, that of C_1 being joined to the point C and that of C_2 to D. In addition the Wagner earthing device (*see* Fig. 185) was applied, z_5 being a variable air condenser (also shielded) and z_6 a variable resistance. Successive adjustment of the main bridge (by varying S) and of the auxiliary bridge (by varying z_5 and z_6) gave balance. The detector was a high resistance telephone. Then with $C_2 = 1{\cdot}240\ m\mu\text{F}$, S was found to be 7,770 ohms, so that the apparent value of C_1 is $0{\cdot}96_4\ m\mu\text{F}$. Removing the lead joining C_1 to the branch point A, balance was again obtained by setting $C_2 = 0{\cdot}16_0\ m\mu\text{F}$, S and the auxiliary bridge being successively adjusted as before, giving $S = 2{,}000$ ohms. Hence the capacitance of the leads connected to C_1 is $0{\cdot}03_2\ m\mu\text{F}$, so that $C_1 = 0{\cdot}93_2\ m\mu\text{F}$.

8. Series Resistance Method. Although De Sauty's method is apparently so simple, it will be found difficult in practice to secure a sharp balance, except when air condensers are being tested. If the condensers have solid dielectrics there will inevitably be dielectric losses therein, and the condensers can no longer be considered perfect. What is done in ordinary rough tests is to adjust Q or S until the indication of the detector is a minimum, and when the two condensers are of similar quality a reasonably good balance can be obtained. If the losses be very different in the two condensers, the minimum is by no means small and is very badly defined; hence accurate settings cannot be obtained.

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* R. T. Glazebrook, "On a method of comparing the electrical capacities of two condensers," *Proc. Phys. Soc.*, Vol. 4, pp. 207–214 (1881). Also *see* F. E. Kester, "The bridge method for comparison of condensers," *Phys. Rev.*, Vol. 24, pp. 120–121 (1907).

† W. H. F. Griffiths, "A wide range portable capacity test set," *Journal Sci. Insts.*, Vol. 7, pp. 199–203 (1930).

The phase displacement between the voltage applied to an imperfect condenser and the current through it is less than $\pi/2$ by an angle θ, called the loss angle of the condenser. Then, as shown on page 190, the imperfect condenser can be replaced by a perfect condenser C, in series with a resistance ρ to account for the losses and to give the same loss angle, $\theta = \arctan \omega C\rho$.

The unsuitability of the above-mentioned minimum balance in accurate work is best realized by the examination of a numerical example.* Suppose C_1 is a paper condenser of 1 microfarad having $\theta_1 = 30'$ corresponding to $\rho_1 = 14{\cdot}545$ ohms. A mica condenser $C_2 = 1{\cdot}05$ microfarad having $\theta_2 = 1'$ and $\rho_2 = 0{\cdot}462$ ohm is used as a standard. S is fixed at 1,000 ohms. A potential difference of 100 volts is applied to the points AB, the frequency being $600/2\pi$. The detector has a resistance of 200 ohms. If the condensers had been perfect, sharp balance and zero current in the detector would have been secured when $Q = 1{,}050$ ohms. Making use of these figures in the equations given on page 60, the current in the detector is found to have the following values as this ideal value of Q is approached, passed through and exceeded.

Q ohms.	Detector current in microamperes
1045·0	20·184
1049·0	14·506
1049·75	14·336
1049·9	14·325
1050·0	**14·323**
1050·1	14·324
1050·25	14·333
1051·0	14·502
1055·0	18·870

From this table it is seen that the current in the detector is never zero, but *the value of Q corresponding to minimum current is the same as if the condensers were perfect.* However, the minimum is so flat that, with a telephone as detector (sensitive, say, to 1 microampere), settings could hardly be made closer than to the nearest ohm, i.e. to about 1 part in 1,000. With a vibration galvanometer the state of affairs is little better, a precision of about 5 in 10,000 being possible.

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* F. W. Grover, "Simultaneous measurement of capacity and power factor of condensers," *Bull. Bur. Stds.*, Vol. 3, pp. 375–376 (1907).

If, however, the loss angles of the condenser branches can be adjusted while, *at the same time*, the ratio of Q to S is varied, the points C and D of De Sauty's bridge can be brought to the same potential *and* phase. This can be secured in a variety of ways, one of the simplest being to join in series with the condenser which has the lesser losses an adjustable resistance of known value. The bridge is then referred to as a *Series Resistance Method* (Fig. 114).

Since it may not be known beforehand which of the two condensers has the smaller dielectric losses, it is usual, in practice, to join a resistance in series with each condenser. Thus, C_1, ρ_1, and C_2, ρ_2, are the two imperfect condensers to be compared, adjustable resistances P_1 and R_1 being connected in series with them as shown. If Q and S be non-inductive resistances, $P = P_1 + \rho_1$ and $R = R_1 + \rho_2$, balance will be secured when

$$S\left(P + \frac{1}{j\omega C_1}\right) = Q\left(R + \frac{1}{j\omega C_2}\right)$$

i.e.
$$C_1/C_2 \quad = S/Q = R/P.$$

The practical process is to make successive adjustments of Q or S and P_1 or R_1 until true balance is attained.

If the loss angle θ_2 of the condenser C_2 be known, this bridge provides a ready means of finding the loss angle θ_1 of C_1. From the balance condition,

$$C_1(P_1 + \rho_1) = C_2(R_1 + \rho_2),$$

whence
$$\omega C_1\rho_1 - \omega C_2\rho_2 \quad = \omega C_2R_1 - \omega C_1P_1$$

i.e.
$$\tan\theta_1 - \tan\theta_2 \quad = \omega C_2\left(R_1 - \frac{S}{Q}P_1\right).$$

Since the loss angles are usually small, this equation is, very approximately,

$$\tan(\theta_1 - \theta_2) \doteqdot \theta_1 - \theta_2 \doteqdot \omega C_2\left(R_1 - \frac{S}{Q}P_1\right),$$

from which θ_1 can be found.

Experimental Example. The following results were obtained at a frequency of 407·1 cycles per second, the detector being a Duddell vibration galvanometer. C_1 was a paper condenser, the capacitance and series resistance of which were to be determined; the resistance P_1 was omitted. The branch AD contained a mica condenser of capacitance $C_2 = 0{\cdot}334_6\ \mu\text{F}$, having a series resistance of 0·30 ohms; R_1 was a low reading variable resistance. Q was a resistance fixed at various values, balance being obtained by successive adjustments of S and R_1, as shown in the table.

Q ohms.	S ohms.	R ohms.	C_1 μF.	ρ_1 ohms.
4000	4785	$5{\cdot}9_0 + 0{\cdot}3_0$	$0{\cdot}400_3$	$5{\cdot}1_7$
2000	2392	$5{\cdot}9_0 + 0{\cdot}3_0$	$0{\cdot}400_2$	$5{\cdot}1_7$
1000	1196	$5{\cdot}9_0 + 0{\cdot}3_0$	$0{\cdot}400_2$	$5{\cdot}1_7$
500	598·4	$5{\cdot}9_0 + 0{\cdot}3_0$	$0{\cdot}400_4$	$5{\cdot}1_7$
200	239·4	$5{\cdot}9_0 + 0{\cdot}3_0$	$0{\cdot}400_4$	$5{\cdot}1_7$
		Average	**$0{\cdot}400_3$** μF.	**$5{\cdot}1_7$** ohms

For the standard condenser the loss angle is given by

$$\omega C_2\rho_2 = \tan\theta_2 = 0{\cdot}00025_7 \doteqdot \theta_2 \doteqdot 53'' ;$$

hence for the condenser under test,

$$\tan\theta_1 = \omega C_2 R_1 + \tan\theta_2 = 0{\cdot}0052_9 \doteqdot \theta_1 \doteqdot 18'.$$

As in the case of De Sauty's bridge, and following the usual principle, it is best to arrange for an equal ratio network; possibility of error is thereby minimized. The method is subject to several sources of error,[*] which must now be briefly noticed, as it is particularly necessary to take precautions to eliminate errors when tests are being made on condensers in which θ is very small and also in the case of capacitance measurements on condensers less than about 0·1 microfarad.

Residual Inductance in Q and S. If λ and μ be the residuals in Q and S, it is easy to show that the balance conditions become

$$\frac{C_1}{C_2} = \frac{S}{Q}\left[1 + \frac{\omega^2\mu C_1 P}{S} \quad \omega^2\lambda C_2 R\right]$$

and $$\tan(\theta_1 - \theta_2) = \omega C_2 R_1 - \omega C_1 P_1 + \omega\left(\frac{\lambda}{Q} - \frac{\mu}{S}\right).$$

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[*] For a full discussion of the sources of error and for applications of the bridge to tests on condensers, *see* F. W. Grover, *loc. cit.*, pp. 378–389 (1907); H. L. Curtis, "Mica condensers as standards of capacity," *Bull. Bur. Stds.*, Vol. 6, pp. 431–488 (1910); F. W. Grover, "The capacity and phase difference of paraffined paper condensers as functions of temperature and frequency," *Bull. Bur. Stds.*, Vol. 7, pp. 495–578 (1911); K. W. Wagner, "Zur Messung dielektrischer Verluste mit der Wechselstrombrücke," *Elekt. Zeits.*, Vol. 32, pp. 1001–1002 (1911); "Die Messung der dielektrischen Ableitung und Kapazitäten mehradrigen Kabel mit Wechselstrom," *Elekt. Zeits.*, Vol. 33, pp. 635–637 (1912). H. J. Macleod, "The variation with frequency of the power loss in dielectrics," *Phys. Rev.*, Vol. 21, pp. 52–73 (1923). W. Hubmann, "Dielektrische Messungen an einen Cellonkondensator bei mittleren Frequenzen und Niederspannung," *Arch. f. Elekt.*, Vol. 20, pp. 371–373 (1928). C. T. Burke, "Bridge methods for measurements at radio frequencies," *G.R. Exp.*, Vol. 7, pp. 1–6 (July, 1932). R. F. Field, "Measurements at low frequencies with the radio frequency bridge," *G.R. Exp.*, Vol. 10, pp. 3–7 (Mar., 1936).

Grover works out a case in which $C_1 = C_2 = 1$ microfarad, $\rho_1 = 49$ ohms, $\rho_2 = 5$ ohms, $Q = S = 1{,}000$ ohms. λ is taken as $-0{\cdot}00004$ henry and μ as $-0{\cdot}0015$ henry, i.e. residual capacitance is the preponderant residual. With $P_1 = 1$ ohm and $R_1 = 45$ ohms, balance is attained at a frequency of 100 cycles per second. With these figures the correction to be applied to the capacitance ratio is only 3 parts in 10^5. On the other hand, $\tan(\theta_1 - \theta_2) = 0{\cdot}0276 - 0{\cdot}00092$, an error of over 3 per cent.

This example serves to show that not only should an equal ratio bridge be arranged, but the branches Q, S should be similar in make-up, so that they have similar residuals. If then a second setting be made with them reversed in the bridge, the mean of the two balances will be little affected by residual errors. The coils should be well adjusted so that error in the ratio of Q and S in inappreciable. The standard C_2 usually has smaller losses than C_1; hence P_1 can be omitted, R_1 being added to degrade the standard to the level of the unknown. The above conditions then may be written

$$C_1/C_2 \doteqdot S/Q$$

and

$$\theta_1 - \theta_2 \doteqdot \omega C_2 R_1 + \omega\left(\frac{\lambda}{Q} - \frac{\mu}{S}\right),$$

where $C_1 \doteqdot C_2$ and $S \doteqdot Q$. Interchanging Q and S let balance be restored by adjusting one or both of them and R_1, all changes being very small. Let Q', S', λ', μ' and R_1' be the new values. Then

$$\theta_1 - \theta_2 \doteqdot \omega C_2 R_1' - \omega\left(\frac{\lambda'}{Q'} - \frac{\mu'}{S'}\right).$$

But $\lambda/Q \doteqdot \lambda'/Q'$ and $\mu/S \doteqdot \mu'/S'$ very closely;

hence by addition of the two balances,

$$2(\theta_1 - \theta_2) \doteqdot \omega C_2(R_1 + R_1'),$$

which eliminates the residuals, as shown by Kind.*

Residuals in P_1 and R_1. Assuming an equal ratio bridge, the above equation shows that for a given value of $\theta_1 - \theta_2$ the difference between R_1 and P_1 will be larger the smaller the value of the capacitance becomes. Since in small condensers quite large values of θ are found, this may necessitate the inclusion of high resistances in series with the condensers. It has been shown on page 89 that such high resistances may have a not inconsiderable equivalent capacitance, represented by a condenser joined in parallel with the resistance. The introduction of these capacitances into the condenser branches will, therefore, introduce an error in the capacitance ratio and in the determination of loss angles.

Grover (*loc. cit.*) shows that the correction due to the capacitance of the series resistances is negligible when the condensers under comparison are large, and, in general, will be unimportant whatever the value of the condensers, provided that they have nearly equal capacitances and loss angles. With small condensers (< 0·001 microfarad) of quite different power factors the corrections may become extremely important. It is with the object of overcoming this source of error that the Series Inductance method (p. 403) was suggested by E. B. Rosa.

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* H. Kind, "Vergleich von Kondensatoren," *E.u.M.*, Vol. 52, p. 292 (1934).

Earth Capacitances. Particularly when testing small condensers, the capacitances of the various branches to earth may prove troublesome. Assuming that balance can be secured (in this connection, *see* p. 528), the method of substitution removes almost completely any troubles due to this cause.

Neglecting residuals, etc., the process is as follows: Let balance be secured with the unknown condenser in the bridge. Then $C_1/C_2 = S/Q$ and $\tan\theta_1 - \tan\theta_2 = \omega C_2\left(R_1 - \frac{S}{Q}P_1\right)$, the two condensers being as nearly equal as possible, so that a symmetrical network is obtained. Now replace the unknown by a standard condenser C'_1 of nearly equal value and loss angle θ_1', re-balancing the bridge by adjustments of P_1 to P_1' and Q to Q', all else being undisturbed. Then $C'_1/C_2 = S/Q'$ and $\tan\theta'_1 - \tan\theta_2 = \omega C_2\left(R_1 - \frac{S}{Q'}P'_1\right)$. Subtracting the tangent formulae and dividing the capacitance ratios gives

$$C_1 = C'_1Q'/Q \text{ and } \tan(\theta_1 - \theta'_1) = \omega C'_1\left(P'_1 - \frac{Q'}{Q}P_1\right),$$

so that C_2 need not be known. Since the arrangement of the bridge is not disturbed between the taking of the two balances—except for the small alterations of P_1 and Q—earth capacitance effects are eliminated. Moreover, the effects of residuals in the branches are also eliminated by the process, so that by using such a "dummy balance" on an equal ratio bridge, settings of very high precision may be obtained.

Wagner and Wertheimer* have also used the series resistance method for the purpose of measuring the time-constants of medium value or high resistance coils. As the time-constant is usually a very small quantity, of the order of 10^{-8} second in coils constructed for bridge work, it is necessary to set up the network in a rather special manner. The condensers C_1 and C_2 are air condensers, so that $\rho_1 = \rho_2 = 0$; C_1 is adjustable in value. P_1 and R_1 are resistance boxes. Q is the coil whose time-constant T_Q is to be found, and S is a standard resistance of a value about equal to the resistance of Q. With the bridge arranged in this way, balance is secured by adjustments of C_1 and P_1. The resistance Q is then removed and replaced by a standard of equal resistance whose time-constant can be calculated from its dimensions, T, say. Re-balance by adjustment of P_1 and C_1 to values P'_1, C'_1. Then it is easy to show that $T_Q = T + PC_1 - P'C'$. It is essential to make allowance for earth capacitances in measurements on such small quantities (*see* p. 542 for the method adopted).

Applications. Walcher† has shown that the conjugate series resistance bridge is very suitable for testing electrolytic condensers, which require a d.c. forming voltage in addition to the a.c. test voltage. In Fig. 114 the alternating voltage is applied to the corners CD and the vibration galvanometer to AB. The polarizing battery is connected to

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* K. W. Wagner and A. Wertheimer, "Über Präzisionswiderstände für hoch-frequenten Wechselstrom," *Elekt. Zeits.*, Vol. 34, pp. 613–616 (1913). K.W. Wagner, *ibid.*, Vol. 36, pp. 606–609 (1915).

† T. Walcher, "Die Methoden der Kapazitätsmessung (Messeinrichtungen und ihre praktische Verwendung)," *E.u.M.*, Vol. 52, pp. 360–366 (1934).

AB and the condensers prevent d.c. flowing in Q and S; d.c. is excluded from the vibration galvanometer by putting a condenser in series with it. The bridge can easily cover a range from 1 to 1,000 μF.

Another interesting application is due to Fehr and Hubmann* who have made a slight modification enabling tests to be made with simultaneous application of currents with frequencies of 50 and 10^6 cycles per second. To do this, the branch AC contains two similar parallel branches each of which consists of an inductor and a test condenser in series. The h.f. supply is applied to the junction of inductor and condenser in each branch; h.f. current flows through the condensers in series and is, by virtue of the symmetry of this auxiliary bridge, excluded from the main bridge. The latter is supplied at 50 cycles per second, and passes l.f. current into the test condensers in parallel; condensers in the h.f. leads exclude the l.f. from the h.f. supply.

9. Parallel Resistance Method. The parallel resistance method illustrated in Fig. 115 is sometimes of service for the comparison of fairly large condensers. Wien† was the first to describe alternating current measurements with the bridge. The two condensers C_1, C_2 are shunted by resistances P, R; Q and S are, as before, the ratio branches. The branch impedance operators are $z_1 = 1\Big/\left(\frac{1}{P} + j\omega C_1\right)$, $z_2 = Q$, $z_3 = S$, $z_4 = 1\Big/\left(\frac{1}{R} + j\omega C_2\right)$; and balance is obtained when

$$S\Big/\left(\frac{1}{P} + j\omega C_1\right) = Q\Big/\left(\frac{1}{R} + j\omega C_2\right),$$

i.e.

$$C_1/C_2 = S/Q = R/P.$$

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* W. Fehr and W. Hubmann, "Leitfähigkeits und Verlustwinkelmessungen an Papierkondensatoren bei zusätzlicher Belastung mit Hochfrequenzströmen," *Elekt. Zeits.*, Vol. 54, pp. 819–822 (1933).

† Max Wien, *loc. cit.*, pp. 696–697 (1891). *See* also J. Hanauer, *loc. cit.* (1898); and J. Cauro, "Sur la capacité électrostatique des bobines, et son influence dans la mésure des coefficients d'induction par le pont de Wheatstone," *Comptes Rendus*, Vol. 120, pp. 308–311 (1895), for examples of the use of the method. For the parent ballistic bridge, *see* W. E. Sumpner, *loc. cit.* (1888), and E. C. Rimington, "Note on comparing capacities," *Proc. Phys. Soc.*, Vol. 9, pp. 60–67 (1888). For an application of the bridge, *see* A. Campbell, *Proc. Roy. Soc.*, Vol. 78, p. 196 (1906). Also see the following: T. Walcher, "Eine kombinierte Wechselstrommessbrücke," *E.u.M.*, Vol. 50, pp. 518–523 (1932). R. Bauder and K. Jahnnsen, "Die Messung der Kapazität handelsüblicher Elektrolytkondensatoren und Untersuchung ihrer Abhängigkeit und Veränderlichkeit," *E.u.M.*, Vol. 50, pp. 581–586 (1932). T. Walcher, *E.u.M.*, Vol. 52, pp. 360–366 (1934). Special uses are described by W. L. Beck and G. F. Shotter, "Cable fault localization," *Elec. Rev.*, Vol. 113, pp. 419–420 (1933). M. A. B. Brazier, "A method for the investigation of the impedance of the human body to an alternating current," *Journal I.E.E.*, Vol. 73, pp. 204–209 (1933). E. W. Smith, "The characteristics of submarine telephone cables at carrier frequencies," *Journal I.E.E.*, Vol. 73, pp. 213–227 (1933).

The resistances Q or S and P or R are successively adjusted until the indication of the detector is reduced to zero.

The construction of the vector diagram is obvious. When the bridge is balanced the currents passing the branch points C and D are clearly in phase with one another and lead on the voltage $\mathbf{e}$ applied to the points A, B. Subtracting from $\mathbf{e}$ the vector $Q\mathbf{i}_C = S\mathbf{i}_D$ gives the vector $\mathbf{e}_1$, which represents the voltage across the branches AC and AD. The current through P is $\mathbf{e}_1/P$ and through the condenser C_1 is $j\omega C_1\mathbf{e}_1$, their sum being the current $\mathbf{i}_C$. Similarly, the sum of $\mathbf{e}_1/R$ and $j\omega C_2\mathbf{e}_1$ is equal to $\mathbf{i}_D$. It is easy, from a comparison of the sides of the vector triangles, to verify the above balance conditions.

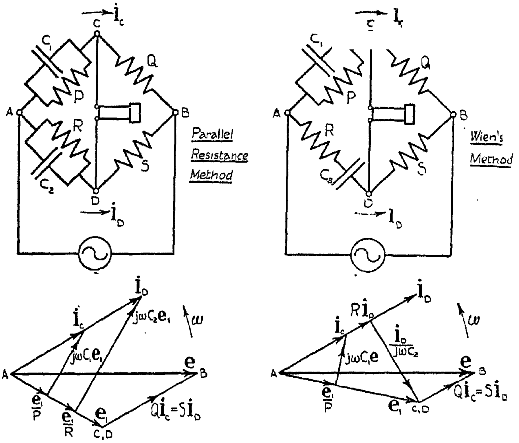

FIG. 115.—PARALLEL RESISTANCE METHOD AND WIEN'S METHOD FOR COMPARING TWO CONDENSERS

From the vector diagram, if ωC_1 and ωC_2 are small compared with $1/P$ and $1/R$, then the condensers contribute only a small component to the currents passing the branch points, $\mathbf{i}_C$ and $\mathbf{i}_D$ coming more nearly in phase with $\mathbf{e}$. Hence if ω, P, and R be fixed, C_1 and C_2 must be large for the method to be sensitive to their presence in the network.

It has been shown on page 190 that the effects of absorption in an imperfect condenser may be represented by a perfect condenser in series with a resistance. Alternatively, it may be represented by a condenser in parallel with a resistance, the loss angle in the two cases being the same. Grover* has shown how the parallel resistance method may be used to find the equivalent capacitance and shunt resistance of an imperfect condenser. In general, the fictitious resistance in parallel with a condenser in which there are dielectric losses will have a very high value; hence, the balancing resistance in parallel with the standard may be inconveniently large. To avoid the use of such large resistances, an artifice due to Wien may be employed. Let C_1, C_2 be the unknown and standard capacitances, their effective shunt resistances being W_1 and W_2. In parallel with C_1 put a resistance P_1 of a value relatively low compared with W_1, say 100,000 ohms or so, according to the circumstances. An adjustable resistance box, R_1, in parallel with C_2, will then be of a reasonable value. The values of C_1 and C_2 should be approximately equal, and the bridge balanced by adjustment of R_1 and Q/S.

Using this notation, $P = 1\Big/\left(\frac{1}{P_1}+\frac{1}{W_1}\right)$ and $R = 1\Big/\left(\frac{1}{R_1}+\frac{1}{W_2}\right)$

From the balance conditions

$$C_1/C_2 = R/P = \left(\frac{1}{P_1}+\frac{1}{W_1}\right)\Big/\left(\frac{1}{R_1}+\frac{1}{W_2}\right),$$

or

$$\frac{1}{P_1C_1}+\frac{1}{W_1C_1}=\frac{1}{R_1C_2}+\frac{1}{W_2C_2}.$$

Multiplying by $1/\omega$, and remembering that the loss angle of a condenser C in parallel with a resistance W is given by $\tan\theta = 1/\omega CW$, the relation becomes

$$\tan\theta_1 - \tan\theta_2 = \frac{1}{\omega C_2R_1}-\frac{1}{\omega C_1P_1}.$$

Since the resistances P_1, R_1 are usually large, they will have self-capacitances c_1 and c_2 in parallel with them, and hence the second balance condition is

$$(C_1+c_1)/(C_2+c_2) = S/Q.$$

When the unknown and standard condensers are similar, c_1 and c_2 will be nearly equal, and will be small, since P_1 and R_1 will be nearly equal. Error due to them can then be neglected.

The method is subject to the same sources of error as the series resistance method, and accurate settings can be obtained by the use of the process of substitution.*

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* *Loc. cit.*, pp. 393–396 (1907).

Experimental Example. An experiment was made at 407·1 cycles per second on the condensers previously tested by the series resistance method (*see* p. 332). C_2 had a capacitance of $0{\cdot}334_6$ μF and an effective shunt resistance of $W_2 = 4{\cdot}56$ megohm. The condensers C_1 and C_2 were shunted by resistances P_1 and R_1 respectively, P_1 being fixed at 1,000 ohms. Balance was secured by successive adjustments of Q, S, and R_1, the values obtained being $Q = 795{\cdot}0$ ohms, $S = 951{\cdot}5$ ohms, $R_1 = 1190{\cdot}5$ ohms. From these results

$$R = 1\Big/\left(\frac{1}{R_1} + \frac{1}{W_2}\right) = 1190{\cdot}2 \text{ ohms},$$

hence $$P = QR/S = 994{\cdot}5 \text{ ohms}.$$

Remembering that $P = 1\Big/\left(\frac{1}{P_1} + \frac{1}{W_1}\right)$, gives $W_1 = 0{\cdot}181$ megohm. Again, $C_1 = SC_2/Q = 0{\cdot}400_5$ μF . From the equations on page 190, the series loss resistance ρ_1 equivalent to W_1 is $\rho_1 = W_1/(1 + \omega^2 C_1^2 W_1^2)$ or $5{\cdot}2_6$ ohms, which should be compared with the value found by the series resistance bridge.

Applications. Ford and Reynolds† have described two precision types of parallel resistance bridge, one for tests between 60 and 1,000 cycles per second, and the other in the range 12 to 1,500 kilocycles per second; connections for the audio-frequency bridge are shown in Fig. 116 (*a*), from which it will be seen that the conjugate of Fig. 115 is used. The doubly-shielded ratios Q, S and the fixed resistor R are 100,000 ohms each; P is a shielded decade box reading to 111,110 ohms. The condensers C_1, C_2 are 1,500 $\mu\mu$F. precision variable air condensers. Principles of the shielding arrangements are discussed in Chapter V. Let $G_P = 1/P$, $G_R = 1/R$ and $G_X = 1/W_X$ be the respective conductances. With *Sw* opened, let balance be secured by adjusting C_1 and P, C_2 being set at a convenient value. Then since $Q = S$

$$G_P + j\omega C_1 = G_R + j\omega C_2$$

Closing *Sw* balance by settings C_1' and G_P', then

$$(G_P' + G_X) + j\omega(C_1' + C_X) = G_R + j\omega C_2$$

Subtracting,

$$G_X = G_P - G_P' = \Delta G_P$$

and $$C_X = C_1 - C_1' = \Delta C_1$$

The radio-frequency bridge is more elaborately shielded and is balanced in a similar way by the use of a differential condenser (*see* Section 9*a*).

Berberich‡ has described a modification of the parallel resistance

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* The theory of parallel substitution in a fully shielded bridge with Wagner earthing device has been studied by R. P. Siskind, "Power factor measurement by the capacitance bridge," *Trans. Amer. I.E.E.*, Vol. 51, pp. 214–221 (1932).

† W. A. Ford and S. I. Reynolds, "Alternating current bridges for measurements of electrical insulating materials," *Gen. Elec. Rev.*, Vol. 36, pp. 99–105 (1933).

‡ L. J. Berberich, "The measurement of the phase angle of shielded resistors," *Physics*, Vol. 3, pp. 296–313 (1932).

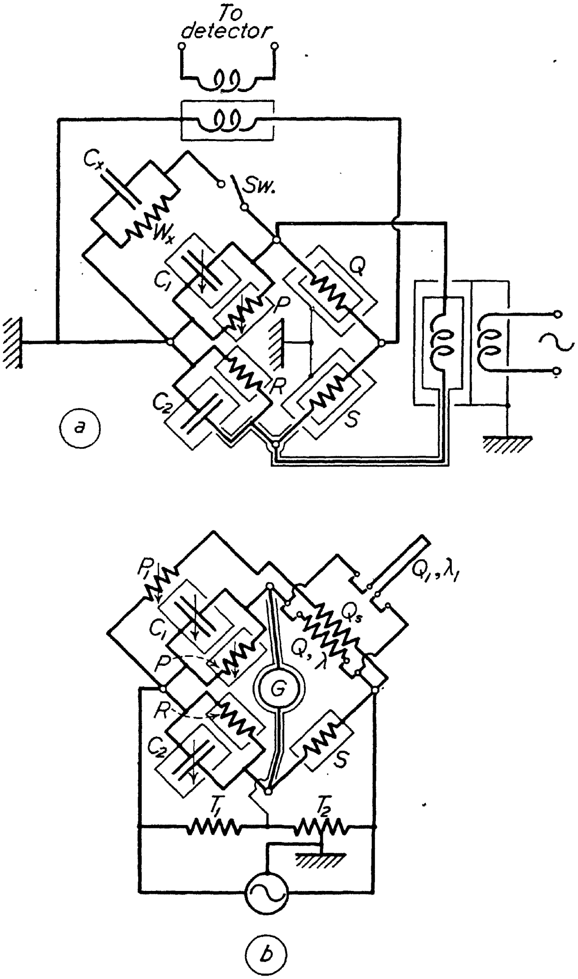

FIG. 116.—FORD AND REYNOLDS'S SHIELDED BRIDGE
BERBERICH'S BRIDGE FOR SHIELDED RESISTORS

bridge to measure the phase-angle of shielded resistors of 1,000 ohms or more, with particular reference to the type in which each element of the resistor is in proximity to an element of a shield at the same potential (see *Instrument Transformers*, pp. 361 and 369; also p. 531 of this book). The circuit is shown in Fig. 116 (*b*), where Q is the resistor of which the phase-angle is to be found and Q_s is its resistance shield; a milliammeter (not shown) could be connected at will in Q or S to enable the currents in them to be regulated to their correct values, the former by adjusting the bridge voltage and the latter by the resistor P_1. By means of the links, Q can be removed from the bridge and replaced by a parallel-wire standard Q_1 of equal resistance and calculable reactance. The earthing device consists of two equal resistors T_1, T_2, the latter earthed at its mid-point; this ensures that at balance the mid-point of Q_1 is at earth potential and that the detector lead and its shield are at equal potentials. When Q is 10,000 ohms the arms S and R are fixed at 10,000 ohms; P is a decade box up to 10,000 ohms; T_1 and T_2 are 1,000 ohms each. The mid-point of the supply is earthed. When Q has any other value S is made equal to it and $T_2 = S/10$, all else being unchanged. The air condenser C_1 reads up to 250 $\mu\mu$F., C_2 being merely to put the reading of C_1 at a convenient part of the scale. Balance is obtained by adjusting P and C_1 first with Q_1 in place and then with Q. Let the values be P, C_1 and P', C_1' respectively. Then if λ_1 is the residual of Q_1, and λ of Q, it is easy to show that

$$\lambda = P'[(\lambda_1/P) + C_1Q_1 - C_1'Q]$$

Time-constants as low as 10^{-8} seconds were satisfactorily measured. The bridge can easily be adapted to test mid-point shielded resistors, i.e. those in which the resistor is contained within a shield box maintained at a potential midway between the terminal voltage of the resistor. All that is necessary is to omit P_1 and Q_s, and to join the shield surrounding Q to the earth point on T_2.

9a. Uses of Differential Condensers. As shown above, several of the methods described in Sections 7, 8, and 9 are used on a difference principle, i.e. the capacitance of an unknown condenser is determined by the difference between two settings of a standard condenser before and after the unknown is inserted in an equal ratio bridge. When the capacitance to be measured is small, this capacitance unbalance is most easily measured by a differential air condenser which adds capacitance to one bridge arm and subtracts an equal amount from the adjacent arm. It usually consists of a bank of semicircular moving plates which can be rotated into the spaces between two sets of semicircular fixed plates; a range of about 600 $\mu\mu$F. each side of a central zero is a very usual value. Alternatively, two ordinary air condensers may be "ganged" together on a common shaft in such a way that one increases and the other decreases in capacitance by equal amounts.

Fig. 117 (*a*) shows the simplest arrangement as applied to the comparison of two nearly equal condensers C_1, C_2 in an equal ratio de Sauty bridge* with $Q = S$; a common application is to the measurement of unbalance capacitance in telephone cables. Source and detector are omitted for simplicity. When balance is secured,

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* R. A. Braden and H. C. Forbes, "A condenser bridge for factory inspection of variable condensers," *Proc. Inst. Rad. Eng.*, Vol. 18, pp. 123–136 (1930), applies such a method to the intercomparison of tuning condensers in radio sets.

$$C_1 + c_1 = C_2 + c_2 \text{ or } C_1 - C_2 = c_2 - c_1,$$

the condenser being calibrated to read $c_2 - c_1$ directly. Rosen* has shown that the range of a given differential condenser can be reduced to any desired value by the arrangement of Fig. 117 (*b*). Converting the star, composed of c, c_1, and c_2 into an equivalent mesh, the branches of the latter will be $c_1c_2/(c_1 + c_2 + c)$, $c_2c/(c_1 + c_2 + c)$, and $c_1c/(c_1 + c_2 + c)$ across CD, DA, AC respectively (see p. 58). The first shunts the points C and D, with no effect on the balance. Balance occurs when

$$C_1 - C_2 = c(c_2 - c_1)/(c_1 + c_2 + c) = c(c_2 - c_1)/(C + c),$$

where $C = c_1 + c_2$ is the sum of the differential capacitances and is constant. Hence the original scale readings must be multiplied by $c/(C + c)$; for example, if $C = 600$ μμF. then $c \doteq 66\frac{2}{3}$ μμF. will give a scale factor of 1/10.

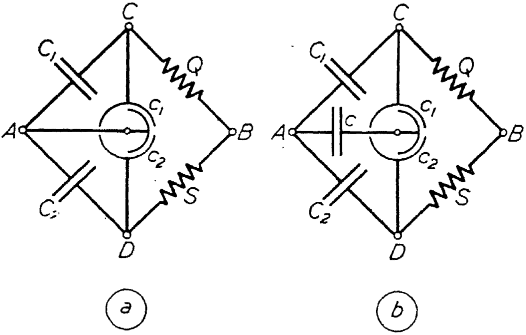

Fig. 117.—Differential Condensers

In a network with two or more terminals the capacitance measured between any pair of terminals includes not only the inter-capacitance or direct capacitance between those terminals but also a series-parallel arrangement of the remaining inter- and earth-capacitances (*see* pp. 9–13). In order to determine the n earth-capacitances and $\frac{1}{2}n(n - 1)$ inter-capacitances of n electrodes the straightforward method is to make $\frac{1}{2}n(n + 1)$ independent measurements (by suitable linking or earthing of terminals) to provide the requisite number of relationships (*see* p. 185). Similar considerations apply to the measurement of the leakage conductances with which, in general, each capacitance is associated.

In many practical cases it is very desirable to be able to measure by a more rapid process the direct capacitance between a pair of electrodes

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* A. Rosen, "A note on differential condensers and resistors," *Journal P.O.E.E.*, Vol. 29, pp. 319–321 (1937).

(inter-capacitance) or that between an electrode and earth (earth capacitance). Such instances occur, e.g. in finding the grid, anode, and filament partial admittances in a vacuum tube, or in measuring the earth and inter-capacitances in a plate condenser with earthed shield or the inter-core and core-earth admittances of a cable. Two simple arrangements have been described, applied primarily to two-electrode systems, but capable of extension to more complex cases.

Fig. 118 shows the bridge described by Hoch* and used in the Bell Telephone Laboratories, New York, for tests on vacuum tubes. The branches Q and S are equal shielded ratio arms; R is a fixed resistor, and P a variable decade box, both being shielded. In the usual case all the resistors are approximately 10,000 ohms each. The condenser C is of three-electrode construction, arranged so that as the capacitance shunting R diminishes that across P correspondingly increases; it may, alternatively, consist of two ordinary shielded air condensers with the rotors geared together to produce the same result. The scale of C is calibrated so that positive readings balance capacitances shunted across R and negative readings those shunted across P.† Let C_x be the direct capacitance to be measured between two electrodes a, b, C_a, C_b being their earth capacitances. With the shielded key K on contact 1, C_b is short-circuited and $C_x + C_a$ is put in parallel with R. Let balance be restored by adjusting C and P to values C_1 and P_1, then

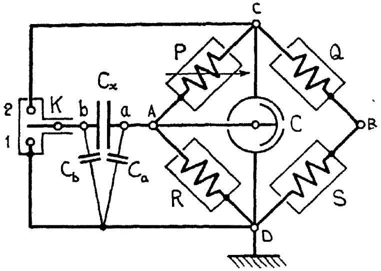

FIG. 118.—HOCH'S BRIDGE FOR DIRECT CAPACITANCE MEASUREMENTS

$$C_1 = C_x + C_a \text{ and } 1/P_1 = G_x + G_a,$$

where G_x and G_a are the leakage conductances of C_x and C_a. Now with the key on contact 2, C_x goes in parallel with P, C_a shunts R, while C_b shunts the detector terminals CD. As C_x now goes on the opposite side we may take it as negative relative to C_a when balance is secured by adjustment of C, P to values C_2, P_2; then

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* E. T. Hoch, "A bridge method for the measurement of inter-electrode admittance in vacuum tubes," *Proc. Inst. Rad. Eng.*, Vol. 16, pp. 487–493 (1928). Also see G. A. Campell, *Bell Syst. Tech. J.*, Vol. 1, pp. 18–38 (1922).

† The source and detector are joined to the bridge through shielded transformers to AB, CD respectively. These, and the additional shielding they add to the ratio arms Q, S, have been omitted to simplify the diagram.

$$C_2 = -C_x + C_a \text{ and } 1/P_2 = -G_x + G_a.$$

From these two conditions,

$$C_x = (C_1 - C_2)/2 \text{ and } G_x = (R_2 - R_1)/2R_1R_2.$$

The corresponding sums give C_a if required and the rearrangement to get C_b will be obvious to the reader.

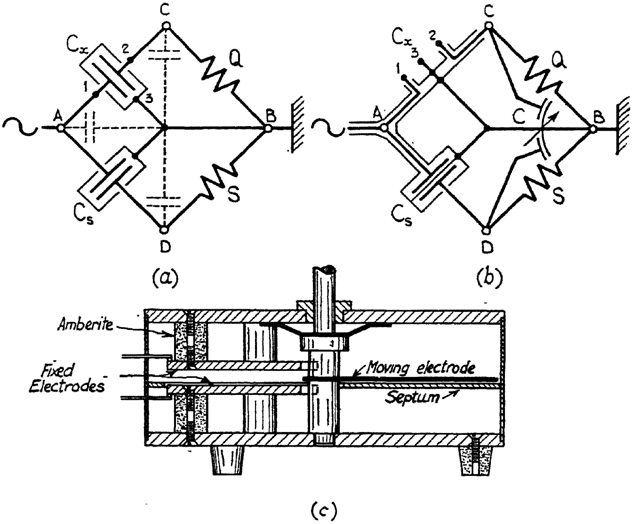

FIG. 119.—ZICKNER'S BRIDGE FOR DIRECT CAPACITANCE MEASUREMENTS

Another method introduced by Zickner* at the Reichsanstalt is shown in principle by Fig. 119 (*a*). In this diagram let it be required to find C_x, the capacitance between electrodes 1 and 2, 3 denoting the surrounding floating earthed shield or neighbouring earthed objects. The corner B is earthed, the source and detector being omitted from the diagram for simplicity. Earth capacitance from 1 will shunt the source branch-points AB, while that from 2 shunts Q. Similarly if C_s is the inter-electrode capacitance of a standard condenser with floating earthed shield, its earth capacitances shunt AB and S respectively. Hence, C_x and C_s are directly compared in the arms AC and AD, while undesired earth-capacitances, shown dotted, shunt the source or the ratio arms.

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* G. Zickner, "Eine Messbrücke für sehr kleine Kapazitäten," *E.N.T.*, Vol. 7, pp. 443–448 (1930). The bridge and special condenser are made by Dr. Max Ulrich G.M.B.H.

The arrangement of the essential features of the bridge is shown in Fig. 119 (*b*). The ratio Q/S can be set at 2,000/20,000, 11,000/11,000 or 20,000/2,000. Balance of the earth capacitances across Q and S and compensation for the loss angle of C_x is effected by shunting both Q and S with a variable air condenser of $1{,}000\mu\mu$F. maximum capacitance; the rotors of the two condensers are geared together so that while one increases in capacitance the other correspondingly decreases. This arrangement is indicated by C in the diagram. In addition fixed condensers of 1,000, 2,000, 3,000 and $4{,}000\mu\mu$F. are provided and can be combined in any desired way up to a maximum of $10{,}000\mu\mu$F. in parallel either with Q or with S. Shielded terminals 1, 2 are provided for the test object, 3 being the corresponding earthed terminal; C_s is a variable air condenser of appropriate magnitude shielded as shown. The entire apparatus is contained in a box provided with an earthed lining; the detector is a telephone with earthed shield. Balance is secured, after selection of an appropriate value for Q/S, by manipulation of C and C_s, shunting Q or S as may be found necessary with additional fixed capacitance; then $C_x = C_sS/Q$.

The condenser C_s may be of the ordinary two-electrode construction, with one fixed and one rotating set of plates, when fairly large capacitances are to be measured. When very small capacitances are in question, however, this construction is less suitable since it is difficult to reduce the minimum capacitance setting to a sufficiently small value, though Oliver has attained considerable success in this direction (*see* pp. 177 and 178). Zickner has designed a low-value condenser with zero initial capacitance, by using the construction shown in Fig. 119 (*c*). In this both electrodes are fixed, each consisting of a semicircular brass plate insulated from the circular earthed casing by short amberite or quartz pillars. The capacitance is varied by turning a third semicircular plate about a central axis, this plate being in electrical connection with the casing as shown. The inner space of the condenser is divided into two compartments by a thin metal septum in which a semicircular opening is cut at the position of the fixed plates. When the moving plate lies completely between the fixed plates no electric flux can pass between the latter, so that the initial capacitance is zero. The solid insulation lies in the earth capacitances between electrodes and the case, so that the inter-electrode capacitance is strictly loss-free. The condenser is constructed with a maximum reading of $10\mu\mu$F. or $20\mu\mu$F., so that the bridge has a range of $0 - 1\mu\mu$F., $10\mu\mu$F., and $100\mu\mu$F., or $0 - 2\mu\mu$F., $20\mu\mu$F., and $200\mu\mu$F. according to the standard condenser employed. The source is a shielded triode generator, giving about 300 volts for tests on small condensers ($2\mu\mu$F.), 200 volts for medium values ($20\mu\mu$F.), and about 100 volts for the larger values, all at 800 cycles per second.

10. Wien's Method. An important method for comparing two condensers was introduced by Wien,* and is illustrated in

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* *Loc. cit.*, pp. 704–707 (1891). L. Hartshorn, *Beama J.*, Vol. 13, pp. 89–99 (1923). T. Walcher, *E.u.M.*, Vol. 52, pp. 360–366 (1934).

Fig. 115. The unknown condenser C_1 is joined in parallel with a resistance P; the standard C_2 is provided with a series rheostat R, Q and S being ratio coils. The operators are $z_1 = 1\bigg/\left(\frac{1}{P} + j\omega C_1\right)$, $z_2 = Q$, $z_3 = S$, $z_4 = R + \frac{1}{j\omega C_2}$ and the balance equation is

$$S\bigg/\left(\frac{1}{P} + j\omega C_1\right) = Q\left(R + \frac{1}{j\omega C_2}\right).$$

Separating components,

$$\frac{C_1}{C_2} = \frac{S}{Q} - \frac{R}{P},$$

and

$$C_1 C_2 = 1/\omega^2 PR.$$

The vector diagram combines the features of the series resistance method with those of the parallel resistance method. The current in the branches ADB, $\mathbf{i}_D$ leads on the voltage $\mathbf{e}$ applied to the points A and B. The vector difference between $\mathbf{e}$ and $S\mathbf{i}_D$ is the voltage $\mathbf{e}_1 = \left(R + \frac{1}{j\omega C_2}\right)\mathbf{i}_D$, which also acts on the branch AC (since C and D are at the same potential). The current $\mathbf{i}_C$ is in phase with $\mathbf{i}_D$ and is compounded of the vectors of current $\mathbf{e}_1/P$ and $j\omega C_1\mathbf{e}_1$.

The principal use of Wien's bridge is to determine the equivalent capacitance C_1 and shunt resistance W_1 of an imperfect condenser, such as a sample of insulation or a length of cable. The resistance P in the diagram is then equal to W_1; C_2 is an air condenser in series with an adjustable resistance R; Q and S are resistance boxes. Balance is obtained by successive alterations of R and Q or S. Solving the above equations, and putting $P = W_1$, gives

$$C_1 = \frac{S}{Q}\,\frac{C_2}{(1 + \omega^2 C_2{}^2 R^2)}$$

and

$$W_1 = \frac{Q}{S}\,\frac{(1 + \omega^2 C_2{}^2 R^2)}{\omega^2 C_2{}^2 R}$$

Now W_1 is usually a large resistance and C_2 is an air condenser of relatively small capacitance. Hence, the ratio of Q to S will tend to become large. The network will have very unequal branches and will be very susceptible to error, due to unsymmetrically distributed earth capacitances. The presence of such error can be detected by reversing the connections to the alternator and noticing if the balance is disturbed. If the change in the adjustments then necessary to restore balance is small, taking the mean of the two results will largely eliminate the error. It is better, however, to eliminate earth capacitance troubles by the Wagner earthing device described on page 542.

When, as is frequently the case in the measurements to which the method is commonly applied, C_1 is small and becomes comparable with the air condenser C_2, a more symmetrical bridge may be secured by shunting C_1 with a resistance P_1 of a value small compared with W_1. This resistance will have a known self capacitance c_1. Referring to the balance conditions, write $C_1 + c_1$ for C_1 and $1/P = (1/W_1) + (1/P_1)$. Solving for C_1, W_1 gives

$$C_1 = \frac{S}{Q} \cdot \frac{C_2}{(1 + \omega^2 C_2{}^2 R^2)} - c_1,$$

and

$$\frac{1}{W_1} = \frac{S}{Q} \cdot \frac{\omega^2 C_2{}^2 R}{(1 + \omega^2 C_2{}^2 R^2)} - \frac{1}{P_1}.$$

Experimental Example. The condensers compared by the series resistance method (*see* p. 332) and by the parallel resistance method (*see* p. 338) were again tested by the Wien bridge. C_1 is the paper condenser whose shunt loss resistance W_1 is to be determined; it is joined in parallel with a resistance $P_1 = 1000$ ohms. C_2 is the mica standard of $0{\cdot}334_6$ μF.; R is composed of the series loss resistance $\rho_2 = 0{\cdot}30$ ohms and a resistance box R_1, so that $R = R_1 + 0{\cdot}30$. Q is fixed at 400 ohms, balance being found (at 407·1 cycles per second) by successive adjustments of S to 940 ohms and of R_1 to 1147·5 ohms. A Duddell vibration galvanometer was used. Then, from above,

$$P = \frac{Q}{S} \cdot \frac{(1 + \omega^2 C_2{}^2 R^2)}{\omega^2 C_2{}^2 R} = 994{\cdot}57 = 1 \Big/ \left(\frac{1}{W_1} + \frac{1}{P_1}\right),$$

whence $W_1 = 0{\cdot}18_3$ megohm. Also, neglecting the capacitance of P_1, $C_1 = 0{\cdot}400_2$ μF. The series resistance ρ_1 corresponding to W_1 is, from the equation on page 190, $5{\cdot}2_2$ ohm, which checks with the values previously found.

In a second trial with $P_1 = 10{,}000$ ohms, $Q = 500$ ohms, $S = 604{\cdot}3$ ohms, and $R_1 = 119{\cdot}9$ ohms. From these values $W_1 = 0{\cdot}18_9$ megohm, $\rho_1 = 5{\cdot}0_8$ ohms, and $C_1 = 0{\cdot}400_2$ μF., which checks with the above test.

Applications. This method has been used in many researches on the properties of imperfect condensers, particularly to find the loss-angle for the dielectric of a high voltage cable. Hanauer,* in 1898, measured the change in C_1 and W_1 with frequency for a number of solid and liquid dielectrics. Monasch,† in 1907, carried out an important and elaborate investigation of the law connecting the dielectric loss in a cable with the applied voltage. High voltages, ranging from 1,000 to 9,000 volts, were used on the bridge, the detector being an optical telephone. Tests were made on quite small samples of cable, having capacitances of the order of 0·001 microfarad; the standard was a cylindrical air condenser specially constructed to withstand the high voltages used, its maximum capacitance being about 230 $\mu\mu$F. Allowance was made for the capacitance of the resistances and for that of the bridge with respect to its surroundings. The point B was earthed. Wien's method has also been used by Campbell and Eckersley,* in 1910, to test the variation of W_1 with frequency for some common dielectrics.

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* *Loc. cit.*, 1898.

† Bruno Monasch, "Ueber den Energieverlust in Dielektrikum in wechselnden elektrischen Feldern," *Ann. der Phys.*, Vol. 22, pp. 905–942 (1907).

Rosen† has investigated the dielectric losses in single- and multi-core cables at voltages up to 30 kilovolts using a modified form of Wien's bridge. He uses the conjugate arrangement with the alternator across CD and the detector across AB. By choice of a suitably large value for Q/S, the voltage across C_2 is reduced and this condenser can now be an ordinary variable mica standard instead of being a special air condenser suitable for extra high voltage. It is now necessary to construct the ratio resistances to withstand the high testing voltage; as these resistances are of large value, some error may be introduced by distributed self capacitance in them. Earth capacitance effects at the detector branch points are eliminated by use of the Wagner earthing device (*see* p. 542).

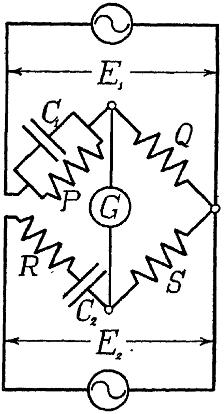

FIG. 120.—ATKINSON'S MODIFICATION OF THE WIEN BRIDGE

In the normal arrangement of the Wien bridge shown in Fig. 115 the resistances Q and S are usually small in comparison with $1/\omega C_1$ and $1/\omega C_2$; hence the condensers are subjected practically to the whole of the applied voltage, which they must both be capable of withstanding. The point B is usually earthed; Q and S should be provided with shields joined to B. It is best to interchange the positions of C_2 and R in the branch AD, and to provide C_2 and C_1 with shields connected to A. By these means it is possible greatly to reduce the disturbing influences of earth capacitance currents at high voltages.

The principal use of the bridge being to measure dielectric losses at high voltages, it is essential in the normal bridge to have a standard condenser to work satisfactorily at the greatest voltage which will be applied. Such condensers are bulky and costly, and Rosen has endeavoured to eliminate their use by adopting the conjugate bridge, as has just been mentioned. The same end is attained in an ingenious way in the bridge due to Atkinson,* shown in Fig. 120, embodying a principle which appears to be new in the technique of bridge measurements. Referring to Fig. 120, the junction of the condenser arms of the Wien bridge is opened and the two halves of the resulting network are supplied at different voltages. For example, E_1 may be several kilovolts and is applied to the branches which contain the sample of cable, while E_2 is of the order of 300 volts and supplies the standard condenser branch of the network. These two voltages are obtained from windings on the same transformer and are in phase with one another; a compensating circuit, not shown in the diagram, enables the exact equality of phase to be accurately adjusted. This can readily be done by balancing the bridge with a standard high voltage air condenser in place of C_1, setting the bridge arms to the calculated balance values and adjusting the phase compensating circuit until the null condition is secured. Atkinson's modification, therefore, applies the high voltage only to the test specimen, and, by confining the voltage applied to the condenser C_2 to a low value, enables normal mica condensers to be used. At the same time it is not necessary to construct any of the resistances to withstand very high voltages; high resistances are not necessary, and the possibility of errors due to the influence of residual capacitance and inductance in them is therefore reduced to a minimum.

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* A. Campbell and J. L. Eckersley, "On the insulation of inductive coils," *Elecn.*, Vol. 64, pp. 350–352 (1910).

† A. Rosen, "The use of the Wien bridge for the measurement of the losses in dielectrics at high voltages, with special reference to electric cables." *Proc. Phys. Soc.*, Vol. 35, pp. 235–252 (1923).

The general theory of four-branch impedance networks with double supply has been worked out by the writer.† For the particular case of Fig. 120 it is shown that if $k = E_1/E_2$ the balance conditions are

$$\frac{C_1}{C_2} = \frac{S}{kQ} - \frac{1}{P}\left\{R + \frac{S}{k}(k-1)\right\} \text{ and } C_1C_2 = 1/\omega^2P\left\{R + \frac{S}{k}(k-1)\right\}$$

Comparing these expressions with those given on page 345 shows that Atkinson's modification is, in effect, to increase Q to a value kQ and R to a value $R + \frac{S}{k}(k-1) = R_1$ say. Solving for C_1 and P

$$C_1 = SC_2/kQ(1 + \omega^2C_2{}^2R_1{}^2) \text{ and } P = kQ(1 + \omega^2C_2{}^2R_1{}^2)/\omega^2C_2{}^2R_1S$$

The frequency term can usually be neglected in comparison with unity; again $\cot\phi = 1/\omega C_1P$ and for power factors less than about 0·1, $\cos\phi \doteqdot \cot\phi$. Hence, finally,

$$C_1 \doteqdot SC_2/kQ \text{ and } \cos\phi \doteqdot \omega C_2R + \omega C_2S - \omega C_1Q$$

Since the ratio k is a large number, Q can be fairly small, so that the greater part of the voltage E_1 is absorbed in C_1. Also C_2 can be large and may conveniently be a subdivided mica condenser suitable for low voltages. Moreover, S can also be small, so that the possibility of error due to distributed capacitance in the resistances is slight.

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* R. W. Atkinson, "A high tension bridge for measurement of dielectric losses in cables," *Elect. J.*, Vol. 22, pp. 58–66 (1925). F. R. Benedict, "Measurement of dielectric power factor," *Elect. J.*, Vol. 31, pp. 239–243 (1934).

† B. Hague, "Measurement of dielectric losses at high voltages. A new type of alternating current bridge," *World Power*, Vol. 4, pp. 81–83 (1925).

The bridge has many uses at low voltages, e.g. in the testing of telephone cables* and in the measurement of losses in samples of dielectric materials. In these cases it is possible to arrange an equal ratio bridge and the conjugate bridge presents advantages from the point of view of shielding. Ford and Reynolds (*loc. cit. ante*) describe the General Radio Co., Type 216, bridge† used over the frequency range of 1,000 to 12,000 cycles per second, embodying a simple shielding arrangement. Referring to Fig. 121 the ratio branches Q and S are 5,000 ohms each; C_1 and C_2 are 1,500 $\mu\mu$F. variable air condensers. The corner A and the mid-point of the source are earthed. C_2 is set at 1,000 $\mu\mu$F., unless C_x is larger than 600 $\mu\mu$F., when a higher setting is appropriate. Balance is secured by varying C_1 and R, using a substitution method. Losses in C_2 can be represented by a very small resistance, negligible by comparison with R; losses in C_1 are represented by a very high resistance P in parallel with it and this may be comparable with the shunt loss resistance W_X of the condenser under test. Let $G_P = 1/P$, $G_X = 1/W_X$; then with *Sw* open,

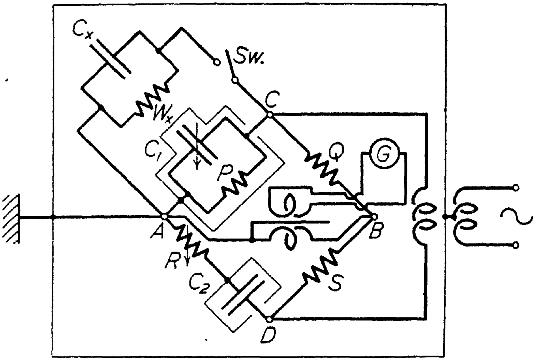

FIG. 121.—FORD AND REYNOLDS'S WIEN BRIDGE

$$G_P + j\omega C_1 = 1/[R + (1/j\omega C_2)] = j\omega C_2/(1 + j\omega C_2 R)$$
$$\doteqdot j\omega C_2(1 - j\omega C_2 R)$$

neglecting $\omega^2 C_2{}^2 R$ in comparison with unity. With *Sw* closed,

$$(G_P + G_X) + j\omega(C_1' + C_X) \doteqdot j\omega C_2(1 - j\omega C_2 R'),$$

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* F. Fischer, "Messung der betriebsmässigen Ableitung von Fernsprechkabeln mit geerdetem und ungeerdetem Bleimantel," *Tel. u. Ferns. Tech.*, Vol. 10, pp. 137–140 (1921). K. Küpfmüller and P. Thomas, "Wechselstrombrücke zum Messen der Scheinwiderstände von Fernsprechkabeln," *Elekt. Zeits.*, Vol. 43, pp. 461–464 (1922).

† C. L. Lyons, "Laboratory measurement of capacity, power factor, dielectric constant, inductance, and resistance by the use of the series resistance bridge," *Journal Sci. Insts.*, Vol. 5, pp. 155–160, 188–194 (1928).

where C_1' and R' are the new balance values. Subtracting gives

$$G_X + j\omega(C_1' - C_1 + C_X) \doteqdot \omega^2 C_2^2(R' - R),$$

or

$$G_X \doteqdot \omega^2 C_2^2(R' - R) = \omega^2 C_2^2\, \Delta R$$

and

$$C_X \doteqdot C_1 - C_1' = \Delta C_1.$$

Siskind (*loc. cit. ante*) has pointed out that earth capacitance of R and the lead joining it to C_2 may make a considerable error in small loss-angle measurements. Sommerman* has used a doubly shielded Wien bridge with Wagner earthing device, in which this and other residual errors are eliminated; his paper should be consulted for full details of circuit and procedure. The bridge operates satisfactorily over a range from 60 to 10,000 cycles per second, and can itself be used to measure the frequency, which enters explicitly into the balance conditions; see below.

Frequency Bridges. The Wien bridge has other uses in addition to the applications described. Thus Ferguson and Bartlett† have shown that it is an excellent substitute for the Maxwell commutator bridge for the purpose of determining capacitance in terms of frequency and resistance, and is much easier to use. Kurokawa and Hoashi‡ have used the bridge as a method of measuring the period $T = 1/f$ of an alternating current, though the suggestion for this purpose appears to be due to D. I. Cone (*loc. cit*, 1920), who ascribes its development to R. C. Mathes. Referring to Fig. 115, let the condenser C_1 have a series loss resistance ρ, then it is not difficult to show that balance will occur when

$$\frac{1}{f} = 2\pi R \frac{(C_1 C_2)^{\frac{1}{2}}}{\left(\frac{S}{Q} - \frac{C_1}{C_2}\right)^{\frac{1}{2}}} \left(1 - \left(\frac{\rho}{R}\right)^2 \cdot \frac{C_1}{C_2} \cdot \frac{S}{Q}\right)^{\frac{1}{2}}$$

$$\text{and } R = P\left(\frac{S}{Q} - \frac{C_1}{C_2}\right) - \frac{C_1}{C_2}\rho$$

Now if C_1 is an air condenser ρ is zero, and will be nearly so for a good mica condenser. Then to a high degree of approximation

$$T = 1/f = 2\pi R(C_1 C_2)^{\frac{1}{2}} \Big/ \left(\frac{S}{Q} - \frac{C_1}{C_2}\right)^{\frac{1}{2}} \text{ and } R = P\left(\frac{S}{Q} - \frac{C_1}{C_2}\right)$$

so that

$$= 2\pi P(C_1 C_2)^{\frac{1}{2}} \left(\frac{S}{Q} - \frac{C_1}{C_2}\right)^{\frac{1}{2}}$$

Take $C_1 = C_2 = 1\ \mu$F., $S/Q = 3{\cdot}533$ and adjust balance by means of P and R; then $T = 10^{-5}P$ and $R = 2{\cdot}533P$. Thus, assuming no losses in C_1 the method is a linear period bridge of wide range.

Contemporaneously with Kurokawa and Hoashi in Japan, Robinson§

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* G. M. L. Sommerman, "Dielectric power-factor measurements at audio and radio frequencies," *Rev. Sci. Insts.*, Vol. 5, pp. 341–345 (1934).

† J. G. Ferguson and B. W. Bartlett, "The measurement of capacitance in terms of resistance and frequency," *Bell Syst. Tech. J.*, Vol. 7, pp. 420–437 (1928).

‡ K. Kurokawa and T. Hoashi, "A linear frequency bridge," *Journal I.E.E., Japan*, No. 437, pp. 1132–1138 (1924).

§ C. Robinson, "A direct-reading frequency meter for telephonic currents," *Journal P.O.E.E.*, Vol. 16, pp. 171–174 (1924).

in England devised a direct reading frequency bridge from the Wien condenser network of Fig. 115. Referring to this diagram, Q, S, C_1 and C_2 are fixed while balance is secured by varying $P = R = r$, then the balance conditions on page 345 reduce to

$$Q/S = C_2/(C_1 + C_2) \text{ and } \omega^2 = 1/r^2C_1C_2.$$

If further $C_1 = C_2 = C$, then

$$Q/S = 0{\cdot}5 \text{ and } \omega = 1/rC, \text{ or } f = 1/2\pi rC.$$

In the actual instrument $C_1 = C_2 = C = 1\mu$F., $Q = 50$ ohms and $S = 100$ ohms.

The variation of $1/P = 1/R = 1/r$ is carried out simultaneously by a special four-decade conductance box, in which the conductances are adjusted so that the dials read frequency directly in thousands, hundreds, tens and units. The range is up to 2,500 cycles per second. A complete diagram of connections with the values of all the necessary resistances is given.

Yet another frequency bridge is described by Field.* The condensers C_1 and C_2 are fixed and equal; the resistances P and R are equal and are simultaneously varied by a single adjustment. Then by fixing the ratio arms so that $S/Q = 2$ the first balance condition on p. 345 is satisfied, and the second condition becomes $f = 1/2\pi C_1P$. The frequency scale covers a range from 20–200 cycles per second, and this can be multiplied by 10 and 100 by changing the condensers. The resistors P and R are wound "tapered" so that equal frequency ratios correspond with equal intervals on the scale, giving equal percentage accuracy throughout the range. The conjugate bridge is used, i.e. Fig. 115 with alternator and detector interchanged, and the point A is earthed.

11. Hay's Method. The late Mr. C. E. Hay† suggested the following modification of Anderson's bridge (*see* p. 379) for the purpose of measuring the small residual capacitance of resistance coils. Q is the resistance whose effective capacitance C_1 is to be found; C_2 is a standard condenser. The rest of the branches are composed of non-reactive resistances, as in Fig. 122.

Comparing this diagram with Fig. 22, it will be seen that $z_1 = P$, $z_2 = 1/\left(\frac{1}{Q} + j\omega C_1\right)$, $z_3 = S$, $z_4 = R$, $z_6 = r$, and $z_7 = 1/j\omega C_2$. Now, from p. 56, the balance equation for the network is

$$z_7(z_1z_3 - z_2z_4) = z_2\{z_6(z_3 + z_4) + z_3z_4\},$$

i.e.

$$SP\left(\frac{1}{Q} + j\omega C_1\right) - R = j\omega C_2[r(R + S) + RS].$$

Separating components, $SP = QR$,

$$C_1 = \frac{C_2}{SP}[r(R + S) + RS].$$

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* R. F. Field, "A bridge type frequency meter," *G.R. Exp.*, Vol. 6, pp. 1–3 (Nov., 1931).

† C. E. Hay, "Alternate current measurements, with special reference to cables, loading coils, and the construction of non-reactive resistances." *Journal, P.O.E.E.*, Vol. 5, pp. 451–454 (1913); also *Professional Papers*, No. 53, pp. 19–21, 36–37.

In practical work it is best to make $R = S$ and hence $P = Q$. Balance is secured by adjustment of P and r, the equation for C_1 reducing to

$$C_1 = \frac{C_2}{P}[2r + R].$$

By re-balancing with R and S interchanged and taking the mean result, small differences between them are eliminated. The effect of residuals in the remaining branches and of capacitance of the bridge to earth may be eliminated by the process of substitution or "dummy" balance. After balancing with the unknown in position, a new balance is obtained when the unknown is replaced by a standard equal in resistance to Q and having calculable residual capacitance. The difference between the unknown and the standard is thus found with a high degree of precision. For further details, reference should be made to the discussion given on page 387 of a similar process applied to resistance coils which have residual inductance.

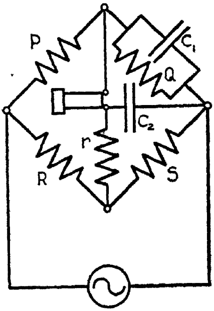

FIG. 122.—HAY'S METHOD FOR MEASURING THE RESIDUAL CAPACITANCE OF A RESISTANCE COIL

12. The Schering Bridge. HIGH VOLTAGE. In 1920, Schering suggested the bridge shown in Fig. 123 as a method for measuring dielectric losses at high voltages, Semm* applying it to this purpose. The bridge proves to have many advantages and to be very versatile. The recent enormous development in high voltage technology has resulted in a very wide use of the method for tests of the losses in cables and in insulators at high working voltages. Moreover, the bridge is also one of the best ways for testing small condensers at low voltages with very high precision, and for many other applications.

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* A. Semm, "Verlustmessungen bei Hochspannung," *Arch. f. Elekt.*, Vol. 9, pp. 30–34 (1921); "Tatigkeitsbericht der Physikalische Technische Reichsanstalt," *Zeits. f. Inst.*, Vol. 40, p. 124–125 (1920). *See* also H. Schering, *ibid.*, p. 124 (1920), for his suggestion of the method. The bridge was described at an earlier date by Phillips Thomas in *United States Patent* No. 1,166,159, "Method of measuring capacity and power-factor," filed 4th Dec., 1913, and accepted 28th Dec., 1915. In spite of this prior description the method has been almost entirely developed in Europe, especially by Professor Schering and those influenced by his work, earlier American practice in h.t. testing tending to be restricted to the Wien bridge. In this connection *see* an interesting appeal to American engineers to follow European practice, made by R. Sparks, "Testing insulation without destroying it," *Elect. J.*, Vol. 27, pp. 229–230, 237 (1930).

Referring to Fig. 123, C_1 is the effective capacitance and ρ_1 the equivalent series loss resistance of the imperfect condenser to be tested. C_2 is a standard condenser; C_3 is a variable condenser. Q and S are non-reactive resistances.

Writing down the branch impedance operators $z_1 = \rho_1 + \dfrac{1}{j\omega C_1}$, $z_2 = Q$, $z_3 = 1\Big/\left(\dfrac{1}{S} + j\omega C_3\right)$, $z_4 = 1/j\omega C_2$, the balance eqn. is $\left(\rho_1 - \dfrac{j}{\omega C_1}\right) = Q\left(\dfrac{C_3}{C_2} - \dfrac{j}{\omega C_2 S}\right)$

whence $\rho_1 = \dfrac{C_3}{C_2} Q$,

and $C_1 = \dfrac{S}{Q} C_2$

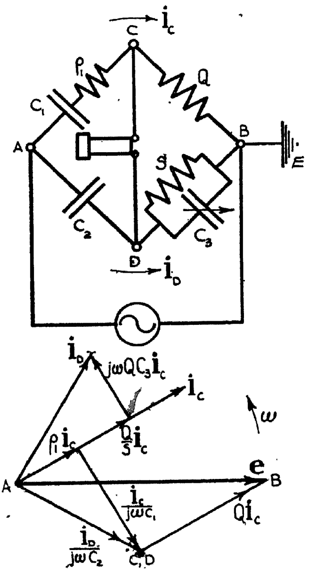

FIG. 123.—THE SCHERING BRIDGE IN ITS SIMPLEST FORM

From this the loss angle θ_1 of the tested condenser is

$$\tan\theta_1 = \omega C_1 \rho_1 = \omega S C_3.$$

If ϕ_1 be the phase angle of the imperfect condenser, the loss angle θ_1 represents the defect of ϕ_1 from exact quadrature; then

$$\cos\phi_1 = \sin\theta_1 \doteqdot \tan\theta_1$$

since θ_1 is usually small. As a rule, in high voltage testing, C_1 and C_2 are small and are, therefore, of enormously high impedance in comparison with the two remaining branches. Hence if E is the voltage applied to the points AB the power lost in the branch AC will be very nearly given by*

$$\text{Power in watts} \doteqdot \left(\frac{E}{Z_1}\right)^2 \rho_1 = \frac{E^2}{\rho_1}\left(\frac{\rho_1}{Z_1}\right)^2 = \frac{E^2}{\rho_1}\sin^2\theta_1$$

$$\doteqdot \frac{E^2}{Q}\cdot\frac{C_2}{C_3}\cdot\sin^2\theta_1 \doteqdot \omega^2 S^2 E^2 \frac{C_2 C_3}{Q}$$

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* G. Hauffe, "Zur Theorie der Scheringschen Brücke," *Arch. f. Elekt.*, Vol. 17, pp. 422–423 (1926). See also G. Benischke, ibid. Vol. 16, pp. 174–176; and criticisms by W. Geyger and H. Schering, ibid. Vol. 17, pp. 423–430 (1926).

In Semm's arrangement of the bridge, C_2 was a Petersen cylindrical air condenser of 50 cm. capacitance suitable for 100 kV. (see Fig. 71); the high voltage electrode was joined to A, the low voltage electrode to D, while the guard cylinders were earthed. Q was fixed at 200 ohms, balance being secured by adjustment of C_3 and S. The condenser C_3 was a three-decade mica standard totalling 1 μF. by steps of 0·001 μF.; S ranged up to 6,000 ohms.

The best method of balancing is to fix S and to adjust Q and C_3. Most tests on cables, etc., are made at 50 cycles per second or $\omega = 100\,\pi$; if S be fixed at the value $1000/\pi$ and C_3 is expressed in microfarads

$$\theta_1 \doteqdot \tan \theta_1 = 0{\cdot}1 C_3 \text{ radians.}$$

Chevalier* gives an investigation of the influence of losses in the standard condenser C_2; if ρ_2 be the series loss resistance of C_2, the balance conditions become

$$C_1 \doteqdot SC_2/Q \text{ and } \cos \varphi_1 \doteqdot \omega SC_3 + \omega C_2 \rho_2.$$

Using this expression in place of that on page 353 enables standard condensers with known losses to be used, e.g. Minos jars, etc.

The simple network shown in Fig. 123 is, subject to certain precautions being observed, suitable for high voltage tests on short lengths of cable, porcelain insulators, and similar apparatus in which the charging current is small, say up to 30 milliamperes. When the current becomes large, as may be the case in testing apparatus near breakdown or in normal tests on long samples of cable, the method is not convenient, since it is not easy to construct a variable non-reactive resistance Q capable of carrying large currents. To overcome this defect the arrangement shown in Fig. 124 (a) is used.† In this, n is a fixed low resistance, r is a fixed resistance of higher value in series with a slide wire s; the sum $n + r + s$ is constant and usually equal to 100 ohms. The resistance ρ is a non-reactive decade box up to 1111 ohms. Balance is now secured by adjustment of ρ and C_3, and by moving the potential contact on the slide wire, a suitable value of n having been chosen. This resistance combination can be converted into its equivalent star by the usual transformation, by means of which it is easy to show that balance occurs when

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* H. Chevalier, "Remarques pratiques au sujet de l'utilisation du pont de Schering," *Bull. Soc. Belge des Elecns.*, Vol. 43, pp. 327–344 (1929).

† "Die Tätigkeit der Physikalische Technische Reichsanstalt in 1923," *Elekt. Zeits.*, Vol. 45, p. 344 (1924). L. Tschiassny, "Die Messgenauigkeit der Scheringbrücke," *Arch. f. Elekt.*, Vol. 18, pp. 248–256 (1927).

$$C_1/C_2 = S(n + r + s + \rho)/n(\rho + \sigma) = S(100 + \rho)/n(\sigma + \rho)$$

$$\tan\theta_1 = \omega S C_3 - \omega S C_2 \frac{r + s - \sigma}{\rho + \sigma} \doteqdot \omega S C_3 \doteqdot 0{\cdot}1C_3$$

taking $n + r + s = 100$, $S = 1000/\pi$ and C_3 in microfarads. The second term in the expression for $\tan\theta_1$ is usually negligible.

Self-contained bridges embodying this device are made by Messrs. Hartmann and Braun, and by the Cambridge Instrument Co. The H. and B. arrangement,* which may be taken as typical, is shown in Fig. 124 (*b*). A rotary switch with seven positions provides values of n equal to 0·3, 1, 3, 10, and 30 ohms respectively on the first five contacts. At the same time a resistance of 69·88 ohms and a slide wire s of 0·12 ohms resistance maintain $n + r + s$ at 100 ohms. These shunts cover currents of 5, 2·5, 0·75, 0·25 and 0·075 amperes. The sixth contact arranges the bridge in the ordinary way for charging currents less than 30 milliamperes. The seventh contact puts 100 ohms in series with ρ; currents greater than 5 amperes are dealt with by using external shunts less than 0·3 ohm connected across the branch CB.

The measurement of large capacitances, up to 10,000 μF. at low voltages, as in the case of dry electrolyte condensers, batteries of condensers for power-factor improvement, etc., presents some difficulty.

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* H. S. Hallo and G. de Zoeten, "Ervaringen met de brug van Schering voor het meten der dielektrische verliezen in hoogspanningskabels," *Tijd. voor Elect.*, Vol. 7, pp. 27–31, 49–54 (1924–25). H. W. L. Bruckman, "La mesure des pertes diélectriques comme moyen de contrôle des réseaux de câbles en service," *Rev. Gén. de l'Él.*, Vol. 17, pp. 881–885 (1925). The following papers may also be consulted for general information on the Schering bridge in both forms. H. S. Hallo, "Mesure des pertes diélectriques dans les isolants," *Gdes. Réseaux*, Vol. 1, pp. 1061–1092 (1925). J. Absil, "Pertes dielectriques, leur mesure dans la technique industrielle," *Bull. [illegible]c. Belge des Elecns.*, Vol. 43, pp. 1–12, 73–81 (1929). A. Palm, "Schering-Messbrücken," *Arch. f. tech. Mess.*, J. 921–3 (Sept., 1932). H. W. Bousman, "A bridge for capacitance and low power-factor measurements," *Gen. Elec. Rev.*, Vol. 35, pp. 295–298 (1932). T. Walcher, *E.u.M.*, Vol. 52, pp. 360–366 (1934).

For self-balancing recording bridges giving a continuous record of capacitance and loss angle with voltage or time, *see* W. Geyger, "Selbsttätige Abgleichung von Komplexen Kompensations-und Brückenschaltungen mit phasenabhängigen Nullmotoren," *Arch. f. Elekt.*, Vol. 29, pp. 842–850 (1935); "Über die Verwendung des C-tgδ-Schreibers in Verbindung mit der Scheringmessbrücke," *ibid.*, Vol. 31, pp. 115–123 (1937); "Kapazitäts und Verlustfaktor-Messbrücke mit selbsttätiger Abgleichung," *Arch. f. tech. Mess.*, J. 924–1 (Aug., 1936). G. Keinath, "Kapazitäts und Verlustmessungen an Kleinkondensatoren," *Arch. f. tech. Mess.*, V. 339–17 (Aug., 1936).

For application of the Schering bridge to corona loss measurements on transmission lines, *see* K. Potthoff, "Koronaverluste an Kupfer- und Aluminiumseilen," *Elekt. Wirts.*, Vol. 30, pp. 526–530 (1931).

The arrangement shown in Fig. 124 (*a*) has been used at high voltages up to 5 μF.; to extend the capacitance range it is necessary to make the low resistor n of very small value. When this is done the residual inductances of the resistances and their connecting leads, especially in n, have considerable effect on the accuracy of the capacitance and loss angle measurement, the simple theory on page 355 requiring modification. Zickner and Pfestorf* have worked out the necessary corrections, and show that such large condensers can be measured with good accuracy when these precautions are taken.

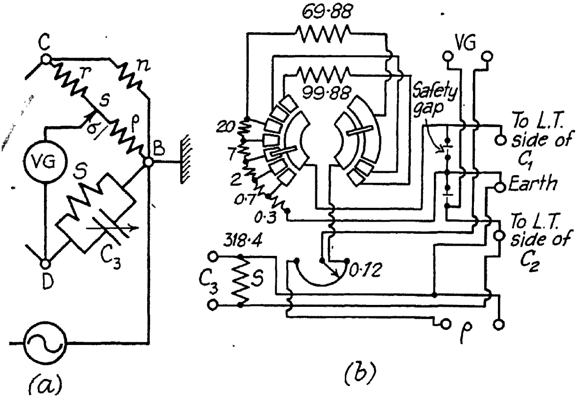

FIG. 124.—MODIFICATIONS OF THE SCHERING BRIDGE FOR LARGE CHARGING CURRENTS

In one or other of the two forms, Fig. 123 or Fig. 124 (*a*), the Schering bridge has been used to measure dielectric losses in all kinds of high-tension apparatus at voltages up to a few hundred kilovolts. It is essential to observe certain precautions in order that the influence of important sources of error may be kept down to a negligible amount.

Safety. The Schering bridge is very safe to operate. The branches *CB* and *BD* have very little voltage across them since their impedances are very small in comparison with those of the condenser branches *AC* and *AD*. For example, if C_1 is about 100 $\mu\mu$F., its impedance at $f = 50$ is 30 megohms; balance can be secured with resistances of a few thousand ohms. Thus the voltage across the resistances is only about one ten-thousandth of that applied to the bridge. Hence the points

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* G. Zickner and G. Pfestorf, "Ueber die Verwendung der Hochspannungsbrücke nach Schering zur Untersuchungen von grossen Kapazitäten," *Zeits. f. tech. Phys.*, Vol. 12, pp. 210–213 (1931). Also *see* R. Bauder and K. Jahnnsen, *E.u.M.*, Vol. 50, pp. 581–586 (1932).

C and D are never more than a few volts above earth potential, so that Q and C_3 may be adjusted with perfect safety to the operator. As an additional safeguard the low voltage portions of the bridge, Q, S and C_3, are often contained in an earthed metal enclosure, the switches effecting the necessary balancing adjustments being provided with carefully insulated knobs.

Although the bridge is safe under normal conditions, this would not be the case if the test condenser C_1 were to break down. In this circumstance the resistance Q would be subjected to the full test voltage, with consequent danger to the operator and possible destruction of the resistance. To avoid this dangerous condition the branches CB and BD are each shunted by an over-voltage safety device. This may consist of a simple needle-plane air gap ; of two small metal plates separated by thin mica or by cigarette paper ; or it may be a neon tube or vacuum arrester. In any case the safety device is arranged to operate when the p.d. across the adjustable branches rises to 100 or 200 volts, so that the bridge is put out of action long before a dangerous condition can be established.

Earth Capacitances and Electric Interference. To a first approximation the earth capacitances of the bridge may be represented by four condensers joining the points A, B, C, D to earth, see page 541 ; since B is directly earthed the earth capacitance current from that point is zero. Similarly the earth capacitance at A has no influence on the bridge balance since it is merely a shunt across the source at AB. There remain, therefore, the earth capacitances at the detector branch-points C and D. Since these points are never more than a few volts above earth potential it follows that the capacitance currents from them must be very small ; in practice they are usually too small to have any appreciable effect on the balance of the bridge. Hence, due to the direct earthing of B and to the dissymmetry of the branch impedances, with consequent low potential of C and D, the influence of earth capacitances in the Schering bridge is slight. This is, however, not true of the effect of cross-capacitances between the various branches.

The direct action of the electric field of the high voltage electrodes of C_1 and C_2 upon the low voltage portions of the bridge, namely, the leads from the low voltage electrodes of C_1 and C_2, the branches CB and BD, and the detector CD, is extremely important and errors due to this cause can only be avoided by the adoption of an appropriate system of electric shielding. Referring to Fig. 125 (*a*), consider first the two low voltage standard branches which contain Q, C_3, and S. These are each provided with a metal shield joined to the earthed point B and are thereby shielded from the high voltage side ; the shields will, however, give to each resistance a distributed capacitive residual, so that the resistance boxes must be calibrated and their residuals determined under the conditions in which they are used. These residuals are usually very small, but in precise work it may be necessary to include their effect in the balance conditions, as is done in Giebe and Zickner's modification of the bridge described below.

Consider now the branch AD which contains the standard high voltage air condenser, the high voltage electrode of which is joined to A. The low voltage electrode is joined to D, its capacitance C_2 with respect to the high voltage electrode being made independent of fringing of the electric field and free from edge losses by a guard electrode. Referring to Fig. 125(b), where the guard electrode is shown earthed, the capacitance between the high voltage and guard electrodes is a shunt across the supply and does not affect the balance. The capacitance between the low voltage and guard electrodes* may in this case introduce a source of error; the diagram indicates that C_g is in parallel with the S, C_3 combination, so that $\tan \theta_1$ is given by $\omega S(C_3+C_g)$ instead of ωSC_3. The error amounts to about 0·01 per cent when S is of the order of 1,000 ohms and increases in proportion to S.

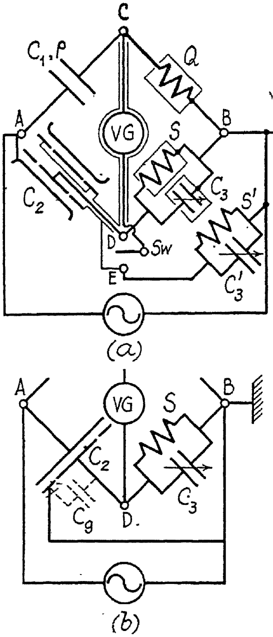

FIG. 125.—SHIELDING OF THE SCHERING BRIDGE

This source of error may be eliminated in the following way.† The guard electrode is not directly earthed; hence C_g is joined to B via the earth capacitance of the guard electrode and still affects the purity of the branch BD. Its effect will clearly vanish if the low voltage and guard electrodes are brought to the same potential; this can be secured by the device shown in Fig. 125(a), where a more comprehensive system of shielding is shown. The low voltage lead to D is taken through a tube connected to the guard electrode, this tube being also connected to the shield surrounding the detector and its leads; C_g now comprises the low-voltage-to-guard electrode capacitance, the capacitance of the l.v. lead to the tube, and that of the branch CD to its shield. The shielding system is connected via one contact E of a two-way switch Sw, to the parallel combination S', C'_3 as shown. With the switch on D the main bridge is balanced; then with the switch on E the capacitances between the high voltage side and the shields are balanced by adjusting S' and C'_3. The process is repeated until balance is undisturbed on switching from D to E; then C, D, and E are at the same potential. Thus the low voltage electrode and the shielding system are now at the same potential, as is required. If the power-factor exceeds 0·01 this auxiliary device may not give balance for the shield effect; in such cases remove C'_3 from S' and shunt it across Q, obtaining the double balance as before. Then $\tan \theta_1 = \omega S C_3 - \omega Q C'_3$. Other devices to secure the same result are given by Rayner in the paper cited.

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* H. L. Curtis, *Journal Amer. I.E.E.*, Vol. 45, pp. 1084–1086 (1926).

† B. G. Churcher and C. Dannatt, "The use of the Schering bridge at 150 kilovolts," *World Power*, Vol. 5, pp. 238–247 (1926). E. H. Rayner, *Journal Sci. Inst.*, Vol. 3, pp. 33–38, 70–77 (1925); pp. 104–106 (1926). *Rev. A.C.E.C.*, No. 121, pp. 1–11 (1929). E. H. Rayner, W. G. Standring, R. Davis, and G. W. Bowdler, "Low power-factor measurements at high voltages," *Journal I.E.E.*, Vol. 68, pp. 1132–1142 (1930). For a more complex shielding system, *see* J. B. Whitehead and F. Hamburger, "The influence of residual air and moisture in impregnated paper insulation," *Trans. Amer. I.E.E.*, Vol. 47, pp. 314–333 (1928). F. Beldi, "Eine Hochspannungsbrücke für Verlustmessungen an Isolierstoffen," *Schweiz. Elekt. Ver. Bull.*, Vol. 33, pp. 197–208 (1930); "Equipment for measuring the dielectric losses in insulating materials and insulators," *Brown-Boveri Rev.*, Vol. 17, pp. 147–154 (1930), gives an entirely different way for regulating the balance of shield admittances, using inductors.

It is often not possible to arrange shielding for the test branch AC, since this will usually consist of some piece of apparatus such as a h.v. bushing or a length of cable. In such cases C must be connected to the low voltage portion of the apparatus, e.g. the cable sheath. When testing samples of cable in the laboratory, the normal bridge often suffices, since the potential of the sheath, which will be earthed when the cable is in service, is but slightly above earth. Modifications of the bridge and its shielding when one side of the test specimen must be solidly earthed will be found on p. 363.

Sensitivity. We have seen that the impedances of C_1 and C_2 are very much greater than those of Q and S; hence it might be expected that the absolute sensitivity of the Schering bridge would be low, since the impedances of the branches deviate so far from the conditions of maximum sensitiveness. However, at high voltages and with a good vibration galvanometer, the available sensitivity is adequate for all practical requirements. Tschiassny (*loc. cit.*) has proved that the sensitivity (i) is proportional to the voltage applied and to the frequency; (ii) increases with C_1 and C_2, and, if they are very dissimilar, is more dependent upon the smaller one; (iii) increases with S and with a low detector impedance. Moreover, the shunted network of Fig. 124 is less sensitive than the simple bridge of Fig. 123. In this connection the paper by Hallo and Zoeten may be consulted with advantage.

The sensitivity conditions have been reconsidered by Schering,* Miller,† Alten,‡ and Bradshaw,§ using more concise mathematics. In the general theory of four-branch networks on p. 77 it is shown that the detector current for any condition of unbalance is

$$\mathbf{i} = \frac{z_1 z_3 - z_2 z_4}{\Delta} e_1 \mathbf{1}$$

where e_1 is the amplitude of the alternator voltage, $\mathbf{1}$ is a unit vector and Δ is the sum of products of the operators z_1, z_2, z_3, z_4, z_5 and z_6 taken three at a time except when three meet in a branch-point. The outstanding feature of the Schering network is that z_1 and z_4 are enormously greater than any of the other operators; hence only those products in Δ will be important which contain z_1z_4 as a factor. In these circumstances, if $z_5 = R_5 + jX_5$ is the detector operator,

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* H. Schering, "Die Empfindlichkeit einer Wechselstrombrücke," *Elekt. Zeits.*, Vol. 52, pp. 1133–1134 (1931).

† J. L. Miller, "Die Empfindlichkeit der Scheringbrücke," *E.u.M.*, Vol. 49, pp. 677–678 (1931).

‡ F. Alten, "Ein graphischer Beitrag zur Schering-Brücke," *Elekt. Zeits.*, Vol. 57, pp. 807–808 (1936).

§ E. Bradshaw, "A note on the sensitivity of the Schering bridge network," *Journal R.T.C.*, Vol. 4, pp. 144–146 (1937).

$$\Delta \doteqdot z_1z_4(z_2 + z_3 + z_5)$$

and

$$\mathbf{i} \doteqdot \frac{z_1z_3 - z_2z_4}{z_1z_4(z_2 + z_3 + z_5)} e_1\mathbf{1} = \frac{(z_3/z_4) - (z_2/z_1)}{z_2 + z_3 + z_5} e_1\mathbf{1}$$

Now from p. 353,

$$z_1 = (1 + j\omega C_1\rho_1)/(j\omega C_1),\ z_2 = Q,\ z_3 = S/(1 + j\omega C_3S) \text{ and } z_4 = 1/j\omega C_2$$

Inserting these values, and neglecting both $\omega^2C_1{}^2\rho_1{}^2$ and $\omega^2C_3{}^2S^2$ in comparison with unity gives

$$\mathbf{i} \doteqdot \frac{\omega(C_2C_3S^2 - C_1{}^2Q\rho_1) + j[C_2S - C_1Q]}{(Q + S + R_5) + j(X_5 - \omega C_3S^2)} \cdot \omega e_1\mathbf{1} \doteqdot \frac{\alpha}{\Delta}\,\omega e_1\mathbf{1}$$

Hence the current is proportional both to the frequency and the voltage. The rate of change of current with respect to any parameter p other than ω is

$$\frac{\partial\mathbf{i}}{\partial p} \doteqdot \frac{\Delta\dfrac{\partial\alpha}{\partial p} - \alpha\dfrac{\partial\Delta}{\partial p}}{\Delta^2} \cdot \omega e_1\mathbf{1}$$

When the bridge is balanced, $C_1Q = C_2S$, $\rho_1C_2 = C_3Q$ and $\tan\theta_1 = \omega C_1\rho_1 = \omega C_3S$; hence when the bridge is near balance $\alpha \rightarrow$ zero and

$$\frac{\partial\mathbf{i}}{\partial p} \doteqdot \frac{1}{\Delta}\frac{\partial\alpha}{\partial p} \cdot \omega e_1\mathbf{1}$$

In the Schering bridge (C_1, ρ_1), z_5 and C_2 are given, S is usually fixed at $1{,}000/\pi$ ohms and balance is secured by Q and C_3. Putting $p = Q$ and $p = C_3$ in turn gives

$$\frac{\partial\mathbf{i}}{\partial Q} \doteqdot -\frac{j\omega C_1}{\Delta}(1 - j\tan\theta_1)e_1\mathbf{1} \text{ with } Q \text{ varied,}$$

and

$$\frac{\partial\mathbf{i}}{\partial C_3} \qquad \frac{\omega^2C_2S^2}{} \cdot e_1\mathbf{1} \text{ with } C_3 \text{ varied.}$$

Proceeding to finite differences, let $\delta Q/Q$ and $\delta C_3/C_3$ be fractional changes in Q and C_3 when near balance; then the change of current from zero is

$$\partial\mathbf{i} \doteqdot -j\omega\frac{C_1Q}{\Delta}(1 - j\tan\theta_1)\frac{\delta Q}{Q}e_1\mathbf{1} \doteqdot -j\omega\frac{C_2S}{\Delta} \cdot \frac{\delta Q}{Q}e_1\mathbf{1}$$

when Q is varied, and

$$\partial\mathbf{i} \doteqdot \frac{\omega^2C_2S^2C_3}{\Delta} \cdot \frac{\delta C_3}{C_3}e_1\mathbf{1} \doteqdot \omega\frac{C_2S}{\Delta} \cdot \tan\theta_1 \cdot \frac{\delta C_3}{C_3}e_1\mathbf{1}$$

for change in C_3. From these expressions it is clear that the sensitiveness depends on the product C_2e_1; to obtain a given current change with a given fractional alteration of Q or C_3 it is necessary that C_2 be increased as the voltage falls. Constructional difficulties limit C_2 to about 100 $\mu\mu$F., and hence it is easier to maintain high sensitiveness at high voltages than at low. The sensitiveness also increases with S, but it is usual for this to be fixed in the normal bridge at $1{,}000/\pi$ ohms. It is advantageous to keep Δ small, which can be assisted by using a low-impedance detector, though the effect of this on Δ is usually swamped by the values of Q and S. For a given detectable change of current with both adjustments

$$\frac{\delta Q}{Q} = \tan \theta_1 \cdot \frac{\delta C_3}{C_3}$$

numerically. Hence a 1 per cent change in C_3 has the same effect on the detector current near balance as a $\tan \theta_1$ percentage change in Q. Since θ_1 is small, it follows that the bridge is much more sensitive to the Q adjustment (capacitance balance) than to change of C_3 (loss-angle balance); hence special care is needed to increase the sensitiveness as much as possible when measuring condensers with very low power-factors.

Voltage measurement. The bridge supply in h.v. testing is taken from a step-up transformer, across the secondary of which C_1 and C_2 form a capacitive load. The test voltage may be measured directly on the h.v. side by means of an electrostatic voltmeter, but such instruments are both delicate and expensive. To avoid their use the secondary voltage is computed by multiplying the primary voltage by the ratio of transformation, but it is essential that the ratio be that proper to the working conditions, since it may change considerably with the amount of capacitive load. Jenss* has shown how to arrange the high voltage Schering bridge so that the voltage applied to the test specimen may be found from the bridge itself without the necessity of special high voltage voltmeters, voltage transformers, or other expensive auxiliaries. This is done by putting a precision condenser of sufficiently large capacitance in series with the much lower valued h.v. air condenser, thus forming an electrostatic potential divider, an electrostatic voltmeter with a maximum reading of a few hundred volts being joined in parallel with the added condenser. Losses in the added condenser affect the balance of the bridge and the accuracy of the capacitive potential division; there is also some alteration necessary in the shielding of the bridge arms. These factors are investigated by Jenss, who shows that the method can be made to give quite accurate results. His method is, however, unnecessarily complex and Schering and Brülle† have used the bridge itself to measure the transformation ratio; their method is similar to that introduced by Yoganandam for voltage transformers (*see Instrument Transformers*, p. 562) and is illustrated in Fig. 126. The test specimen $C_1\rho_1$ is put across the secondary winding, as also is the series combination of the branches *AD* and *DB*. An auxiliary mica condenser $C_4\rho_4$ in series with Q is joined across the primary winding. By retaining the h.v. condensers across the secondary, the capacitance current taken from the transformer will be practically the same as when the losses in C_1 are being measured. If the polarity of the primary and secondary windings are as shown, when the detector carries no current the voltages across *CB* and *DB* must be equal. The impedances across the primary and secondary are respectively,

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* H. Jenss, "Über die Messung hoher Spannungen in der Brücke nach Schering," *Elekt. Zeits.*, Vol. 52, pp. 7–8 (1931).

† H. Schering and H. Brülle, "Die Bestimmung der Hochspannung bei Verlustfaktormessungen mit der Brücke," *Elekt. Zeist.*, Vol. 54, pp. 51–54 (1933).

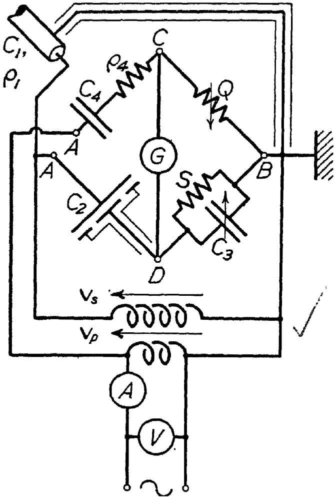

FIG. 126.—SCHERING AND BRÜLLE'S METHOD FOR VOLTAGE RATIO MEASUREMENT

$$[1 + (Q + \rho_4)j\omega C_4]/(j\omega C_4) \text{ and } [1 + j\omega S(C_2 + C_3)]/[j\omega C_2(1 + j\omega C_3 S)]$$

and at balance,

$$\frac{j\omega C_4 Q}{1 + (Q + \rho_4)j\omega C_4}\mathbf{V}_p = \frac{j\omega C_2(1 + j\omega C_3 S)}{1 + j\omega S(C_2 + C_3)} \cdot \frac{S}{1 + j\omega C_3 S} =$$

Hence

$$\frac{}{\mathbf{V}_p} = \frac{C_4 Q}{C_2 S} \cdot \frac{1 + j\omega S(C_2 + C_3)}{1 + j\omega C_4(Q + \rho_4)}$$

The voltage ratio is

$$K_v = \frac{V_s}{V_p} = \frac{C_4 Q}{C_2 S}\sqrt{\left[\frac{1 + \omega^2 S^2(C_2 + C_3)^2}{1 + \omega^2 C_4^2(Q + \rho_4)^2}\right]}$$

Now C_2 is small compared with C_3 and ρ_4 with Q; also the squared terms are small compared with unity; thus

$$K_v \doteqdot \frac{C_4Q}{C_2S}[1 + \tfrac{1}{2}(\omega^2S^2C_3{}^2 - \omega^2C_4{}^2Q^2)] \doteqdot \frac{C_4Q}{C_2S}$$

to a high degree of accuracy. Full details of shielding and other precautions are given in Schering and Brülle's paper.

Field Strength Measurements. Jenss* has made a special application of the Schering bridge, with source and detector interchanged, to find the distribution of electric field strength at points on the surface of an earthed conductor. Around any given point an annular cut is made through the surface, the small detached disc being held in place by an insulator. The capacitance measured between any such disc and the h.v. electrode is proportional to the electric field strength on the disc; the rest of the surface acts as a guard electrode and is connected to the earthed shield surrounding the resistance arms of the bridge.

HIGH VOLTAGE BRIDGE WITH EARTHED TEST CONDENSER. In the normal type of Schering bridge considered above, the junction of the two resistance arms, point B, is earthed. As these branches have only a small impedance compared with the h.v. condenser branches, the l.v. plates of C_1 and C_2 will be slightly above earth potential and practically the full p.d. is impressed on these condensers. In practice many instances arise where one electrode of the test condenser C_1 is necessarily at earth potential, e.g. in cable testing where the lead sheath is earthed† and in testing h.v. bushings‡ where the central ring is fixed to the earthed tank of a switch or transformer. In such cases it is not permissible also to earth B and a new technique must be adopted.

The most obvious way to meet the new condition is to put the earth point at A instead of B, giving the so-called "inverted" Schering bridge introduced by Bormann and Seiler.§ A shield joined to the h.v. corner B defines the stray capacitances between the h.v. parts of the bridge and the l.v. sides of

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[*] H. Jenss, "Über die Messung der elektrischen Feldstärke an Leiteroberflächen," *Arch. f. Elekt.*, Vol. 26, pp. 471–490 (1932).

[†] L. Hartshorn, "The measurement of power losses in dielectrics," *Beama J.*, Vol. 13, pp. 89–99 (1923) considers methods of earthing for tests on two-core cables.

[‡] P. Rosenlöcher and E. Rühlemann, "Aussere Einflüsse bei der Messung dielektrischer Verluste von Hartpapierdurchführungen und ihre Beseitigung," *Arch. f. Elekt.*, Vol. 22, pp. 21–30 (1929). O. Kautzmann, "Die Messung dielektrischer Verluste mit der Scheringschen Messbrücke an Hartpapierdurchführungen und Generatoren in Anlagen," *Elekt. Zeits.*, Vol. 50, pp. 1401–1403 (1929).

[§] Eliz. Bormann and J. Seiler, "Dielektrische Verlustmessungen an Drehstromkabeln bei betriebsmässiger Beanspruchung," *Elekt. Zeits.*, Vol. 49, pp. 239–247 (1928); particular attention is given to three-phase cables.

C_1 and C_2, as shown in Fig. 127 (*a*). It will be observed that the adjustable elements Q and C_3 are at the full test voltage above earth, and must be provided with fully insulated control handles to avoid danger to the operator. This greatly complicates the constructional features of the bridge, but in spite of this difficulty satisfactory apparatus has been made in America* for the testing of bushings and cables *in situ*; double shielding is used, an earthed shield enclosing the h.v. shield shown in the diagram.

These difficulties are removed by transferring the earth point to C as shown by Bormann and Seiler† in an earlier paper. Referring to Fig. 127 (*b*) earth capacitances from the corners D, B, and A act as shown. The capacitance c_D shunts the detector and has no effect on the balance; c_A is in parallel with C_1 and c_B with Q, and these introduce errors. By means of a safety gap it is ensured that B never rises more than a few volts above earth, so that the effect of c_B on Q is very small and attention can be confined to elimination of the influence of c_A. This can be done by balancing the bridge as shown, obtaining values C_0 and $\tan\theta_0$ for the combination in the branch AC. Now remove the test specimen and rebalance, giving c_A and $\tan\theta_A$. Then it is not difficult to show that with close approximation,

$$C_1 \doteqdot C_0 - c_A \text{ and } \tan\theta_1 \doteqdot (C_0\tan\theta_0 - c_A\tan\theta_A)/(C_0 - c_A)$$

It is essential, however, that the earth and intercapacitances remain definite and this can only be secured by appropriate shielding.‡ Fig. 127 (*c*) shows diagrammatically an arrangement of shields intended to define and localize the capacitances and leakances between the bridge arms and between the primary winding, secondary winding, and tank of the supply transformer. Dielectric losses between the secondary winding and the insulated tank are considerable, and can be eliminated

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* C. F. Hill, T. R. Watts and G. A. Burr, "Portable Schering bridge for field tests," *Elec. Eng.*, Vol. 53, pp. 176–182 (1934), describes an arrangement for 13·8 kV.; F. R. Benedict, "Measurement of dielectric power factor," *Elect. J.*, Vol. 31, pp. 239–243 (1934), operates up to 300 kV.

† Eliz. Bormann and J. Seiler, "Dielektrische Verlustmessungen an einem vertegten Hochspannungskabel," *Elekt. Zeits.*, Vol. 46, pp. 114–115 (1925).

‡ L. G. Brazier, "Dielectric loss-angle measurement of multi-core high-tension cables, with special reference to the Schering bridge," *Journal I.E.E.*, Vol. 69, pp. 757–770 (1931). C. Dannatt, "Insulation testing *in situ*. The Schering bridge with earthed specimen," *Elecn.*, Vol. 114, pp. 483–485 (1935). C. Dannatt and F. S. Edwards, "Schering bridge equipment for dielectric loss measurements on installed apparatus," *M.V. Gaz.*, Vol. 17, pp. 11–13 (1937). "Directions for the measurement of the power-factor of cables at power frequencies and voltages up to 200 kV.," *Journal I.E.E.*, Vol. 77, pp. 845–850 (1935).

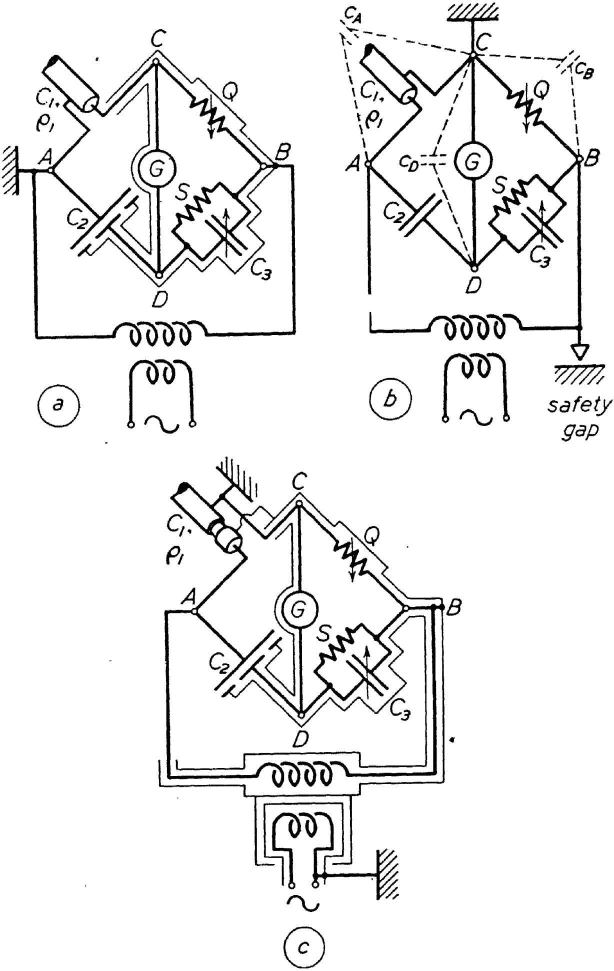

FIG. 127.—SCHERING BRIDGE WITH EARTHED TEST CONDENSER

from the bridge measurements by enclosing the primary in an earthed shield and the secondary in a shield joined to the l.v. corner B; the tank is joined to this corner also.

The measurement of the phase-defect or dielectric loss-angle in three-core cables has been investigated by Brazier in an extensive and important research. It is shown that any partial capacitance in such a cable cannot be considered to have associated with it a definite loss-angle, this varying with the dielectric stress distribution. Measurements made with one stress cannot be used to predict the loss-angle with another stress; nor can three-phase results be precisely estimated from tests with single-phase currents. A very complete analysis of the connections and operation of a Schering bridge arranged to measure any desired partial admittance (inter-core or core-sheath) is given, the theory of the bridge being fully worked out and all practical precautions discussed. Dannatt describes a portable testing outfit for use up to 5 kV.; it is pointed out that preliminary signs of impending failure in h.v. insulators can easily be detected by measuring the dielectric loss at quite low voltages, 5 kV. being quite adequate to show up any abnormality. The shielding system is similar to Fig. 127 (c) except that an earth shield surrounds the bridge shield. The last paper cited is important since it contains the E.R.A. recommendations for cable testing by the Schering bridge up to 200 kV.; Fig. 127 (c) is based upon the circuit shown therein.

End Effects. Tests on long lengths of cable, say 200 yards or more, can be made without any serious error; it is usual in such cases to modify the Q branch in the way shown in Fig. 124 (a) to deal with the large currents involved. It is often necessary to make tests on much shorter lengths, sometimes only 1 yard, and then considerable error may arise from end effects. The earthed sheath is always stripped back some little distance, laying bare the insulation, and it is found that losses in the cable end may easily swamp the true losses in the cable itself when short specimens are used. These losses are due (i) to leakage over the high surface resistivity of the bared insulation, and in the distributed capacitance between this and the core of the cable; (ii) non-uniformity of the electric field near the end of the earthed sheath; (iii) ionization of the air near the end by influence of neighbouring h.v. conductors. Accurate measurements can be made if suitable guard rings of a length not less than eight times the insulation thickness, are applied at the ends to by-pass the surface effects from the bridge.* In its simplest form the guard ring can be made either by cutting a gap in the lead sheath or by winding metal foil on the stripped insulation; a simple ring of this kind is shown in Fig. 127 (*c*) connected to the shielding system of the bridge so that the end currents do not enter the bridge arms but are passed *via* the shields to point *B*. More refined methods aim also at relieving electric stress concentration on the end by reinforced insulation, oil-filled metal end cones, condenser-cone terminations, and the like; full details will be found in the references.

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* R. Dieterle, "Die Schützerdung bei der dielektrischen Verlustmessung an Hochspannungskabeln," *Arch. f. Elekt.*, Vol. 11, pp. 182–188 (1922). E. H. Salter, "Guarding and shielding for dielectric loss. Measurements on short lengths of high tension power cable," *Trans. Amer. I.E.E.*, Vol. 48, pp. 1294–1299 (1929). R. A. Brockbank, "Errors in power-factor measurement due to terminal losses on short lengths of cable," *Journal I.E.E.*, Vol. 70, pp. 281–292 (1932). L. Tschiassny, "Fehlerhafte Verlustwinkelmessungen infolge von Kapazitätsschiebungen während der Periode der angelegten Wechselspannung," *Arch. f. Elekt.*, Vol. 27, pp. 675–680 (1933). J. K. Webb, "The condenser cone. A new device for use in connection with high-tension cable terminations and joints," *Elect. Comm.*, Vol. 12, pp. 92–103 (1933).

LOW VOLTAGE BRIDGES. In addition to its great importance in high voltage measurements* the Schering bridge is one of the best ways of testing condensers of all values at voltages up to a few hundred, and with the highest precision. Giebe and Zickner† have made an extensive and precise research in which the losses in standard mica, fixed and variable air, and other laboratory condensers were tested in a bridge sensitive enough when testing a 0·1 μF. capacitance to enable a loss-angle so small as 1 second to be detected, corresponding with a power factor of 0·000005, while the error in the capacitance measurement was less than 1 part in 10^6. In such precise work it is necessary to allow for all residual errors and to provide very efficient shielding arrangements.

Referring to Fig. 123, let Q and S have small residuals, so that their phase-displacements are ϕ_q and ϕ_s respectively instead of zero. Then very nearly $C_1/C_2 = S/Q$ and $\theta_1 = \omega SC_3 + \phi_q - \phi_s$. The error in the loss-angle measurement may be made very nearly zero by arranging Q and S to be as nearly equal as possible. Even if they are quite unequal the error may be entirely eliminated by the use of a substitution process. To effect this shunt Q with a condenser C_4. Then, if C_2 has a loss-angle θ_2, adjust the bridge for balance; then

$$C_1/C_2 = S/Q,\ \theta_1 - \theta_2 = \omega SC_3 - \omega QC_4 + \phi_q - \phi_s$$

Now replace C_1 by a standard free from loss; calling this C_n, adjust C_n and C_3 to re-balance; then if ΔC_3 be the change in C_3,

$$C_n/C_2 = S/Q,\ -\theta_2 = \omega S(C_3 - \Delta C_3) - \omega QC_4 + \phi_q - \phi_s$$

if Q and S are unchanged. Thus $C_1 = C_n$ and $\theta_1 = \omega S \,.\, \Delta C_3$, completely removing all residual effects.

Giebe and Zickner used a completely shielded bifilar bridge (*cf.* Fig. 111), shown diagrammatically in Fig. 128 (*a*), in which all the residuals were kept small and symmetrical, and all the earth and cross-capacitances were rendered quite definite by the shields.

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* For further examples see E. S. Lee, "Testing high tension, impregnated paper insulated, lead covered cable," *Journal Amer. I.E.E.*, Vol. 44, pp. 156–164 (1925). K. Draeger, "Ueber Verlustwinkel-und Kapazitätsmessungen an Porzellan Isolatoren," *Elek. Zeits.*, Vol. 46, pp. 683–688 (1925). L. Hentschel, "Ueber das dielektrische Verhalten ölgetrankter Papiere," *Arch. f. Elekt.*, Vol. 15, pp. 138–173 (1926).

† E. Giebe and G. Zickner, "Verlustmessungen an Kondensatoren," *Arch. f. Elekt.*, Vol. 11, pp. 109–129 (1922).

Hartshorn* has also used the Schering bridge for low voltage tests, in particular to measure very small capacitances, such as the inter-electrode capacitances of thermionic valves; the arrangement is shown in Fig. 128 (*b*).

In this diagram C_x is the small condenser to be tested, connected to the bridge by rigid leads. C_1 and c_1 are condensers arranged in series as a low reading standard (see page 177); the former consists of a 0·01 μF. mica condenser in parallel with a variable air condenser of 0·002 μF., while the latter is a 0·001 μF. mica standard; all the condensers are shielded. This arrangement gives a magnification of about 160, so that changes of about 1/100 $\mu\mu$F. in the combination correspond with readings on the standard air condenser to the nearest 1 $\mu\mu$F. The

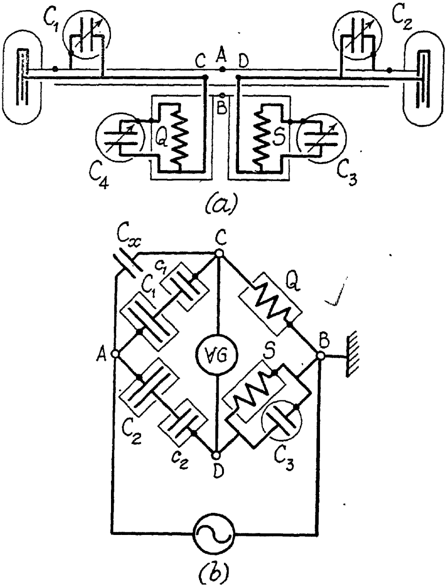

FIG. 128.—MODIFICATIONS OF THE SCHERING BRIDGE FOR HIGH PRECISION MEASUREMENTS

condensers C_2 and c_2 are duplicates of those in the first branch. Q and S are equal 5,000 ohm resistances, and C_3 is a low value variable air condenser. The minimum capacitance of C_3 is balanced by a small capacitance in parallel with Q, usually a short length of flex joining Q to the bridge. Balance is first secured with the condenser C_x connected, and then with it removed ; the change in capacitance of the C_1c_1 combination is then equal to C_x.

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* L. Hartshorn, "A method of measuring very small capacities," *Proc. Phys. Soc.*, Vol. 36, pp. 399–404 (1924). L. Hartshorn and T. I. Jones, "The inter-electrode capacities of thermionic valves," *Exp. W. and W. Eng.*, Vol. 2, pp. 263–273 (1925). L. Hartshorn, "The input impedance of thermionic valves at low frequencies," *Proc. Phys. Soc.*, Vol. 39, pp. 108–123 (1927).

Earth capacitances have a negligible effect. The earth capacitances of the shields of C_1 and C_2 are a shunt across the source. Those of the shields of c_1 and c_2 are shunted across Q and S and are balanced against one another. Since the impedances of AC, AD are enormously greater than those of the other branches the detector branch points are only slightly above earth potential, and " head effect " will be negligible. Adequate sensitivity is attained by supplying the bridge at several hundred volts. Hartshorn shows that the bridge gives a true measure of inter-capacitance and in illustration measures the inter-capacitance between two spheres 1 cm. radius and 2·25 cm. apart, the value being about 0·7 $\mu\mu$F.

Application to Tests on Solid Dielectrics. Hartshorn* and his associates at the National Physical Laboratory have applied his bridge to investigate the dielectric properties of varnished cloth. For tests at low voltage gradients (up to 50 V. per mil) the arrangement of Fig. 128 (*b*) is used with the following modifications: branch AC, the specimen C_x in parallel with a variable air condenser C_1 of 2,000 $\mu\mu$F. maximum capacitance; branch AD a similar variable condenser; Q and S are fixed resistors of 1,000 ohms each; C_3 a mica condenser up to 1 μF. by steps of 0·001 μF. in parallel with a variable air condenser up to 0·001 μF.; Q is shunted by a small condenser to balance the initial capacitance of C_3. The shields of C_1 and Q are joined to C and those of C_2, C_3, and S to D. The point B is not earthed; a Wagner earth (*see* p. 542) is joined across the source and regulated so that C and D are at earth potential when balance is secured, without any direct earth on the main bridge. An earthed enclosure provided with temperature and humidity controls surrounds C_x. A substitution method is used to find C_x and its loss-angle; about 200 volts is applied to the bridge. For higher voltage gradients (up to 230 V. per mil) AC contains only the test specimen and the condenser across Q is omitted, the other branches being as just described. Point B is earthed and the shields of Q, S, C_3 and C_2 are connected as shown in Fig. 128 (*b*); the bridge voltage is about 2,000. A somewhat similar bridge, shielded and earthed as in Fig. 128 (*b*), has been applied to tests on cable compounds and papers by Kouwenhoven and Baños,† who have given a very detailed analysis of sources of error due to the shielding system; an a.c. galvanometer is used. The range of power-factor is 0·00007 to 0·16 and considerable precision is claimed.

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* L. Hartshorn, "The dielectric properties of varnished cloth at low voltage gradients," *Journal I.E.E.*, Vol. 70, pp. 417–435 (1932). L. Hartshorn and E. Rushton, "The dielectric properties of varnished cloth at high voltage gradients," *Journal I.E.E.*, Vol. 75, pp. 631–642 (1934).

† W. B. Kouwenhoven and A. Baños, "A high sensitivity power-factor bridge," *Trans. Amer. I.E.E.*, Vol. 51, pp. 202–210 (1932).

The question of proper electrodes* for tests on sheet materials has been very fully investigated with the aid of the Schering birdge. The best form undoubtedly consists of hollow, flat-faced cast-iron containers between which the insulating sheet is clamped; contact is made with the surfaces of the sheet by mercury filling the cavities in the castings. A thin film of graphite between the mercury and the sample may improve the contact. Dannatt and Goodall have shown that, unless the mercury is clean, there may be surface effects which completely mask the true power-factor of the specimen. If mercury electrodes are not available, tin-foil attached with a trace of vaseline forms a good substitute. At audio frequencies a graphite layer (diluted "aquadag" painted on the surface) backed by a metal plate is satisfactory, but should not be used at radio frequencies.

Gemant† has used the bridge to investigate the harmonics which appear in the currents flowing in samples of insulating materials; these harmonics are different for different materials and seem to be a characteristic of a given material. The Schering bridge being balanced for the fundamental, an oscillograph connected to the detector branch points *via* a two-stage resistance-capacitance coupled amplifier will give the wave shape of the desired harmonics.

Application to Oil Testing. The Schering bridge has been applied to test the power-factor of transformer and cable oil as a function of electric stress, the earliest results being recorded by Riley and Scott.‡ A shielded bridge of the type shown in Fig. 127 (*c*) with balanced shielding was used by Gillies.§ By far the most exhaustive investigations so far published have been made in America by Balsbaugh‖ at the Massachusetts Institute of Technology and by Berberich** of the Socony-Vacuum Oil Co.

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* C. Dannatt and S. E. Goodall, "The permittivity and power-factor of micas," *Journal I.E.E.*, Vol. 69, pp. 490–496 (1931). *See* also, B. G. Churcher, C. Dannatt and J. W. Dalgleish, "Contact effects between electrodes and dielectrics," *ibid.*, Vol. 67, pp. 271–290 (1929). L. Hartshorn, W. H. Ward, B. A. Sharpe and B. J. O'Kane, "The effects of electrodes on measurements of permittivity and power-factor on insulating materials in sheet form," *Journal I.E.E.*, Vol. 75, pp. 730–736 (1934). T. I. Jones, "Mercury electrodes for measurements on solid dielectrics at radio frequencies," *Journal I.E.E.*, Vol. 74 pp. 179–186 (1934).

† A. Gemant, "Oszillographie von Strömen in Isolierstoffen," *Arch. f. Elekt.* Vol. 23, pp. 683–694 (1930).

‡ T. N. Riley and T. R. Scott, "Insulating oils for high voltage cables," *Journal I.E.E.*, Vol. 66, pp. 805–830 (1928).

§ J. F. Gillies, "Conduction through transformer oil at high field strengths," *Journal I.E.E.*, Vol. 76, pp. 647–655 (1935).

‖ J. C. Balsbaugh and P. H. Moon, "A bridge for precision power factor measurements on small oil samples," *Trans. Amer. I.E.E.*, Vol. 52, pp. 528–535 (1935). J. C. Balsbaugh and A. Herzenberg, "Comprehensive theory of a power-factor bridge," *Journal F. Inst.*, Vol. 218, pp. 49–98 (1934). J. C. Balsbaugh, N. D. Kenney and A. Herzenberg, "The M.I.T. power-factor bridge and oil cell," *Elec. Eng.*, Vol. 54, pp. 272–279 (1935). R. F. Field, "A large capacitance oil cell," *G.R. Exp.*, Vol. 9, pp. 7–8 (Apr., 1935).

** W. G. Horsch and L. J. Berberich, "A cell for routine electrical measurements on insulating oils," *Rev. Sci. Insts.*, Vol. 5, pp. 194–196 (1934). L. J. Berberich, "Measuring equipment for oil power-factor," *Elec. Eng.*, Vol. 55, pp. 264–268 (1936). G. Keinath, "Messung des Verlustfaktors in Olen," *Arch. f. tech. Mess.*, V. 942–2 (July, 1936).

These workers use a fully shielded symmetrical network of the Hartshorn type, and have investigated and corrected for every type of residual and shielding error. Particular attention has been paid to the design of suitable shielded cells to contain the oil sample, Balsbaugh favouring Pyrex glass and Berberich monel metal. Their papers should be consulted for complete details of theory and construction.

Application to Recording of Small Movements. Schulze and Zickner,* by using a special condenser in which the plates are capable of relative movement, convert small mechanical movements into capacitance changes in a Schering network. The unbalance voltage in the detector branch is applied to the grid of a rectifier triode tube, in the anode circuit of which is an electrical recorder; it is possible to secure a linear relation between the capacitance changes and the motion of the recorder pen, thus securing a continuous record of any small mechanical movements to which the capacitance variations are due. The arrangement has many applications, e.g. in the measurement of movements in loaded structures, the displacement of lathe cutting tools, etc.

Application at Radio Frequencies. Dye and Jones† at the National Physical Laboratory have used the Schering bridge for precision measurements at frequencies up to 10^6 cycles per second. The conjugate symmetrical bridge is used, with source across CD and detector across AB, in combination with a Wagner earth connection to bring the detector branch-points to zero potential. The ratio arms Q and S are equal 1,000 ohm resistors with a series of tappings; equal variable air condensers C_3 are shunted between the point B and the similar tapping on each ratio arm, the shields of the condensers being joined to B. The branch C_2 is a variable air condenser and C_1 a similar condenser, both with their shields joined to A; C_1 can be replaced by C_x, the condenser under test. The Wagner earthing device consists of two similar arms with condensers and resistors in parallel. Let $Q/\rho = S/\rho$ be the tapping across which the condensers C_3 are shunted. Then with C_x in position balance is obtained by successively adjusting C_2 and the condenser C_3 shunting Q, followed by adjustment of the Wagner earth arms until the main and auxiliary bridges balance simultaneously. Replacing C_x by C_1 the double balance is restored by regulating C_1 and C_3; let C_3' be the new value, then if θ_x is the loss-angle of C_x and θ_1 that of C_1 it is easy to prove that

$$C_x = C_1 \text{ and } \theta_x - \theta_1 = \omega Q(C_3' - C_3)/\rho^2.$$

The advantage of the tapped ratio arm is now apparent; for a given angle difference, the observed change $C_3' - C_3$ varies inversely as the frequency, hence by fixing $Q = S$ and varying ρ the same capacitance change can be retained at all frequencies. For simplicity $\rho = 10^n$ where n is an integer.

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* A. Schulze and G. Zickner, "Eine Methode zur Registrierung von Kapazitätsänderungen und ihre Anwendung zur laufenden Beobachtung physikalischer Vorgänge," *Arch. f. Elekt.*, Vol. 24, pp. 111–126 (1930).

† D. W. Dye and T. I. Jones, "A radio-frequency bridge for impedance and power-factor measurements," *Journal I.E.E.*, Vol. 72, pp. 169–181 (1933).

All leads, the resistors Q and S, the shields of C_3 and the Wagner arms are contained in an earthed shield. The source is a shielded tuned-anode triode oscillator with toroidal coils, a range from 10 kilocycles to 1,200 kilocycles being obtainable. The detector is a shielded heterodyne, consisting of a tuned h.f. stage receiving the bridge output on a screen-grid tube, a rectifying stage and two l.f. amplifying stages supplying a telephone. The original paper should be consulted for full details of procedure.

13. Fleming and Dyke's Method. On page 328 it has been pointed out that for sensitivity the ratio branches of a simple condenser bridge must have high resistance, since the impedances of the condenser branches are high, especially when small condensers are to be tested at low frequencies. For example, suppose C_1 and C_2 in Fig. 114 are each of the order of 0·001 microfarad, and the frequency is about 1,000 cycles per second. Then the impedance, $1/\omega C$, is of the order of 150,000 ohms. For a sensitive arrangement, therefore, Q and S must each be of the order of 100,000 ohms or so. Now it is not very easy to produce resistances of this magnitude which will be free from the effects of residual inductance or capacitance, so that the theory of the bridge becomes complicated by the consideration of the residuals in the ratio branches. However, it is quite easy to make variable air condensers of small capacitance (about 0·002 microfarad), and as these are free from losses, they may be used as high impedance ratio branches in a condenser bridge without introducing any residual error.

Fleming and Dyke* have applied this artifice to the Wien bridge of Fig. 115 in a research on the alternating current conductivity of dielectrics, their arrangement being drawn in Fig. 129. The branch AC contains the imperfect condenser which is to be tested, C_1 being its effective capacitance and P its effective shunt resistance. The branch AD contains a variable air condenser in series with a small adjustable resistance R. C_2 and C_3 are variable air condensers to act as ratio branches.

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* J. A. Fleming and G. B. Dyke, "On the power factor and conductivity of dielectrics when tested with alternating currents of telephonic frequency at various temperatures," *Journal, I. E. E.*, Vol. 49, pp. 323–431 (1912). The idea of a four condenser bridge seems to be due to Nernst, *Ann. der Phys.*, Vol. 60, p. 600 (1897). For its use at radio-frequencies, see *Dictionary of Applied Physics*, Vol. 2, p. 132. Also see H. J. Mau, *Arch. f. Elekt.*, Vol. 31, pp. 473–487 (1937).

The impedance operators are $z_1 = 1\Big/\left(\frac{1}{P} + j\omega C_1\right)$, $z_2 = 1/j\omega C_2$, $z_3 = 1/j\omega C_3$, $z_4 = R + \frac{1}{j\omega C_4}$; and the balance equation

$$C_2/C_3 = \left(R - \frac{j}{\omega C_4}\right)\left(\frac{1}{P} + j\omega C_1\right).$$

Separating components,

$$\frac{C_1}{C_4} = \frac{C_2}{C_3} - \frac{R}{P},$$

and $C_1C_4 = 1/\omega^2 PR$.

The vector diagram follows at once from that for the Wien bridge, as a comparison of Figs. 129 and 115 will show.

Solving these equations for C_1 and P gives,

$$C_1 = \frac{C_2}{C_3} \cdot \frac{C_4}{1 + \omega^2 C_4^2 R^2}$$

$$P = \frac{C_3}{C_2} \cdot \frac{1 + \omega^2 C_4^2 R^2}{\omega^2 C_4^2 R}.$$

Hence the frequency must be known and constant, and the wave form must be pure if telephones are used to detect balance.

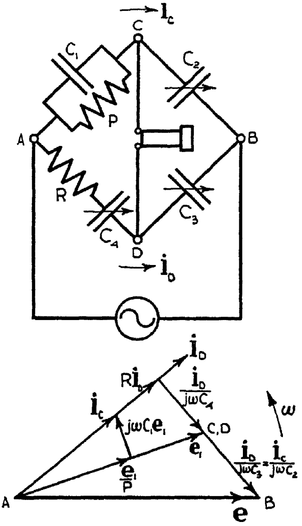

FIG. 129.—FLEMING AND DYKE'S FOUR CONDENSER BRIDGE

In practical working, the condensers C_1, C_2, C_3, C_4 are arranged to be as nearly equal as possible. A suitable value of C_4 being chosen, balance is secured by successive adjustments of C_2/C_3 and R. The condensers should not be too close together or mutual capacitances will exist between them and vitiate the results. Nor must the body of the observer be brought too near the condensers when balance is nearly obtained. All connections should be made of fine wire, so that their capacitance is reduced to a negligible amount.

Experimental Example. Fleming and Dyke describe the following illustrative experiment carried out at 4,400 cycles per second, a high resistance telephone being used to find the balance point. An air condenser C_1 of 430 $\mu\mu$F. was shunted with a resistance $P = 2{\cdot}15$ megohm. At balance, $C_4 = 476$ $\mu\mu$F., $C_2 = 1130$ $\mu\mu$F., $C_3 = 1216$ $\mu\mu$F., and $R = 2975$ ohms. Using these values, $C_1 = 443$ $\mu\mu$F., and $P = 2{\cdot}10$ megohm, which are in good agreement with the actual values. A large number of tests on small imperfect condensers are described in their paper, to which the student is referred for further details.

If a symmetrical bridge be arranged, as suggested, the effects of earth capacitances between the branches and earth will be small. After balance has been obtained, the connections to the alternator should be reversed. Balance will, in general, be slightly disturbed and should be restored by small adjustments of C_2/C_3 and R. The mean of the two settings may be taken as the correct result.

In accurate tests, C_2, C_3, and C_4 should be provided with shields. The resistance R should then be connected in the branch AD on the detector side of C_4, not on the alternator side as shown in the figure; the shield of C_4 should be joined to one terminal of the resistance R, whose other terminal is joined to D. The shields of C_2 and C_3 should be connected to C and D respectively. C_1 should, if possible, be provided with a shield joined to the branch point C. Earth capacitance effects at the points CD are eliminated by the Wagner earthing device (p. 542). These precautions of shielding should be taken in the case of any bridge in which small air condensers are used. The detector should have a high impedance or should be connected through a step-up transformer to the bridge.

Fleming and Dyke carried out an extensive research by this method. Their source of current was a 900 cycle Crompton alternator with a very impure wave form. Pure waves were obtained for use on the bridge by means of a " wave filter " which resonated a chosen harmonic in the alternator wave. By this means a range of 900 to 5,000 cycles per second was covered. As the tests had reference to telephonic work, low voltages were used on the bridge (4 to 5 volts). The detector was a high resistance telephone. Very consistent results were obtained, their paper being well worthy of study by those intending to make power factor measurements on small condensers, especially at high frequencies.

NETWORKS CONTAINING RESISTANCE, SELF-INDUCTANCE AND CAPACITANCE

14. Maxwell's Method. A large class of networks, in which self-inductance and capacitance are compared, is developed from a ballistic method introduced by Maxwell.* Fig. 130 shows Wien's† arrangement of the Maxwell bridge for use with alternating current.

The branch impedance operators are $z_1 = P + j\omega L$, $z_2 = Q$, $z_3 = S/(1 + j\omega CS)$, $z_4 = R$, so that balance occurs when

$$S\,\frac{(P + j\omega L)}{(1 + j\omega CS)} - QR = 0\,;$$

i.e. when

$$SP = QR = L/C.$$

These balance conditions are illustrated by the vector diagram, the construction of which is self-evident.

In practice, Q, R, and S are non-inductive resistances. If L and C are both fixed in value, balance must be secured by successive adjustments of Q/S and R. As these two adjustments interfere, balancing the bridge is often a tedious process. Considerable simplification results if the branch AC contains a variable inductance or if the condenser be sub-divided. It is then possible to fix two of these resistances and to balance by successive adjustment of the third resistance and either the inductometer or the condenser, as the case may be.

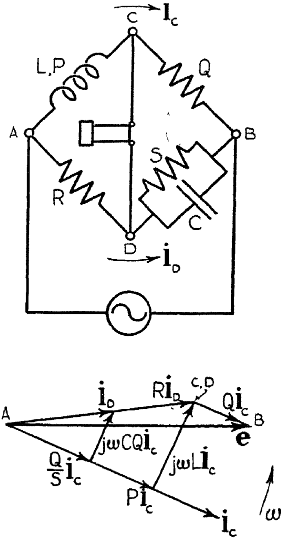

FIG. 130.—MAXWELL'S METHOD FOR COMPARING SELF-INDUCTANCE WITH CAPACITANCE

Experimental Example. The following results were obtained in a test at 407·1 cycles per second in which a mica condenser C of $0{\cdot}334_6$ μF, was compared with a coil of about 40 mH. A resistance of 350 ohms was joined in series with the coil, balance being obtained by successive adjustments of Q, S, and R, using a Duddell vibration galvanometer as detector. The values found were $Q = 346{\cdot}7_6$ ohms, $S = 342{\cdot}3_9$ ohms, and $R = 350{\cdot}5_0$ ohms, whence $L = 40{\cdot}66_7$ mH. Also, $P = 354{\cdot}97$ ohms, so that the effective resistance of the coil is 4·97 ohms.

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* *A Treatise on Electricity and Magnetism*, 1st Edn., Vol. 2, pp. 377–379 (1873). For further descriptions of this ballistic method, *see* E. C. Rimington, "Self-induction and its measurement," *Tel. J.*, Vol. 21, (1887). W. E. Sumpner, "The variation of the coefficients of induction," *Proc Phys. Soc.*, Vol. 9, pp. 235–259 (1888). A. Russell, "Measuring coefficients of induction," *Elecn.*, Vol. 33, pp. 5–6 (1894).

† Max Wien, "Messung der Inductionsconstanten mit dem 'optische Telephon,'" *Ann. der Phys.*, Vol. 44, pp. 689–712 (1891).

C. E. Hay,* in England, and H. Curtis and F. Grover,† in America have described the use of Maxwell's method to measure the small residual inductances of resistance coils. It is necessary to make allowance for residuals in the branches Q, R, and S, P being the resistance of which the residual inductance L is to be measured. Hay's procedure is then as follows. Let R and S be ratio coils having equal resistances and residuals, and suppose Q to be a constant inductance rheostat. With L, P in the bridge, balance by adjustment of C and Q. Then replace L, P by a standard of nearly equal resistance and calculable inductance L', e.g. a parallel wire resistance, re-balancing by adjustment of C and slight alteration of Q to a value Q'. Then it is easy to prove, since the inductance of the branch CB remains unchanged, that $L = L' + R(CQ - C'Q')$; or assuming the standard to have a resistance exactly equal to that of the coil under test, $L = L' + QR(C - C')$, to a very high degree of approximation. Residual effects are thus very nearly, if not entirely, eliminated.

Grover and Curtis slightly modify this procedure by fixing the resistances Q, R, and S. A rheostat is then included in the branch AC, the bridge being balanced by successive adjustments of this rheostat and the condenser. If ΔC be the change of capacitance necessary to secure balance when the test coil is substituted for the standard, and Δl be the change of inductance corresponding to the adjustment of the rheostat in the branch AC, then $L = L' + QR \, . \, \Delta C - \Delta l$.

The method has been applied by Goldstein‡ to measure the iron losses of a transformer. The exciting winding forms the branch AC and, when the bridge is balanced, L is the effective inductance and P the loss resistance of the transformer on open circuit; neglecting the copper loss, P represents the iron loss. Measure V, the voltage across AC; then the watts supplied to that branch will be

$$W = V^2P/(P^2 + \omega^2L^2)$$

But $$P = QR/S \text{ and } L = CQR, \text{ so that}$$

$$W = V^2\frac{S}{QR} \cdot \frac{1}{(1 + \omega^2C^2S^2)}$$

In practice R is of the order 0·1 to 1 ohm and Q is large; the conjugate arrangement of alternator and detector is used, with D earthed. The detector is a vibration galvanometer combined with a filter circuit to cut out harmonic currents. Tests of a 20,000 kVA., 50-cycle transformer, gave $V = 3{,}000$ V., $R = 0{\cdot}3067$ ohm, $S = 12{,}670$ ohm,

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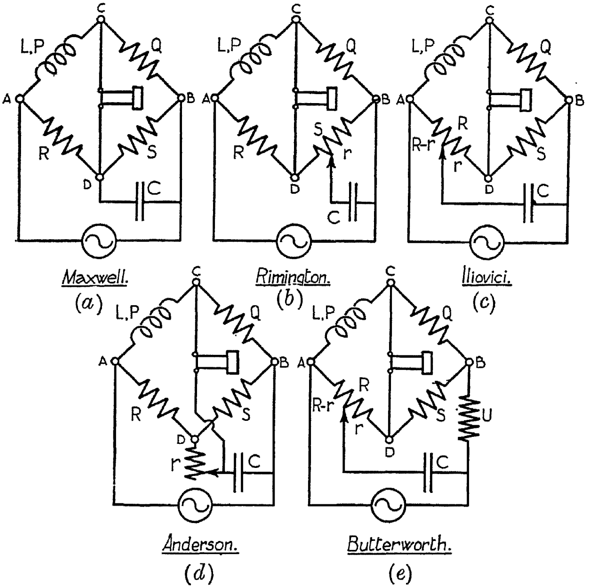
* C. E. Hay, "Alternate current measurements, with special reference to cables, loading coils, and the construction of non-reactive resistances," *Journal P.O.E.E.*, Vol. 5, pp. 451–454 (1913); also *Professional Papers*, No. 53, pp. 19–22.

† F. W. Grover and H. L. Curtis, "The measurement of the inductances of resistance coils," *Bull. Bur. Stds.*, Vol. 8, p. 462 (1913).

‡ J. Goldstein, "Eisenverlustmessungen nach ein Brücken-methode," *Elekt. Zeits.*, Vol. 45, pp. 1270–1272 (1924).

$Q = 248{,}000$ ohm, and $C = 1{\cdot}269$ μF., giving $W = 56{,}200$ W.; a watt-meter test gave 53,000 W.

15. Modifications of Maxwell's Method. The apparatus required for the measurement of a self-inductance by Maxwell's method is simple and would be found in any laboratory. If, therefore, some means could be devised to remove the troublesome interference of the balance adjustments, the method would have great practical convenience. In Fig. 131 several modifications of Maxwell's bridge are shown, having for their common object the simplification of the balancing procedure. It is proposed in succeeding sections to describe each of these bridges in detail, and to point out their particular advantages.



Maxwell. (*a*) Rimington. (*b*) Iliovici. (*c*)

Anderson. (*d*) Butterworth. (*e*)

FIG. 131.—MODIFICATIONS OF MAXWELL'S METHOD FOR COMPARING SELF-INDUCTANCE WITH CAPACITANCE

16. Rimington's Method. The arrangement shown in Fig. 131 (*b*) was introduced independently by E. C. Rimington* and C. Niven,† in 1887, as a modification of Maxwell's ballistic bridge. Their procedure is to balance the bridge for steady current, so that $SP = QR$, and then to adjust the tapping on the resistance S until the bridge is also balanced for make and break. When this ballistic balance is secured, then $L = CPr^2/S$. Rimington has shown that the balance obtained in this way is for aggregate zero quantity through the galvanometer, so that the method is not primarily suitable for alternating current.

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* E. C. Rimington, *loc. cit.*; also "On a modification of a method of Maxwell's for measuring the coefficient of self-induction," *Proc. Phys. Soc.*, Vol. 9, pp. 26–32 (1888).

† C. Niven, "On some methods of determining and comparing coefficients of mutual induction," *Phil. Mag.*, 5th series, Vol. 24, pp. 225–238 (1887).

It was shown later by Dalton* that Rimington's method can be used with alternating current, provided that the initial steady current balance condition be abandoned. Let P, Q, and S be fixed, balance being obtained by successive adjustments of R and r. Then, writing $z_1 = P + j\omega L$, $z_2 = Q$, $z_3 = (S - r) + [r/(1 + j\omega Cr)]$, $z_4 = R$, the balance equation is

$$(P + j\omega L)\left\{(S - r) + \frac{r}{1 + j\omega Cr}\right\} = QR$$

Separating the components gives for the balance conditions,

$$LC = \frac{SP - QR}{\omega^2 r(S - r)},$$

$$L/C = \frac{r}{S}\{Pr - (SP - QR)\},$$

nce L may be found in terms of the resistances and ω. A knowledge frequency becomes necessary, and it should be observed that L, C, and all the resistances are fixed, balance is only possible at requency. The method is not convenient in practice.

is easily seen from the above equations why the condition $SP = QR$ admissible when the bridge is used with alternating current. the first equation, if $SP = QR$ then $\omega^2 = 0$; so that balance is possible for very slow alternations, i.e. ballistically. The second tion then gives $L/C = r^2 P/S$. If, in addition, $S = r$, the first tion is satisfied whatever be the value of ω, provided $SP = QR$. method is then Maxwell's, in which continuous balance is always sible, and $L/C = SP$.

17. Iliovici's Method. A modification of Maxwell's method, aving some resemblance to Rimington's arrangement, is own in Fig. 131 (c). This network has been introduced independently by Iliovici,† Butterworth,‡ Orlich,§ and Dalton‖ various dates between 1904 and 1914.

Comparing this diagram with Fig. 22, Iliovici's method is seen to be a six-branch network for which the balance equation is given on page 56 as

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* J. P. Dalton, "On a new continuous balance method of comparing an ductance with a capacity," *Phil. Mag.*, 6th series, Vol. 27, pp. 37–44 (1914).

† M. Iliovici, "Sur une méthode propre à mesurer les coefficients de self induction," *Comptes Rendus*, Vol. 138, pp. 1411–1413 (1904).

‡ S. Butterworth, "On the vibration galvanometer and its application to inductance bridges," *Proc. Phys. Soc.*, Vol. 24, pp. 75–94 (1912).

§ E. Orlich, *Kapazität und Induktivität*, p. 260.

‖ J. A. Dalton, *loc. cit.*

$$z_7(z_1z_3 - z_2z_4) = z_2\{z_6(z_3 + z_4) + z_3z_4\}.$$

Substituting $z_1 = P + j\omega L$, $z_2 = Q$, $z_3 = 1/j\omega C$, $z_4 = R - r$, $z_6 = r$, $z_7 = S$, gives

$$S[P + j\omega\{L - QC(R - r)\}] = Q\{R + j\omega Cr(R - r)\}.$$

Separating components,

$$SP = QR,$$

and

$$L = \frac{CQ}{S}(R - r)\ (S + r)$$

are the balance conditions. These do not interfere with one another and are free from the objections to those of Rimington's method.

In practice, it is best to use an equal ratio bridge, setting $P = Q$ and $S = R$. Balance is obtained by adjustment of the condenser tapping r and small alteration of S or Q. For rapid work, the resistance R should be a slide wire, upon which the position of tapping r can be conveniently adjusted.

Experimental Example. An experiment was made using the coil and condenser compared by Maxwell's method (*see* p. 375), the detector and frequency remaining the same. The branch AD was made up of two decade resistances joined by a short slide wire, the total value of R being 2,000 ohms. Q was fixed at 1,000 ohms and a resistance of 910 ohms was joined in series with the coil L. At balance, $r = 1935{\cdot}6$ ohms and $S = 2185{\cdot}3$ ohms, so that $L = 40{\cdot}63_5$ mH. Also, $P = 915{\cdot}2$ ohms, which makes the effective resistance of the coil equal to 5·2 ohms.

18. Anderson's Method. Of all the modifications of Maxwell's method that introduced in 1891 by A. Anderson* is the most important and the most useful. Originally used ballistically, H. Rowland,† in 1898, adapted the method for use with alternating current, the arrangement of the network being shown in Figs. 131 (*d*) and 132.

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* A. Anderson, "On coefficients of induction," *Phil. Mag.*, 5th series, Vol. 31, pp. 329–337 (1891).

† H. Rowland, "Electrical measurement by alternating currents," *Amer. J. Sc.*, 4th series, Vol. 4, pp. 429–448 (1897); *Phil. Mag.*, 5th series, Vol. 45, pp. 66–85 (1898).

Anderson's method appears to have been first used for the accurate measurement of inductance by Prof. J. A. Fleming and W. C. Clinton,* in 1903. These investigators employed a commutator arrangement of the secohmmeter type, supplying the bridge with interrupted current and using a ballistic galvanometer to detect balance. Prof. Fleming,† in 1905, described experiments in which inductances of the order of 20 millihenrys were measured, the source of current being an electromagnetic buzzer and the detector a telephone. In both these uses of the bridge the supply of current is of an irregular wave form and is not a true alternating current. As mentioned above, H. Rowland, in 1898, had used a regular alternating source of supply, his detector being an electrodynamometer. W. Stroud and J. H. Oates,‡ in 1903, using similar apparatus, recorded measurements of inductance lying between a few millihenrys and about a henry. The full advantages of Anderson's bridge were not realized, however, until 1905, when E. Rosa and F. W. Grover,§ of the U.S. Bureau of Standards, applied the vibration galvanometer to the network, thereby converting it into a method for measuring inductance with very high precision.‖ The discussion of the bridge which follows is largely adapted from their paper on the subject and from other publications of the Bureau.

Referring to Fig. 132, let the impedance operators of the various branches be $z_1 = P + j\omega L$, $z_2 = Q$, $z_3 = S$, $z_4 = R$, $z_6 = r$, $z_7 = 1/j\omega C$. Putting these values in the balance equation on p. 56 gives, as the symbolic condition for balance

$$SP - QR + j\omega LS = j\omega CQ\{r(S + R) + SR\}.$$

Separating the components, there will be no current in the detector when

$$SP = QR,$$

and

$$L = CQ\left[r\left(1 + \frac{R}{S}\right) + R\right]. \qquad (a)$$

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* J. A. Fleming and W. C. Clinton, "On the measurement of small capacities and inductances," *Proc. Phys. Soc.*, Vol. 18, pp. 386–409 (1903).

† J. A. Fleming, "Note on the measurement of small inductances and capacities, and on a standard of small inductance," *Proc. Phys. Soc.*, Vol. 19, pp. 160–172 (1905).

‡ W. Stroud and J. H. Oates, "On the application of alternating currents to the calibration of capacity boxes, and to the comparison of capacities and inductances," *Phil. Mag.*, 6th series, Vol. 6, pp. 707–720 (1903).

§ E. B. Rosa and F. W. Grover, "Measurement of inductance by Anderson's method, using alternating currents and a vibration galvanometer," *Bull. Bur. Stds.*, Vol. 1, pp. 291–336 (1905).

‖ Also *see* A. de F. Palmer, "An inductance and capacity bridge," *Phys. Rev.*, Vol. 23, pp. 55–63 (1906). C. Günther, "Eine neue Messbrücke zur Bestimmung von Selbstinduktionen," *Zeits. f. Inst.*, Vol. 46, pp. 623–631 (1926); *Zeits. f. Inst.*, Vol. 47, pp. 249–256 (1927). J. Krönert, "Wechselstrom-Brücken vom Anderson-Typ," *Arch f. tech. Mess.*, J. 921–1 (Aug., 1931). R. D. Salmon, "Inductance measurement. The Owen and Anderson bridge networks," *Elecn.*, Vol. 113, pp. 827–829 (1934).

Making use of the first of these conditions in the second, the inductance is given by

$$L = C[r(Q + P) + QR], \quad . \quad . \quad . \quad (b)$$

when the condenser is perfect and the resistances are free from residuals. The bridge can be balanced by independent adjustment of one of the branches (P, Q, R, or S) and r. Moreover, the range of L covered by a given bridge and condenser is very considerable, since r may have any desired value.

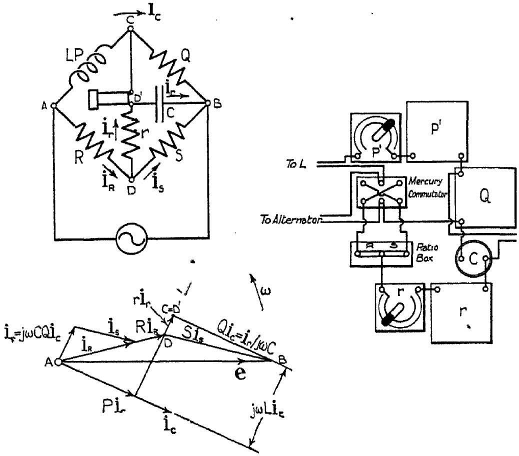

FIG. 132.—ANDERSON'S METHOD FOR COMPARING SELF-INDUCTANCE WITH CAPACITANCE

The vector diagram for the balanced bridge is easily constructed. In Fig. 132 let the vector AB denote the voltage $\mathbf{e}$, across the points A and B. In the branch ACB a current $\mathbf{i}_C$ is flowing, of magnitude and phase given by $\mathbf{i}_C = \mathbf{e}/z_{AB}$, so that $\mathbf{e} = (P + j\omega L)\mathbf{i}_C + Q\mathbf{i}_C$. The potential of the point C is thus represented by the point C on the diagram of vectors; and since no current flows in the detector, C is also the potential of the point D'. The potential difference across the condenser is $Q\mathbf{i}_C$, so that the current flowing through it must be $\mathbf{i}_r = j\omega CQ\,\mathbf{i}_C$. This current also flows in the resistance r, producing a drop of potential $r\mathbf{i}_r$; the vector sum of this and the vector $Q\mathbf{i}_c = \mathbf{i}_r/j\omega C$ being the potential difference across the resistance S. Thus the magnitude and phase of the current $\mathbf{i}_s$ is determined, and the rest of the diagram may be easily completed.

In setting up Anderson's bridge, it is desirable to arrange the various resistances so that the greatest sensitivity is attained. Assuming the bridge to be adjusted so that it is nearly balanced, and that a vibration galvanometer is used, Mr. Butterworth* has shown that, for the greatest sensitivity to a change in r, the resistances should be arranged so that $Q = P$, $S = R = \frac{1}{2}P$ and $L/C = 2P^2$. At the same time, the frequency should be low to allow the galvanometer to work at the most sensitive part of its range. Thus, the conditions for best sensitivity of Anderson's bridge make it an equal ratio method, which is precisely what is required to reduce residual errors to a minimum.

A convenient practical arrangement of the necessary apparatus is shown in Fig. 132. The branches R and S are equal coils in a ratio box, and are connected to the bridge through a reversing switch, so that, by taking two balances with their positions interchanged, any small differences of the residual inductances of R and S may be eliminated in the mean. The resistance Q is composed of a non-inductive box, and is set to a fixed value; the rheostat r consists of a resistance box and a fine adjustment rheostat. The condenser may be a suitable mica standard and, as will be shown later, need only be fairly good, provided that its calibration with alternating current be known. The remaining branch contains the coil, the inductance of which is to be determined, together with a resistance box similar to Q and a fine adjustment rheostat (P'). The following is then the best procedure to adopt—

(i) Choose a reasonable value of C, so that L/C makes $P = Q$ of reasonable magnitude. Set Q to this value.

(ii) Make $S = R = \frac{1}{2}P = \frac{1}{2}Q$.

(iii) Adjust successively the resistances P' in the inductance branch and r until balance is attained.

(iv) Calculate L. Since $P = Q$, Equation (b) gives

$$L = CQ[R + 2r].$$

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* S. Butterworth, "On the vibration galvanometer and its application to inductance bridges," *Proc. Phys. Soc.*, Vol. 24, p. 83 (1912).

(v) Repeat with the positions of R and S interchanged; re-calculate L, and take the mean of the two values as the true inductance of the coil.

It may be found in practical working that the resistance r has a fairly large value. As this resistance lies in the detector circuit, it may, when large, considerably reduce the sensitivity of setting. In such a case the sensitiveness can be increased by interchanging the source and detector and increasing the voltage applied to the bridge in order to allow for the effect of r, which will then lie in series with the source. This modification is virtually that introduced by Stroud and Oates.

Experimental Example. The coil previously tested by Maxwell's and by Iliovici's methods was compared with a mica condenser of $0{\cdot}334_6\ \mu$F. capacitance at a frequency of 407·1 cycles per second, using a Duddell vibration galvanometer and an arrangement of apparatus similar to that shown in Fig. 132. Since L is about 40 mH., $Q = \sqrt{L/2C}$ must be about 240 ohms to satisfy the sensitivity condition. Hence, Q was set to values of 200, 250, and 300 ohms; $R = S$ were kept at 100 ohms each. For each value of Q balance was found by adjusting successively the resistance r and the resistances P' in series with L. After each balance the ratio coils were reversed in the bridge, but it was not found that any change in balance was caused thereby. The observations are given in the table—

Q ohms.	r ohms.	P' ohms.	L mH.	Eff. res. of Coil ohms.
200	$253{\cdot}9_0$	$194{\cdot}8_7$	$40{\cdot}67_5$	$5{\cdot}1_3$
250	$193{\cdot}1_0$	$244{\cdot}9_2$	$40{\cdot}67_1$	$5{\cdot}0_8$
300	$152{\cdot}6_7$	$294{\cdot}9_1$	$40{\cdot}68_8$	$5{\cdot}0_9$
		Average	**$40{\cdot}67_8$** mH.	**$5{\cdot}1_0$** ohms

19. Sources of Error in Anderson's Method. Anderson's method is particularly applicable to precise measurements of inductance; as the method has often been used for such measurements, it is important to examine the various possible sources of error and to determine their effects upon observations obtained from the bridge.

Residual Inductance or Capacitance in the Resistances. The various resistances used to make up the bridge may have slight residual inductance or capacitance, and error in the measurement of L may be introduced thereby. If R and S be made equal, slight differences in their resistances or their residuals can be eliminated in the mean of two balances obtained by reversing their positions in the bridge. However, differences between P and Q can not be so eliminated, since these branches are not made up in a similar way ; P is composed of the resistance of the coil to be measured and a resistance box, whereas Q is merely a resistance box. Hence only the non-inductive part of P can balance the residuals of a corresponding number of similar coils in Q. Moreover, the resistance of L is that of a copper coil, whereas the rheostats in P and Q are usually of manganin ; hence there may be a drift in the balance due to temperature differences. Butterworth* has overcome the difficulty of balancing the residuals of P and Q, and also the effects of temperature by making up the branch Q of a rheostat exactly similar to that in P, together with a non-inductive copper coil equal in resistance to the inductance L. The latter and the balancing coil are enclosed in a wool-lined box so that they may have the same temperature.

The residual inductance of a resistance may be positive or negative, according to whether inductance or capacitance effects predominate. If l_P, l_Q, l_R, l_S, and l_r be the residual inductances (plus or minus) of P, Q, R, S, and r, it is easy to prove† that

$$L = CQ\left[r\left(1+\frac{R}{S}\right)+R\right]+\frac{1}{S}\left[l_RQ-l_PS-l_SP+l_QR\right] \qquad . \ (c)$$

$$-\frac{\omega^2 C}{S}\left[rl_Q(l_R+l_S)+Q(l_Rl_S+l_Sl_r+l_rl_R)+Sl_Q(l_R+l_r)+Rl_Q(l_S+l_r)\right].$$

Comparing the first term with Equation (a) it will be seen that it represents the value obtained for the inductance when all the resistances are assumed perfect ; call it L_0. Then, if α and β be written for the corrections introduced by the residuals,

$$L = L_0 + \alpha - \beta.$$

It is found in practice that β is negligible, except at the highest frequencies ; hence, since β alone contains l_r, *a small residual in r does not introduce any important error.* The correction α includes two terms, namely, $(l_QR - l_PS)$ and $(l_RQ - l_SP)$. If R and S be made equal and reversed, the second term is eliminated from the mean, leaving a correction $\alpha = l_Q - l_P$. Hence the make-up of Q and P should be as nearly the same as possible, so that α is a very small quantity.

Rosa and Grover have calculated the magnitude of the errors to be expected, the results being tabulated below.

Inductance to be Measured.	C	$R=S$	l_R	l_S	$P=Q$	l_P	l_Q	r	α
Milli-henrys.	Micro-farads.	Ohms.	Micro-henrys.	Micro-henrys.	Ohms.	Micro-henrys.	Micro-henrys.	Ohms.	Milli-henrys.
100	1·0	250	+ 2	−2	250	−2	+ 2	75	0·008
10	0·4	100	+ 1	−1·0	100	−1	+ 1	75	0·004
1	0·1	50	+ 0·5	−0·5	100	−1	+ 1	25	0·004
0·1	0·05	20	+ 0·5	−0·5	50	−0·5	+ 0·5	10	0·0045
0·01	0·02	20	+ 0·5	−0·5	20	−0·5	+ 0·5	2·5	0·002

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* S. Butterworth, " On the use of Anderson's bridge for the measurement of the variations of the capacity and effective resistance of a condenser with frequency," *Proc. Phys. Soc.*, Vol. 34, pp. 1–7 (1921).

† *See* Rosa and Grover, *loc. cit.*, pp. 304–311. The student should deduce it from the equations on p. 56.

The preceding theory shows that—

(i) Residuals in R and S are eliminated by making them equal and reversing them in the bridge.

(ii) Small residual inductance in r produces an inappreciable error.

(iii) The difference between the residuals of P and Q should be made as small as possible—especially when measuring low inductances—by making up these branches of similar coils.

Experiments are also given to verify the results predicted by the theory. From this table it is clear that large errors may be introduced into the measurement of small inductances, unless the resistances are wound so that they have the smallest possible residuals. If the residual of the rheostat in series with the inductive coil is known, accuracy can be attained by balancing the bridge with Q, S, and R fixed in value, (i) when the inductance is in place, and (ii) when it is removed and its resistance compensated for by a coil of known residual inductance. The difference then gives the inductance of the coil to be measured.

Errors Due to the Leads. The inductance or capacitance of the leads connecting L to the bridge will be included in the measured value of the inductance. In precise work, it is necessary to allow for the effect of the leads and to arrange them so that they introduce only a very small error (*see* p. 522).

Self Capacitance of L. As shown on p. 319, the self capacitance of the inductance under test will affect the measured value by an amount depending on the true value of the inductance, its self capacitance, and the square of the frequency. In accurate work, it is necessary to make some allowance for the effect, more especially at high frequencies. For example, in a certain coil which has a true inductance of 1 henry and a capacitance of 10^{-10} farad, the measured value at 112 cycles per second will be 1·00005 henry, which is an inappreciable correction. But if the frequency is 1,120, then the measured value becomes 1·005 henry, which is an important correction.

Imperfection of the Condenser C. In the above discussion, the condenser has been assumed perfect, i.e. its insulation resistance has been taken as infinite, and the current through the condenser has been assumed to lead on the applied potential difference by exactly a quarter of a period. Now, in practice, the insulation resistance of a condenser, though usually very high, is certainly not infinitely great. Moreover, in condensers which have solid dielectrics there is always a certain amount of energy absorption, so that the phase displacement is less than $\pi/2$. It is necessary, therefore, to examine the errors which these two factors will introduce.

Taking first the effect of insulation resistance, a leakage across the condenser can be represented by a resistance, R_l, in parallel with the condenser; R_l is seldom less than 1,000 megohms in a good mica condenser. In the analysis preceding Equation (a) substitute for $z_7 = 1/j\omega C$ the term $R_l/(1 + j\omega CR_l)$. Then the balance equation is

$$R_l\{SP - QR + j\omega LS\} = Q\{r(S + R) + SR\} \cdot \{1 + j\omega CR_l\}$$

Separating components gives

$$SP = QR + \frac{Q}{R_l}\{r(S + R) + SR\} = QR + \frac{L_0 S}{CR_l}$$

and $$L = CQ\left[r\left(1 + \frac{R}{S}\right) + R\right] = L_0.$$

The second balance condition is identical with that obtained when the condenser is perfect; hence *leakage has no effect upon the measured value of inductance, and only a small effect on the measured value of P.*

The phase displacement due to the absorption of energy in an imperfect dielectric can be accounted for by a resistance ρ in series with the condenser. In a good mica condenser the phase displacement will fall short of 90 degrees by a very small angle, so that ρ is a small resistance. In the analysis before Equation (a), put $z_7 = \rho + \frac{1}{j\omega C}$; the balance equation is

$$(1 + j\omega C\rho)\,[SP - QR + j\omega LS] = j\omega CQ[r(S + R) + SR].$$

Separating the components makes

$$SP - QR = \omega^2 LCS\rho$$

and $$L = CQ\left[r\left(1 + \frac{R}{S}\right) + R\right] - \frac{C\rho(SP - QR)}{S}$$

In the second equation, the first term is the value of the measured inductance when the condenser is perfect, namely, L_0. Substituting from the first equation in the last term of the second, gives

$$L = L_0 - \omega^2 C^2 \rho^2 L.$$

Now the angle by which the phase displacement falls short of $\pi/2$ is $\theta = \arctan \omega C\rho$ (*see* p. 190) so that

$$L_0 = L(1 + \tan^2 \theta).$$

The angle θ is usually small, so that very nearly

$$L = L_0\,(1 - \tan^2\theta).$$

If in a good mica condenser θ were half a minute of arc, the correction would only amount to 2 parts in 10^8; it would be allowable for θ to attain a value of 5 or 6 minutes without producing any marked error. Hence, *absorption in the dielectric has usually a negligible effect.*

20. Special Uses of Anderson's Method. As described above, Anderson's method has been treated as a means of measuring self-inductance in terms of capacitance. As such, it has a very large range, from the smallest to the largest inductance, depending entirely upon the choice of the various branches of the network.

By a suitable arrangement of Anderson's bridge, the very small inductance of resistance coils can be measured.[*] It is essential in this case to make some allowance for the residual effects in the bridge itself, these being of a magnitude comparable with that of the inductance under test. The procedure is as follows. A standard is first prepared having a resistance as nearly as possible equal to that of the coil to be tested ; this standard is made in a form such that its residuals may be calculated (*see* p. 118). Inserting this standard in AC (Fig. 132) in series with a rheostat, and, if necessary, an auxiliary inductance (so that r may have a reasonable value), let balance be obtained by adjustment of P and r. Substituting the unknown for the standard, re-balance by adjustments of P and r as before. If $\triangle r$ be the change of r and $\triangle l$ the alteration of inductance of the rheostat in AC, it is easy to show that

$$\triangle L = 2CQ\triangle r - \triangle l,\dagger$$

where $\triangle L$ is the difference between the inductance of the standard and the unknown, the bridge having equal ratio branches, $R = S$ Residual errors are thus entirely eliminated.

If a standard self-inductance is available, Anderson's method becomes a very convenient way to measure a condenser with high precision. Butterworth[‡] has recently shown that the variation of capacitance and effective series resistance of a condenser with frequency can be determined from tests made in a specially arranged Anderson bridge.

21. Butterworth's Method.[§] A bridge which is very suitable for the measurements of small inductances is shown in Fig. 131 (*e*), embodying the features of Iliovici's method with those of the conjugate to Anderson's method. The trouble experienced in Iliovici's bridge is that the tapping for the resistance r, by which inductive balance is secured, is made upon one of the resistances entering into the resistance balance, namely, R. Hence R is preferably a slide wire if rapid work is to be done. The use of a slide wire can be avoided if the Anderson principle be adapted to the bridge, so that R may be an ordinary resistance box.

By putting cyclic currents in the four meshes of the network and solving the resulting equations, no current will flow in the detector when

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[*] F. W. Grover and H. L. Curtis, "The measurement of the inductance of resistance coils," *Bull. Bur. Stds.*, Vol. 8, pp. 461–462 (1913). *See* also A. H. Taylor and E. H. Williams, "Distributed capacity in resistance boxes," *Phys. Rev.*, Vol. 26, pp. 417–423 (1908).

[†] The student should verify this result by reference to Equation (*c*), p 384.

[‡] S. Butterworth, "On the use of Anderson's bridge for the measurement of the variations of the capacity and effective series resistance of a condenser with frequency," *Proc. Phys. Soc.*, Vol. 34, pp. 1–7 (1921). *See* also B. V. Hill, "The variation of apparent capacity of a condenser with the time of discharge and the variation of capacity with frequency in alternating current measurements," *Phys. Rev.*, Vol. 26, pp. 400–405 (1908).

[§] S. Butterworth, "A method of measuring small inductances," *Proc. Phys. Soc.*, Vol. 24, pp. 210–214 (1912).

$$SP = QR,$$

and
$$L = \frac{(R-r)}{S}\left[(Q+S)U + Q(S+r)\right]C.$$

Thus, by choice of $(R-r)$, inductances of any value can be measured by means of a single condenser; and also inductive balance is secured independently of resistance balance by adjustment of U.

To determine the best conditions for working the bridge, let V be the total resistance of the source between the branch points A and B, and G that of the detector, reactances being neglected. Then Butterworth has shown that the condenser has little influence on the sensitivity, and that the best values of the branch resistances are approximately

$$S = \quad Q = \sqrt{PG\cdot\frac{P+V}{P+G}} \text{ and } R = \sqrt{PV\cdot\frac{P+G}{P+V}},$$

where U is included in V and, for greatest sensitivity, should be kept small (*see* page 82).

In practice, R and S should be set to the values for maximum sensitivity, suitable values of P, C, and r being chosen. Balance should then be obtained by successive adjustments of Q and U. If a vibration galvanometer be used, the frequency should be low.

Experimental Example. The following results were obtained at a frequency of 407·1 cycles per second, using a Duddell vibration galvanometer. The coil L was joined in series with a resistance box, but in three tests P consisted of the resistance of the coil alone. S was fixed at 1,000 ohms throughout; R consisted of a $4{\cdot}08_1$ ohm slide wire in series with a resistance box set to some definite value. The position of the slider was set to give a desired value of r. Balance was found by successive alterations of Q and U, using a condenser of 0·1010 μF. The results are tabulated below—

R ohms.	r ohms.	P ohms.	Q ohms.	U ohms.	L μH.
$4{\cdot}08_1$	$2{\cdot}72_1$	$0{\cdot}64_6$	157	188	$51{\cdot}4_4$
$4{\cdot}08_1$	$3{\cdot}49_8$	$0{\cdot}64_6$	157	615	$51{\cdot}1_5$
$4{\cdot}08_1$	$3{\cdot}49_8$	$1{\cdot}64_6$	403	332	$51{\cdot}2_6$
$14{\cdot}08_1$	$12{\cdot}72_1$	$0{\cdot}64_6$	45·9	315	$51{\cdot}5_8$
				Average	**$51{\cdot}4_8$ μH.**

By this method, Butterworth has been able to measure a coil of 20 microhenrys with an agreement among several observations within 1 per cent of the mean at a frequency of 100, C being 0·1 microfarad. A Duddell vibration galvanometer for which $G \doteqdot 200$ ohms was used.

22. Methods for Large Inductances.

When the time-constant of a coil is large it becomes a matter of difficulty to measure the inductance and effective resistance of the coil in an ordinary inductance bridge. For example, "loading coils," used in telephony for the purpose of improving speech transmission are constructed so that they have a large inductance, although the amount of wire in them, and therefore their resistance and losses, is maintained small. In a typical case, a coil which had an inductance of about 140 millihenrys had a resistance of only 6 ohms, the time-constant being about 1/40 second. Coils of much higher inductance, sometimes up to 40 henrys, are constructed for use in cable telegraphy, and have even larger time-constants.*

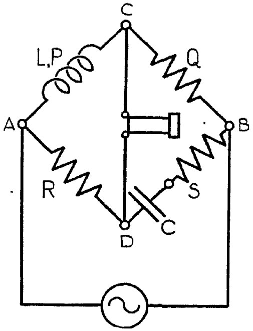

FIG. 133.—HAY'S METHOD FOR MEASURING LARGE INDUCTANCES

C. E. Hay† has devised a simple method for the determination of the inductance and effective resistance of loading coils, the arrangement of the bridge being shown in Fig. 133. The branch AC contains the coil to be tested, CB and AD being composed of non-inductive resistances. The resistance S should be capable of fine adjustment, as also should the condenser C. The latter should be composed of mica condensers with a continuously variable air condenser in parallel with them for attaining final balance.

Taking the branch impedances as $z_1 = P + j\omega L$, $z_2 = Q$, $z_3 = S + \frac{1}{j\omega C}$, and $z_4 = R$, the balance condition will be

$$(P + j\omega L)\left(S + \frac{1}{j\omega C}\right) = QR\,;$$

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* In contrast, it should be noted that reactance coils used in power stations have time-constants as high as 1 second.

† C. E. Hay, "Measurement of self-inductance and effective resistance at high frequencies," *Elec. Rev.*, Vol. 67, pp. 965–966 (1910).

or

$$L/C = (QR - SP),$$

$$LC = P/\omega^2 S.$$

Solving these equations for L and P gives

$$L = QR\frac{C}{(1 + \omega^2 C^2 S^2)},\ P = QR\frac{\omega^2 C^2 S}{(1 + \omega^2 C^2 S^2)},\ L/P = 1/\omega^2 CS,$$

and the time-constant is $1/\omega^2 CS$. Large values are easily provided for by keeping S small.

In practice, Q is fixed; R, S, and, if necessary, C also, are varied until balance is secured. Since the results involve ω, the frequency must be maintained constant and be accurately measured. If telephones are used to detect balance, the wave form must be pure. Insulation of the whole apparatus should be very good; and capacitance errors should be avoided by suitable shielding of the branches, and by taking the bridge current from a well-insulated transformer with an earthed shield between the primary and secondary windings.

Experimental Example. Two sets of tests were made, (i) on a 40 mH. coil of d.c. resistance $5{\cdot}1_7$ ohm at 15° C., and (ii) on a short thick coil of 0·6 henry and 54·1 ohm d.c. resistance at 15° C. In both cases a Duddell vibration galvanometer was used, current being supplied to the bridge at 407·1 cycles per second from a triode oscillator and shielded transformer. L, P is the coil under test, Q a fixed resistance; S and R are each composed of a resistance box in combination with a 1 ohm rheostat. C is a mica condenser of $0{\cdot}334_8$ μF. capacitance; its series loss resistance of 0·30 ohm is included in the tabulated values of S.

(i) 40 mH. Coil

Q ohms.	R ohms.	S ohms.	L mH.	P ohms.
400	$304{\cdot}6_5$	$58{\cdot}4_5$	$40{\cdot}67_4$	$5{\cdot}2_0$
500	$243{\cdot}7_3$	$58{\cdot}4_5$	$40{\cdot}67_5$	$5{\cdot}2_0$
600	$203{\cdot}0_8$	$58{\cdot}4_5$	$40{\cdot}67_1$	$5{\cdot}2_0$
700	$174{\cdot}0_6$	$58{\cdot}4_5$	$40{\cdot}66_7$	$5{\cdot}2_0$
		Average	**$40{\cdot}67_2$** mH.	**$5{\cdot}2_0$** ohms.

The time-constant is thus 0·0078 second.

(ii) 0·6 Henry Coil

Q ohms.	*R* ohms.	*S* ohms.	*L* henry.	*P* ohms.
700	2599	$71{\cdot}5_0$	$0{\cdot}606_5$	$94{\cdot}9_3$
800	2274	$71{\cdot}5_0$	$0{\cdot}606_4$	$94{\cdot}9_2$
900	2021	$71{\cdot}5_0$	$0{\cdot}606_3$	$94{\cdot}9_1$
1000	1819	$71{\cdot}5_0$	$0{\cdot}606_4$	$94{\cdot}9_1$
		Average	**$0{\cdot}606_4$** henry	**$94{\cdot}9_2$** ohms

The time-constant of the coil is 0·0064 second. The very great difference between the effective resistance to a.c. and the d.c. resistance of this coil is particularly to be noticed.

A measurement of increasing importance is the determination of the effective inductance and resistance of iron-cored choking coils and transformers, the windings of which carry a comparatively large direct current, upon which is superposed an alternating current ripple. Landon* and Hartshorn† have independently shown that Hay's bridge is very convenient for the purpose. The arrangement of the network is shown in Fig. 134. The coil to be tested had an inductance of about 20 henrys and carried direct currents up to 250 milliamperes with a superposed alternating current of 1 to 5 milliamperes at 100 cycles per second. The resistances *R* and *Q* have values of 100,000 ohms and 200 ohms respectively; *C* is a mica condenser of 1 μF. capacitance, variable in steps of 0·001 μF.; *S* is a resistance box giving a total of 10,000 ohms in steps of 1 ohm. Balance is secured by adjustment of *C* and *S*. It will

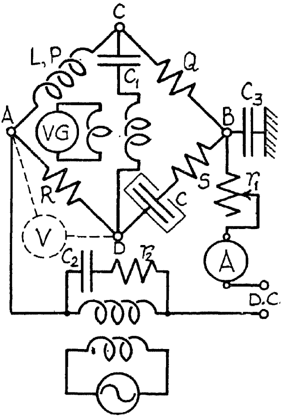

Fig. 134.—Hay's Bridge Arranged for Measurements with Superposed Direct and Alternating Currents

be seen from the positions of the condensers C, C_1, C_2, and C_3 that the direct current passes only through the branches AC and CB in series; since the d.c. resistance of the coil was about 400 ohms, the direct current of 250 milliamperes necessitates a supply of over 100 volts. To avoid unduly large volt-drop and power loss in Q the low value of 200 ohms was chosen. The condenser C prevents the d.c. from passing through BD and AD; the condenser C_1 keeps the d.c. out of the detector circuit. Thus the whole direct current passes through the coil L, P and can be read upon the ammeter, its strength being regulated by the rheostat r_1. The detector is a tuned vibration galvanometer coupled to the bridge by a step-up "output transformer" of the type used with loud speakers; C_1 is regulated to resonate the transformer and greatly increase the sensitivity, a few tenths of a μF. being requisite at 100 cycles per second. The a.c. can be regulated independently of the d.c. by the condenser C_2 of a few μF. and the rheostat r_2. The magnitude of the a.c. is found from the reading of an electrostatic voltmeter connected across R; since this resistance carries only an alternating current, the a.p.d. across it when the bridge is balanced is equal to that across the coil. The alternating current is then $I = V\sqrt{1 + S^2C^2\omega^2}/QRC\omega$, or very nearly $I \doteqdot V/\omega CQR$.

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* V. D. Landon, "A bridge circuit for measuring the inductance of coils while passing direct current," *Proc. Inst. Rad. Eng.*, Vol. 16, pp. 1771–1775 (1928).

† L. Hartshorn, "The measurement of the inductances and effective resistances of iron-cored coils carrying both direct and alternating current," *Journal Sci. Insts.*, Vol. 6, pp. 113–115 (1929). Also see L. E. Ryall, "The measurement of inductance and effective resistance of loading coils," *Journal P.O.E.E.*, Vol. 21, pp. 307–310 (1929), and M. Kobayashi, "Method of impedance measurement of coils with superposed direct current," *Journal I.E.E. Japan*, No. 490, pp. 527–530 (1929).

The balancing procedure is then as follows. Regulate the d.c. to the required value by the use of r_1. The bridge is then balanced approximately with the voltmeter connected across R and any convenient value of the a.c. The reading of the voltmeter corresponding with the desired a.c. is then calculated and the reading adjusted to this value by means of C_2 and r_2. The voltmeter is then disconnected, the bridge finally balanced, the voltage being checked and adjusted if necessary afterwards.

The corner B is earthed through a condenser C_3 of about 10 μF. It would be desirable to earth it directly so that the detector circuit CD should be at a potential not very different from zero, CB and BD being of low impedance. Such a direct connection to earth is not usually permissible, since it would involve earthing one side of the d.c. supply. The use of the condenser C_3 overcomes this difficulty, since it insulates the d.c. supply from earth, but, having a low impedance, acts almost as effectively as direct earthing to the a.c. source. All the apparatus is of standard type, except Q, which has to dissipate about 12 watts and must be larger than the coils usually used in resistance boxes.

Hay's method can also be used as an excellent frequency bridge* of long range, as described by Kurokawa and Hoashi and later by Soucy and Bayly. It is not difficult to show that the balance conditions may be written as

$$f = P/2\pi\left\{LC\left(QR - \frac{L}{C}\right)\right\}^{\frac{1}{2}} \text{ and } S = \frac{1}{P}\left(QR - \frac{L}{C}\right)$$

If L, C, Q, and R are fixed the denominator is constant and $f = kP$; balance is secured by adjusting P and S. Hence the method is a linear frequency bridge. These workers suggest $Q = R = 594{\cdot}39$ ohms, $C = 1\ \mu$F; then with $L = 100$ mH., $f = P$; while with $Q = R = 187{\cdot}96$, $C = 1\ \mu$F., and $L = 10$ mH., $f = 10P$. The first arrangement has a range from 25 to 5,000 cycles per sec., and the second arrangement from 500 to 5,000 cycles per sec. without P or S assuming extreme values.

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* K. Kurokawa and T. Hoashi, "A linear frequency bridge," *Journal I.E.E., Japan*, No. 437, pp. 1132–1138 (1924). C. J. Soucy and B. de F. Bayly, "A direct reading frequency bridge based on Hay's bridge circuit," *Proc. Inst. Rad. Eng.*, Vol. 17, pp. 834–840 (1929), discovered the method independently.

A further method* for inductances with high time-constant is shown in Fig. 135; the circuit resembles Hay's method of Fig. 133 with S made zero and an additional condenser in parallel with R. Balance will occur if

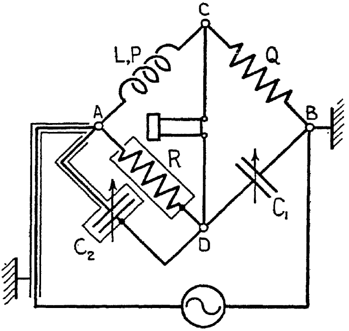

FIG. 135.—BRIDGE FOR LOADING COILS

$$(P + j\omega L)/j\omega C_1 = QR/(1 + j\omega C_2 R),$$

which on separating the components gives

$$P = \omega^2 L C_2 R,$$

$$L + P C_2 R = C_1 Q R.$$

From the first condition, if δ is the phase defect of the coil from 90° lag,

$$\tan \delta = P/\omega L = \omega C_2 R,$$

and from the second condition

$$L = C_1 Q R/(1 + \tan^2 \delta).$$

Small values of δ, i.e. large time-constants, correspond with small values of R.

In loading coils $\tan \delta$ is of the order of 0·015. With $R = 2{,}000$ ohms, $Q = 100$ ohms, $\omega = 5{,}000$ ($5{,}000/2\pi = 796$ cycles per sec.), balance is secured by varying C_1 and C_2 only; if the condensers are read in microfarads

$$L \doteqdot 0{\cdot}2\ C_1 \text{ henry and } \tan \delta = 10 C_2 \doteqdot \delta \text{ radians.}$$

If θ_1 be the loss angle of C_1, φ_Q of Q and φ_R of R, residual effects will have an influence upon the measurement of δ represented by

$$\tan \delta = \omega C_2 R + \theta_1 - \varphi_R - \varphi_Q$$

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* "Eine neue Brücke für Pupinspulen," *Elekt. Zeits.*, Vol. 48, p. 1086 (1927).

The impedance of L being large compared with Q the detector will come approximately to earth potential since B is earthed. The elements C_2 and R are shielded, their shield-earth capacitances shunting C_1 and adding to its initial capacitance. Earth capacitance from A merely shunts the source; that from C shunts Q and gives it a residual included in the angle φ_Q. It is easily possible, if desired, to superpose direct current in the coil by the arrangement shown in Fig. 134 for Hay's bridge.

23. Wirk's Bridge. When coils of high time-constant, i.e. of low phase defect or loss-angle, are measured in Maxwell's bridge there is some practical difficulty in choosing a sufficiently large value for S. The time constant is

$$L/P = CS,$$

and the phase defect $\tan \delta = P/\omega L = 1/\omega CS$.

For example, if $\tan \delta = 10^{-4}$, $\omega = 10^4$ ($f = 1{,}600$ cycles per second) and C is an air condenser of 1,000 $\mu\mu$F. (10^{-9} F.), then S will have the impracticable value of 10^9 ohms; even if C is 1 μF., S attains a megohm. The difficulty is surmounted in Fig. 133 by substituting a series combination of C and S for the parallel combination; S is then of small value. The same result is attained in Fig. 135 by removing S and putting a condenser in parallel with R.

Wirk* has described an ingenious artifice shown in Fig. 136 (*a*), where source and detector have been omitted for simplicity. Transforming the star composed of T, U, and V into its equivalent mesh t, u, and v, gives the network of Fig. 136 (*b*). Comparing this with Fig. 130, t is in parallel with the detector and does not affect the balance, v replaces S and the parallel combination of Q and u replaces Q ; with these substitutions the transformation of Wirk's bridge is identical with that of Maxwell.

Using the star-mesh transformation given on p. 58

$$u = (TU + UV + VT)/U \text{ and } v = (TU + UV + VT)/V$$

Hence v can be made as large as required by making V sufficiently small. If Q' denote the parallel combination of Q and u,

$$Q' = Q(TU + UV + VT)/[U(Q + T) + V(T + U)],$$

and the balance conditions are

$$vP = Q'R = L/C;$$

and $$L/P = Cv \text{ with } \tan \delta = P/\omega L = 1/\omega Cv.$$

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* A. Wirk, "Über die Messung kleinster Winkeldifferenzen von Scheinwiderständen," *E.N.T.*, Vol. II, pp. 61–66 (1934). "Absolute Verlustfaktorbestimmung bei der dielektrischer Werkstoffprüfung," *Zeits. f. tech. Phys.*, Vol. 15, pp. 487–491 (1934).

The above discussion assumes Q to be a perfect resistor. If this is not so, the bridge measures the difference between the loss-angles of P and Q. Wirk shows that by suitable arrangement of the bridge it is possible to measure angle differences as low as 10^{-10} radian. The bridge is fully shielded, a Wagner earth is provided, and R is replaced by a second condenser; both condensers have air dielectric and are shielded. The shields are joined to the Wagner branch-point, so that only the loss-free inter-capacitances of the condensers enter into the bridge balance.

It will be seen by reference to Fig. 136 (*a*) that Wirk's bridge is a particular case of a general type of eight-branch network in which four impedances z_1, z_2, z_3, z_4 are mesh connected between the points AC, CB, BD, and DA, and four additional impedances z_A, z_B, z_C, z_D are star connected between the points A, B, C, D and a common point. The general theory is used in discussing the earth admittances of a Wheatstone network, *see* p. 541, but Wirk's bridge is the only one of this class at present in use for measuring purposes.

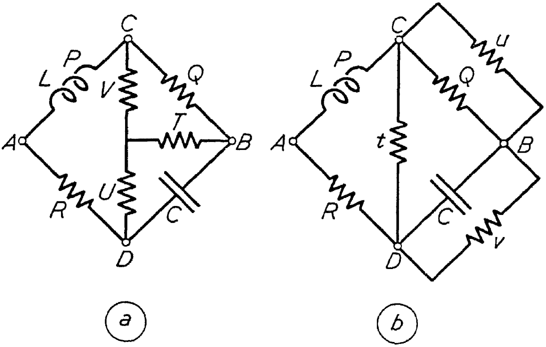

FIG. 136.—WIRK'S BRIDGE FOR LARGE TIME-CONSTANTS

24. Series Condenser Methods. In the methods described above the inductance and condenser which are to be compared are situated in *different* branches of the network. A few bridges will now be noticed, in which the inductance and condenser are put in series in the *same* branch of the network; they are essentially resonance or tuned arm bridges, *see* p. 309.

Haworth* has introduced a bridge for the measurement of the equivalent capacitance and series resistance of an electrolytic cell

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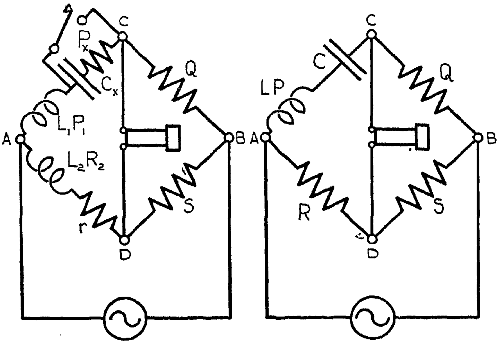
* H. F. Haworth, "The measurement of electrolytic resistance using alternating currents," *Trans. Far. Soc.*, Vol. 16, pp. 365–391 (1921).

(Fig. 137). In practical working, C_x, P_x are respectively the equivalent capacitance and resistance to be determined ; L_1 is a variable standard of self-inductance of the Ayrton-Perry type ; Q, S, and r are non-inductive resistances, Q and S being made equal. To balance, the electrolytic cell is first short-circuited ; the inductance L_2 is inserted in AD to balance the minimum reading of L_1, and r is adjusted until the detector indicates zero. The cell is now inserted in AC, and balance is restored by adjustment of L_1 and r. Assuming $Q = S$, it is obvious that P_x is equal to the change in r, while $C_x = 1/\omega^2 L$, L being the change in value of L_1. It is clear that the method is applicable to the measurement of the capacitance and series resistance of a condenser by using the same procedure.



Haworth's Method. Resonance bridge.

FIG. 137.—SERIES CONDENSER METHODS FOR COMPARING SELF-INDUCTANCE AND CAPACITANCE ; HAWORTH'S METHOD AND THE RESONANCE BRIDGE

Haworth's bridge is really a modification of the well-known resonance bridge* (Fig. 137). In this, three of the branches are non-inductive resistances. The fourth consists of a condenser and inductance in series. When L, C, or ω are adjusted so that the condenser and inductance are in resonance $\omega L = 1/\omega C$ and the branch AC is non-reactive. If then $SP = QR$ the bridge is balanced. The method can be used to measure with considerable accuracy and sensitiveness the effective resistance of a coil or condenser. In practice, Q and S are equal ratio coils and R is a fixed resistance. The branch AC contains the inductive coil L_1, P_1, in series with an adjustable self-inductance l of the Ayrton-Perry type, a resistance box r and the condenser C, the series loss resistance of the latter being ρ. With L_1 and C short-circuited, l and r are adjusted to values l_o and r_o to secure balance. (If l does not go down to sufficiently low values it may be necessary to insert a small balancing coil in AD, as in Haworth's modification.) L_1 and C are now inserted, and balance is restored by alteration of the adjustments to values l_1, r_1. Then

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* Grüneisen and Giebe, *Zeits. f. Inst.*, Vol. 30, pp. 147–148 (1910), have used the method to compare an inductance with a standard air condenser. *See* also D. I. Cone, "Bridge methods for alternating current measurements," *Journal Amer. I.E.E.*, Vol. 39, p. 643 (1920).

$$P_1 + \rho = r_o - r_1$$

and

$$(L_1 + l_1 - l_o)C\omega^2 = 1.$$

Thus if P_1 be known, then ρ can be found, and conversely.

In both these methods* it is essential that the frequency be constant and known. A vibration galvanometer forms the best detector, though telephones can be used if the wave form of current in the bridge be pure.

The simple resonance bridge is not convenient for small inductances at low frequencies, since this requires excessively large condensers. The defect is overcome by a modification due to Starr.† Q and S are equal; L is the coil to be measured and P its resistance; C is as large as possible and variable. A fixed condenser C_2 is put in series with the variable resistor R. Then the balance conditions are

$$P = R$$

and

$$L = (C_2 - C)/\omega^2 C_2 C \risingdotseq \Delta C/\omega^2 C_2^2$$

the approximation holding when the condensers are nearly equal. As an example, if $L = 1$ mH. and $f = 1{,}000$ cycles per second the simple resonance bridge requires $C = 25\ \mu$F. With $C_2 = 0{\cdot}1\mu$F. $\risingdotseq C$, a range of 1 mH. is covered by $\Delta C \risingdotseq 400\ \mu\mu$F., which can conveniently be an air condenser in parallel with C. If C_2 is 1 μF. $\risingdotseq C$, $\Delta C \risingdotseq 0{\cdot}04\ \mu$F., and if C_2 is 4 μF., $\Delta C = 1\ \mu$F.; quite good sensitivity is to be obtained even in the last case. It is sometimes more convenient in practice to fix C and to vary C_2.

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* For an additional modification applied to this bridge for tests on electrolytic cells, *see* A. H. Taylor, "A method for the determination of electrolytic resistance and capacity, using alternating currents," *Phys. Rev.*, Vol. 24, pp. 402–406 (1907). Balance is secured independently of frequency. Also *see* H. V. Higgitt, *Elecn.*, Vol. 90, p. 114 (1923).

† A. T. Starr, "A bridge for small inductances," *Journal Sci. Insts.*, Vol. 10, pp. 361–362 (1933).

The resonance bridge has been used by Batcheller[*] to test the quality of welded joints. For this purpose the inductance is a coil with a U-shaped iron core, the magnetic circuit of which is completed by the welded specimen. The bridge is supplied from an oscillator and low-pass filter; the detector is an amplifier and vacuum tube voltmeter. By moving the coil along the specimen a visual indication of unbalance is given when any discontinuity, such as a faulty weld, is encountered. The apparatus succeeds in locating bad welds with a reliability of 8 out of 9 joints tested.

The tuned arm or resonance bridge can also be used as an excellent method of measuring frequency.[†] Q and S are equal resistances ; L is a variable self-inductometer and C a standard condenser. Balance is secured by varying L and R, when $f = 1/2\pi\sqrt{LC}$. The method is useful at medium frequencies, since at low values large coils and condensers become necessary. At high values, where the telephone must be used as a detector, the wave form must be very pure if absolute silence is to be secured; alternatively a suitable filter may be used. This difficulty does not arise when the vibration galvanometer with its high frequency selectivity can be used, as at frequencies lower than about 500.

The tuned-arm or resonance bridge has been used at the Reichsanstalt by Giebe and Alberti[‡] for precise measurements of frequency over the range 160 to 300,000 cycles per second with an accuracy within 0·2 per cent. The inductances used are specially wound to have low eddy current losses and small self-capacitance; the condensers are of the type shown on page 170, and are measured by an absolute method. All the necessary allowances for residual effects are made.

Wave Analysis. An important use of the bridge is to determine the degree of purity of an alternating voltage wave-form,[§] a statement of which is now frequently required as an acceptance condition for electrical machinery. Referring to Fig. 137, the voltage to be analysed is applied to the points AB and the detector is a low reading dynamometer voltmeter. With Q and S fixed L and R are adjusted until the voltmeter gives a minimum reading; then if L and C are resonating at fundamental frequency, there can be no p.d. of that frequency between the points CD. The reading on the voltmeter will be due to residual currents produced by the harmonics in the applied voltage, i.e. to the "harmonic residue" $\sqrt{v_3^2 + v_5^2 + \ldots}$. Now at resonance AC has the minimum impedance to the fundamental and an extremely high impedance to the harmonics. Hence the harmonic currents in the voltmeter are much the same as if the harmonic voltages alone were applied to AB with AC opened. The voltmeter, being independent of wave form errors, can be calibrated by opening AC and applying known small voltages to AB for each value of R.

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[*] J. R. Batcheller, "A bridge method of testing welds," *Elec. Eng.*, Vol. 51, pp. 781–783 (1932).

[†] See S. Chiba, *Journal I.E.E., Japan*, No. 405, pp. 294–300 (1922), for modifications of the tuned arm bridge for high or low frequencies. For general uses of the resonance bridge see A. Tobler, *Journal Télégraphique*, Vol. 44, pp. 117–123 (1920).

[‡] E. Giebe and E. Alberti, "Absolute Messung der Frequenz elektrischer Schwingungen. Ein Normalfrequenz-oder Wellenmesser," *Zeits. f. tech. Phys.*, Vol. 6, pp. 92–102, 135–145 (1925).

[§] G. Belfils, "Mesure du 'résidu' des courbes de tension par la méthode du pont filtrant," *Rev. Gén. de l'Él.*, Vol. 19, pp. 523–529 (1926). I. Wolff, "The alternating current bridge as a harmonic analyser," *Journal Opt. Soc. Amer.*, Vol. 15, pp. 163–170 (1927) W. G. Radley, "Interference between power and communication circuits. Summary of recent information (1926–29)," *Journal I.E.E.*, Vol. 69, pp. 1117–1148 (1931). G. Belfils, "Note sur la mesure du résidu des courbes de tension alternative par la méthode du pont filtrant," *Rev. Gén. de l'Él.*, Vol. 31, pp. 267–270 (1932).

In the apparatus described by Belfils for use on the works test-bed, L is a stranded wire, concentric-coil inductometer of the Ayrton-Perry type, C is about 20 μF., $Q = 85$ ohms, and $S = 160$ ohms for use at 50 cycles per second. The detector is a 7·5 volt dynamometer voltmeter of 250 ohms resistance, and the p.d. at AB is 110 volts. The full theory of the bridge is given in the papers cited.

Chiodi* has described further modifications of the tuned arm bridge used by Belfils for the determination of the "harmonic residue" of a voltage wave-form. By an addition to the circuit resonance with any given harmonic can be obtained, and this separated from the residue, giving a complete analysis of the wave shape. In an additional paper he shows how to use the bridge to measure the degree of irregularity of the d.c. from a rectifier or other source of not strictly steady voltage.

Experimental Example. To illustrate one of the uses of the bridge a coil and condenser were compared at 407·1 cycles per second, using a Duddell vibration galvanometer to detect balance. As the lowest reading of the variable inductance was 3 mH., a coil of about 4 mH. was included in AD. Q and S were 100 ohms each, R was set at $101{\cdot}2_1$ ohms, and C was $2{\cdot}103_8$ μF. The following readings were obtained. $r_0 = 90{\cdot}33$ ohms, $l_0 = 4{\cdot}35$ mH., $r_1 = 83{\cdot}83$ ohms, $l_1 = 36{\cdot}35$ μH. Thus $L_1 + l_1 - l_0 = 1/C\omega_2 = 72{\cdot}65$ mH., whence $L_1 = 40{\cdot}6_5$ mH. Also $P_1 + \rho = 6{\cdot}50$ ohms; since ρ was known to be 1·31 ohms, $P_1 = 5{\cdot}1_9$ ohms.

25. Parallel Condenser Methods. The resonance principle can be applied to a bridge which has three non-inductive branches by a parallel connection of the inductance and condenser in the fourth branch.

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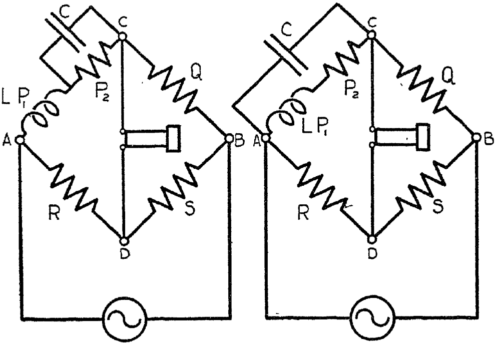
* C. Chiodi, "Modifiche al ponte filtrante di Belfils e suo adattamento all' analisi armonica delle tensione," *L'Elettro.*, Vol. 15, pp. 166–174 (1928). "Ponte per la misura della irregolarita di una tensione non perfettamente continua," *L'Elettro.*, Vol. 15, pp. 757–764 (1928). "Pont filtrant pour la mesure du 'residu' des courbes de tension alternative non parfaitement sinusoïdale et des courbes de tension continue non parfaitement constante et leur analyse harmonique," *Gdes. Réseaux*, Vol. 1, pp. 135–154 (1929).

One of the oldest methods in this category is Pirani's bridge (Fig. 138), originally devised for ballistic use. In Pirani's method* the inductance is connected in series with a shunted condenser. When the constants of the various branches and the frequency are adjusted to values which will make the branch AC non-reactive and at the same time balance† the bridge, then

$$\frac{L}{C} = \frac{P_2^2}{(1+\omega^2 C^2 P_2^2)} \text{ and } S\left[P_1 + \frac{P_2}{1+\omega^2 C^2 P_2^2}\right] = QR.$$

The frequency must be accurately known when L and C are under comparison. With a known inductance and capacitance, the bridge can be used for frequency measurement. As in the case of Rimington's method (p. 377), it will only be possible to use the bridge ballistically if the resistance balance be obtained with direct current, i.e. if $S(P_1+P_2) = QR$ be enforced, then $\omega = 0$ of necessity.



Pirani's Method. Wien's Method.

FIG. 138.—PARALLEL CONDENSER METHODS FOR COMPARING SELF-INDUCTANCE AND CAPACITANCE; PIRANI'S AND WIEN'S METHODS

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* O. de A. Silva, "Sur la méthode de Pirani," *Écl. Élect.*, Vol. 50, pp. 113–116 (1907); C. H. Lees, "On Vaschy's or Pirani's method of comparing the self-inductance of a coil with the capacity of a condenser," *Phil. Mag.*, 6th series, Vol. 18, pp. 432–436 (1909); J. P. Kuenen, "On Pirani's method of measuring self-inductance," *Phil. Mag.*, 6th series, Vol. 19, pp. 439–441 (1910); E. C. Snow, "On Pirani's method of measuring the self-inductance of a coil," *Proc. Phys. Soc.*, Vol. 21, pp. 634–640 (1910).

† J. P. Dalton, "On a new continuous balance method of comparing an inductance with a capacity," *Phil. Mag.*, 6th series, Vol. 27, pp. 37–44 (1914).

In a method introduced by Niven,* and used with alternating current by Max Wien,† the condenser is shunted across the branch AC as in Fig. 138. If $P = P_1 + P_2$ be the total resistance of AC, it is easy to prove that balance will occur when

$$LC = \frac{1}{\omega^2}\left(1 - \frac{SP}{QR}\right),$$

and

$$L/C = PQR/S.$$

From these equations L and C may be independently found in terms of resistance and frequency. Balance is attained by adjustment of the rheostats in the four branches, such values being chosen as will make the bridge approximately of unity ratio, i.e. $P \doteqdot R$, and $Q \doteqdot S$.

In a practical example, Wien compares an inductance, nominally of 0·2 henry, with a condenser of about 1 microfarad. The frequency being 249 cycles per second, balance was secured when $P = 331{\cdot}9$ ohms, $Q = 570$ ohms, $R = 23{\cdot}503$ ohms, $S = 20{\cdot}887$ ohms. Solving the above equation, it is found that $L = 0{\cdot}1993$ henry and $C = 0{\cdot}992$ microfarad.

Dongier‡ has shown how the measurement of L in terms of C can be made by a combination of Pirani's and Wien's methods in which frequency is eliminated. Let Pirani's bridge be balanced, so that $L = CP_2^2/(1 + \omega^2 C^2 P_2^2)$, by adjustment of P_2, R and S. Now change the connections of C so that it shunts the whole of the branch AC, as in Wien's method. Adjust P_2, R, and S until balance is secured; then, if P be the resistance of AC, the above equations give $L = C(P^2 + \omega^2 L^2)$. Eliminating ω^2 gives $L = CPP_2$.

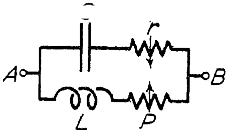

Fig. 139.—General Parallel Resonance Arm

Experimental Example. The following results were obtained at 407·1 cycles per second, using a Duddell vibration galvanometer and $C = 0{\cdot}334_6\ \mu$F. In *Pirani's method*, with $Q = 100$ ohms, balance was found by successive adjustments of P_2, R, and S to the values $P_2 = 365{\cdot}2_0$ ohms, $R = 337{\cdot}8_8$ ohms, and $S = 99{\cdot}9_0$ ohms. From these, $L = 40{\cdot}65_4$ mH. and $P_1 = 5{\cdot}2_1$ ohms. In *Wien's method*, balance was secured when $Q = 100$ ohms, $P_2 = 327{\cdot}5_8$ ohms, $R = 363{\cdot}0_8$ ohms, and $S = 99{\cdot}4_4$ ohms. Taking $P_1 = 5{\cdot}2_1$ ohms, $P = 332{\cdot}7_9$ ohms, and hence $L = 40{\cdot}65_6$ mH. Using this value of P and $P_2 = 365{\cdot}2_0$ ohms in *Dongier's method* gives $L = 40{\cdot}66_5$ mH.

General Parallel Resonance. The more general type of parallel resonance arm shown in Fig. 139 has some interesting properties that have been used by several workers.* It is not difficult to show that the impedance operator for this circuit is

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* C. Niven, " On some methods of determining and comparing coefficients of mutual induction," *Phil. Mag.*, 5th series, Vol. 24, pp. 225–238 (1887).

† Max Wien, " Messung der Inductionsconstanten mit dem 'optische Telephon,' " *Ann. der Phys.*, Vol. 44, pp. 707–708 (1891); " Über einen Apparat zum variiren der Selbstinduction," *Ann. der Phys.*, Vol. 57, p. 257 (1896). Also see O. Zwierina, " Präzisionsmessbrücken für Gleich-und Wechselstrom," *Messtech.*, Vol. 5, pp. 38–44 (1929).

‡ R. Dongier, " Sur la mésure des coefficients de self-induction au moyen du Téléphone," *Comptes Rendus*, Vol. 137, pp. 115–117 (1903).

$$z = \frac{(P - \omega^2 LCr)(1 - \omega^2 LC) + \omega^2 C(L + CrP)(P + r)}{(1 - \omega^2 LC)^2 + \omega^2 C^2 (P + r)^2}$$

$$+ j\omega \cdot \frac{(L - CP^2) - \omega^2 LC(L - Cr^2)}{(1 - \omega^2 LC)^2 + \omega^2 C^2 (P + r)^2} = R_1 + jX_1$$

For the circuit to be non-reactive the second term must vanish, i.e.

$$(L - CP^2) - \omega^2 LC(L - Cr^2) = 0.$$

This will occur at all frequencies if $P^2 = r^2 = L/C$. Also, if P and r are *any* pair of equal resistances, X_1 will be zero when $(1 - \omega^2 LC)$ vanishes. Putting $X_1 = 0$ and $P = r$ in general,

$$z \equiv R_1 = \frac{(1 - \omega^2 LC)^2 + 2\omega^2 C(L + Cr^2)}{(1 - \omega^2 LC)^2 + 4\omega^2 C^2 r^2}$$

In the special cases—

(i) Put $L = Cr^2$; then, from the condition for zero reactance at any frequency, $L = CP^2$ also. For this special condition

$$P = r = \sqrt{(L/C)} \text{ the value of } z \equiv R_1 = r = \sqrt{(L/C)}$$

i.e. the circuit has a resistance r independent of the frequency.

(ii) At the frequency which would give resonance with the coil and condenser in series, $\omega^2 = 1/LC$; putting this in the expression for R_1

$$z \equiv R_1 = (L + Cr^2)/2Cr,$$

which has a minimum value when $r^2 = L/C$. That is, when the frequency has the series resonance value, the resistance of the circuit passes through the minimum value $R_1 = \sqrt{(L/C)}$ when $r^2 = L/C$. This principle has been used by Boner to measure capacitance or inductance in terms of frequency and resistance at frequencies up to 500 kilocycles per second. A shielded equal ratio bridge is used, Q and S being 300 ohm bifilar loops. The resistors P and r are similar loops with adjustable sliders by which they can be varied. The coil and condenser are first put in series and adjustments are made to obtain series resonance balance; then $\omega^2 = 1/LC$. Putting them in parallel, r is set to an arbitrary value and balance obtained by P and R. This is repeated for several values of r until R reaches a minimum; then $R^2 \equiv R_1^2 = L/C$. From these relations, $C = 1/\omega R$.

Wave Analysis. The parallel resonance bridge has also been used[*] for analysis of the "harmonic residue" of voltage waves, the Wien form shown in Fig. 138 being preferred. For $f = 24$ cycles per second, $L = 4{\cdot}59$ H., $C = 9{\cdot}53$ μF., $P = 20$ ohms, $R = 24{,}000$ ohms, and $Q = S = 50{,}000$ ohms; the harmonics are all about halved, and their phase relations only very slightly altered. The detector may be a moving coil instrument with cuprous oxide rectifier, an electric harmonic analyser, or an amplifier operating an oscillograph.

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* W. E. Forsythe, "On a method of comparing inductance and capacity," *Phys. Rev.*, Vol. 1, pp. 463–465 (1913). T. M. Ganguli, "On a method of comparing inductance and capacity," *Phil. Mag.*, Vol. 49, pp. 165–167 (1925). T. B. Vinycombe, "Note on a method of comparing inductance and capacity," *Phil. Mag.*, Vol. 50, pp. 439–444 (1925). C. P. Boner, "The measurement of capacitance and inductance in terms of frequency and resistance at radio frequencies," *Rev. Sci. Insts.*, Vol. 1, pp. 243–259 (1930).

26. Grover's and Owen's Methods. F. W. Grover[†] has described the method shown in Fig. 140, originally suggested by E. B. Rosa for the purpose of finding the power factor of a small imperfect condenser.[‡] As shown on page 330, it is not possible to balance a simple De Sauty bridge when the condensers are imperfect—i.e. have dielectric losses—unless some means be provided to bring the potentials of the detector branch points into phase. Grover's method effects this by means of inductances in place of the resistances of the series resistance method. Since the condensers which are to be compared stand alone in the branches CD and BD, and balance is secured by adjustment of the remaining branches, the process of balancing does not affect the capacitance of the condenser branches. Hence the method is suitable for tests on small condensers.

Referring to Fig. 140 (a), L_1 and L_2 are standard inductances, either or both being of the variable pattern. If necessary, they are each connected in series with a suitable non-inductive rheostat. C_1 and C_2 are the unknown and standard condensers, ρ_1 and ρ_2 being their equivalent series resistances. Let balance be secured by adjustment of P or R and L_1 or L_2. The branch impedance operators being $z_1 = P + j\omega L_1$, $z_2 = \rho_1 - \frac{j}{\omega C_1}$, $z_3 = \rho_2 - \frac{j}{\omega C_2}$, $z_4 = R + j\omega L_2$, balance will be secured when

$$(P + j\omega L_1)\left(\rho_2 - \frac{\omega C_2}{j}\right) = (R + j\omega L_2)\left(\rho_1 - \frac{j}{\omega C_1}\right).$$

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[*] H. M. Wagner "A note on fundamental suppression in harmonic measurements," *Proc. Inst. Rad., Eng.*, Vol. 23, pp. 85–88 (1935). H. E. Linckh, "Eine neue Brückenschaltung zur Kurvenformmessung," *Phys. Zeits.*, Vol. 38, pp. 105–111 (1937).

[†] F. W. Grover, "Simultaneous measurement of the capacity and power-factor of condensers," *Bull. Bur. Stds.*, Vol. 3, pp. 389–393 (1907). H. L. Curtis, "Mica condensers as standards of capacity," *Bull. Bur. Stds.*, Vol. 6, pp. 436–438 (1910). F. W. Grover, "The capacity and phase difference of paraffined paper condensers as functions of temperature and frequency," *Bull. Bur. Stds.*, Vol. 7, pp. 498–499 (1911).

[‡] For application of this method at low frequency and at voltages up to 10,000, *see* C. A. Butman, "Flexible and accurate method for dielectric tests," *Elec. World*, Vol. 71, pp. 502–506 (1918). For a shielded bridge operating up to 500 kilocycles per second, see J. A. Stratton, "A high-frequency bridge," *Journal Opt. Soc., Amer.*, Vol. 13, pp. 471–494 (1926).

Separating components, the two conditions for balance are

$$\frac{C_1}{C_2} = \frac{R}{P} + \frac{\omega^2 C_1}{P}(L_1\rho_2 - L_2\rho_1)$$

and $$R\rho_1 - P\rho_2 = \frac{L_1}{C_2} - \frac{L_2}{C_1}$$

It is possible to show that if L_1, L_2 are not unduly large, the correction due to the second term in the first of these equations is very small, so that $C_1/C_2 = R/P$ to a high degree of approximation. Substituting this condition in the second equation, and remembering that the loss angle of an imperfect condenser is $\omega C\rho$ (*see* p. 190), gives

$$\tan\theta_1 - \tan\theta_2 = \omega\left(\frac{L_1}{P} - \frac{L_2}{R}\right)$$

Since θ_1 and θ_2 are small, this can be written as

$$\tan(\theta_1 - \theta_2) \doteqdot \tan\phi_P - \tan\phi_R,$$

ϕ_P and ϕ_R being the phase angles of the branches AC and AD. Hence the phase-difference of the unknown condenser is given in terms of the known phase-difference of the standard.

This method is most conveniently used as a substitution method. The bridge is balanced with the unknown condenser C_1 and a standard C_2; the unknown being removed, an auxiliary standard C'_1 is

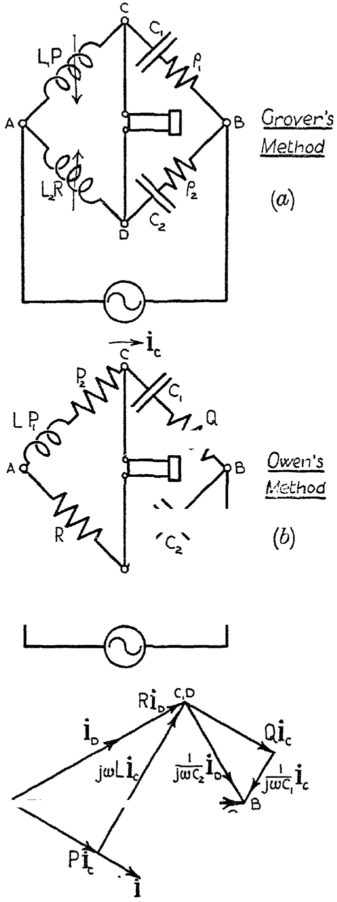

FIG. 140.—GROVER'S METHOD FOR COMPARING TWO CONDENSERS, AND OWEN'S METHOD FOR THE MEASUREMENT OF SELF-INDUCTANCE

inserted in its place and balance restored by adjustment of L_1 and P, all else remaining unaltered. If θ'_1 be the phase difference of the auxiliary standard, L'_1 and P'_1 the balance values, then

$$\tan\theta_1 - \tan\theta'_1 = \omega\left(\frac{L_1}{P} - \frac{L'_1}{P'}\right) \doteqdot \tan(\theta_1 - \theta'_1).$$

By this process, residual errors in the various branches and errors due to earth capacitances from the bridge to earth are eliminated. Settings of great precision can be attained.

The method is most suitable for tests on condensers smaller than 0·01 microfarad at frequencies over 100 cycles per second, the difference between the unknown and the auxiliary condenser, $\theta_1 - \theta'_1$, being less than 1 degree.

D. Owen* has shown that Grover's bridge can be modified in such a way that it becomes a very useful means of measuring self-inductance in terms of standard condensers. Referring to Fig. 140 (*b*), the inductance to be tested is put in the branch AC in series with an adjustable non-inductive resistance P_2, the total resistance of the branch being $P = P_1 + P_2$. The branch AD consists of a second non-inductive resistance, R; CB contains a condenser and a resistance box. The fourth branch is composed of a standard condenser C_2, assumed to be perfect. Comparing this diagram with that for Grover's bridge, it will be seen that they are very similar; in fact, $P = P_1 + P_2$, $\rho_1 = Q$, $\rho_2 = 0$, $L_2 = 0$ in Grover's network will convert it into that of Owen. Putting these values in the balance equations gives

$$C_1(P_1 + P_2) = C_2R,$$

and

$$L = C_2QR$$

for balance.

The practical procedure is as follows: C_1, C_2, and R being fixed, balance is attained by successive adjustments of P_2 and Q. The process is easy and rapidly convergent, since the two conditions of balance are independent; the balance does not depend on frequency or wave form.

The vector diagram for the balanced bridge is very simply constructed. The current $\mathbf{i}_D$ in the branches ADB leads on the voltage applied to AB by an angle $\arctan(1/\omega C_2R)$, the two components of $\mathbf{e}$ being $R\mathbf{i}_D$ and $(1/j\omega C_2)\mathbf{i}_D$. The vector $R\mathbf{i}_D$ is also equal to the potential difference between A and C, namely $(P+j\omega L)\mathbf{i}_C$, where $\mathbf{i}_C$ is displaced from $\mathbf{e}$ by an angle $\arctan\left[\left(\omega L-\frac{1}{\omega C_1}\right)/(P+Q)\right]$. Similarly, the vector $(1/j\omega C_2)\mathbf{i}_D$ is equal to $\left(Q-\frac{j}{\omega C_1}\right)\mathbf{i}_C$. The above balance conditions then immediately follow from the geometry of the triangles in the vector diagram.

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* D. Owen, "A bridge for the measurement of self-induction in terms of capacity and resistance," *Proc. Phys. Soc.*, Vol. 27, pp. 39–55 (1915). R. D. Salmon, "Inductance measurement. The Owen and Anderson bridge networks," *Elecn.*, Vol. 113, pp. 827–829 (1934).

Experimental Example. In a test at 407·1 cycles per second, the following values were obtained. The condensers had values $C_2=0{\cdot}334_6$ μF. (mica standard) and $C_1=0{\cdot}400_3$ μF. (paper). The latter had a series loss resistance of 5·20 ohms, which is included in the tabulated values of Q. A Duddell vibration galvanometer was used.

R ohms.	P_2 ohms.	Q ohms.	L mH.	P_1 ohms.
100	$78{\cdot}3_9$	1216·2	$40{\cdot}69_4$	$5{\cdot}1_8$
200	$161{\cdot}9_6$	608·0	$40{\cdot}68_7$	$5{\cdot}1_7$
300	$245{\cdot}5_2$	405·3	$40{\cdot}68_5$	$5{\cdot}1_9$
		Average	**$40{\cdot}68_9$ mH.**	**$5{\cdot}1_8$ ohms**

There are a variety of residual effects which may cause error in tests of high precision, especially on small inductances. Such errors can be very nearly eliminated by making use of an auxiliary balance with the coil L removed, as will now be shown.

The sources of error are, briefly—

(i) In AC; residual inductance in P_2 and in the leads joining L to the bridge. These modify the operator for the branch to the value $z_1=P+j\omega(L+l_P)$ where l_P is the residual.

(ii) In CB; residual inductance l_Q in Q and in the leads joining C_1 to the bridge, also loss resistance ρ_1 in the condenser. The operator is $z_2=Q+\rho_1+j\left(\omega l_Q-\frac{1}{\omega C_1}\right)$.

(iii) In BD; residual inductance of the leads joining C_2 to the bridge, l_2. Resistance of these leads and absorption in the condenser represented together by ρ_2, make the operator $z_3=\rho_2+j\left(\omega l_2-\frac{1}{\omega C_2}\right)$.

(iv) In DA; residual inductance l_R, making $z_4=R+j\omega l_R$.

Using these values to calculate the balance conditions, it is easy to show that

$$R/C_1=(P/C_2)+\omega^2[l_R(Q+\rho_1)+l_QR-(L+l_P)\rho_2-l_2P]$$

$$C_2QR=L+l_P+C_2(\rho_2P-\rho_4R)-l_R(C_2/C_1)+\omega^2C_2[l_Ql_R-l_2(L+l_P)]$$

in general.

Balance is obtained by adjustment of P_2 and Q. Let the coil L be short-circuited. Let the values of P and Q requisite for balance be P' and Q_0, l'_P and l'_Q being the new residuals. Then, from the second condition,

$$C_2 Q_0 R = l'_P + C_2(\rho_2 P' - \rho_1 R) - l_R(C_2/C_1) + \omega^2 C_2[l'_Q l_R - l_2 l'_P].$$

Subtracting this from the equation obtained with L in place,

$$C_2 R(Q-Q_0) = L + (l_P - l'_P) + C_2\rho_2(P-P') + \omega^2 C_2[l_R(l_Q - l'_Q) - l_2(l_P - l'_P) - l_2 L].$$

Now in most cases the third and fourth terms on the right-hand side are of negligible value, so that

$$L = C_2 R(Q - Q_0) - (l_P - l'_P).$$

The correction term is easily allowed for if P_2 be a rheostat of calculable inductance. If P_2 be of constant inductance, then the correction is zero. In any case, if the time constant of L be $> 10^{-4}$ the correction is negligible. It is essential, however, to make this correction and also to make the auxiliary balance when small inductances are being measured.

In practice, Owen has shown that the range of the bridge is very wide even with limited apparatus. Thus, with C_1 and C_2 each equal to $\frac{1}{3}$ microfarad, and values of R ranging from 1 to 200 ohms, a range of inductance between 2 microhenrys and 0·5 henry can be satisfactorily covered. P_2 and Q should each contain fine adjustment—preferably constant inductance—rheostats. The bridge is extremely convenient and has the great practical advantage that only resistance adjustments are used. Also there are no inductances other than the one under test, thus avoiding magnetic interference. Turner* has devised a self-contained set for rapid measurements of inductance between 1/10 microhenry and 1/10 henry, the apparatus being made by Messrs. H. Tinsley & Co.

Application of Shielded Bridge. The Owen bridge has been developed for the exact measurement of inductance and effective resistance at telephonic and carrier frequencies by the Bell Telephone Laboratories of New York. For this purpose a shielded bridge has been designed and is used for precise testing of coils under works conditions in the Western Electric Co. and other manufacturing plants of the A.T. and T. organization. The Owen bridge is particularly suitable for this purpose; the balance adjustments are independent and are made with variable resistors; it is easy to arrange the bridge to read inductance values directly; and residual errors can be made very small.

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* L. B. Turner, "Everyday measurements of inductance and capacity in the wireless laboratory," *Rad. Rev.*, Vol. 1, pp. 585–590 (1920).

In the arrangement described by Ferguson* the difference process described on page 407 is adopted, and inductance can be measured with high accuracy and effective resistance with fair accuracy at frequencies up to 3,000 cycles per second provided there is no coupling between any of the four arms or between any of these and the source or detector. Since there is only one inductance in the bridge, namely L in Fig. 140 (*b*), it is easy to eliminate magnetic coupling by using toroidal input and detector transformers. The reduction of electric coupling by appropriate shielding is, however, more troublesome. An ideal system of shields should confine capacitance effects to the branch points AB, CD, where they will shunt the source and detector respectively, and as a shunt to C_2, where they can be included in the assumed value of that quantity. Such a system can be devised but is rather difficult to carry out in a practical bridge construction. Ferguson's bridge, therefore, is constructed with the simpler system shown in Fig. 141. In this, in addition to shield capacitance effects across the diagonal branch-points and across C_2, capacitances are also introduced across R, Q, and the whole arm BC. The latter may be made negligible since it consists only of the capacitance of one condenser lead to the shield. The capacitances across R and Q do not enter as first order errors in the value of L but affect the measurement of P_1. The earth point is at D; if P_2 is only a small part of the impedance of the arm AC this is practically equivalent to the earthing of one terminal of L since C and D are at the same potential when balance occurs. This is true provided L is not of considerable physical size, with consequent large distributed earth capacitance. To obtain different ranges for inductance measurement the mica condensers used for C_2 can be given values of 0·6 and 0·06μF., while R can be varied in five multiples or submultiples of 1000/0·6 or

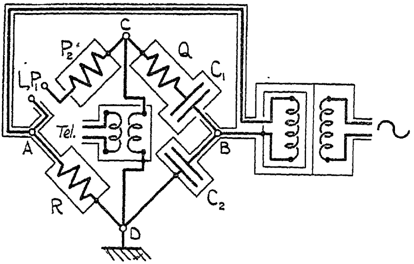

FIG. 141.—SHIELDED OWEN BRIDGE

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* J. G. Ferguson, "Measurement of inductance by the shielded Owen bridge," *Bell Syst. Tech. J.*, Vol. 6., pp. 375–386 (1927). S. J. Zammataro, "Development of the impedance bridge," *Bell Lab. Rec.*, Vol. 7, pp. 150–154 (1928); "Impedance bridges," *Bell Lab. Rec.*, Vol. 8, pp. 167–170 (1929). T. Slonczewski, "Measuring inductance with a resistor," *Bell Lab. Rec.*, Vol. 12, pp. 77–80 (1933). H. T. Wilhelm, "A self-contained bridge for measuring both inductive and capacitive impedances," *Bell Lab. Rec.*, Vol. 12, pp. 181–184 (1934).

1,667 ohms, thus making the bridge direct reading in millihenrys. The value of C_1 is $0{\cdot}6\mu$F. Both Q and P_2 are shielded dial decade boxes variable in 0·01 ohm steps to 10,000 ohms. Inductances ranging from 0·1 to 3 henrys can be measured with an error less than 0·1 per cent, and for 10 henrys the accuracy is better than 0·25 per cent. The overall accuracy of effective resistance measurements is about 2 per cent.

The shielded bridge has been still further improved by Bartlett* for measurements over a range of 5mH. to 10 henrys at acoustic and carrier frequencies up to 50,000 cycles per second. A more complete shielding system is provided, by means of which it is possible to earth directly one terminal of L. The accuracy of inductance measurements is within ± 0·03 per cent at speech frequencies and ± 0·05 per cent at carrier frequencies.

Mr. Ferguson has pointed out to the writer that the Owen bridge used in connection with a tuned-arm bridge forms a ready method for the absolute measurement of inductance or capacitance in terms of resistance and time. For example, if L and C_2 are compared in an Owen bridge $L/C_2 = QR$; the same coil and condenser in a tuned-arm bridge give $LC_2 = 1/\omega^2$ from which $L = \sqrt{QR}/2\pi f$ or $C_2 = 1/2\pi f\sqrt{QR}$, and $QR = \omega^2 L^2 = 1/\omega^2 C_2^2$. This process should be compared with that described on p. 402 for the same purpose, using series and parallel tuned arms.

27. Hartshorn's Modification of the Kelvin Double Bridge. Four-terminal low resistors are frequently used in a.c. measurements and it is often necessary to know their residual inductances. The determination of the residuals can be made in a variety of ways. Thus Campbell has described methods dependent upon the use of variable mutual inductances (see p. 443), while a variety of methods have been tried at the Bureau of Standards.† The most convenient method in practice is the Kelvin double bridge shown in general form in Fig. 27, page 63. Here z_3 and z_4 are two four-terminal resistances, one being a standard; z_1, z_2 are the main ratios and z_5, z_6 the auxiliary ratios, all being non-reactive resistances. The necessary phase compensation may be effected by including self-inductances in series with two of the ratio resistances,‡ say z_2 and z_6. Alternatively mutual inductance between the connecting impedance z_7 and the detector branch may be used, as has been done by Déguisne.[*]

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* B. W. Bartlett, "A shielded bridge for the measurement of inductance in terms of resistance and capacity," *Journal Op. Soc. Amer.*, Vol. 16, pp. 409–418 (1928).

† F. Wenner, E. Weibel, and F. B. Silsbee, "Methods of measuring the inductance of low resistance standards," *Bull. Bur. Stds.*, Vol. 12, pp. 11–21 (1916). F. Silsbee, "A study of the inductance of four-terminal resistance standards," *Bull. Bur. Stds.*, Vol. 13, pp. 375–421 (1917).

‡ C. H. Sharp and W. W. Crawford, "Some recent developments in exact alternating current measurements," *Trans. Amer. I.E.E.*, Vol. 29, pp. 1517–1541 (1911). F. Wenner, "The four-terminal conductor and the Thomson bridge," *Bull. Bur. Stds.*, Vol. 8, pp. 559–610 (1912). S. J. Barnett, "The measurement of inductances and capacities with the Thomson double bridge," *Phys. Rev.*, Vol. 34, pp. 74–75 (1912).

In practice, the use of self or mutual inductances in a double bridge is troublesome, owing to the great care necessary to avoid errors arising from stray magnetic fields. To overcome this defect Schering[†] at the Reichsanstalt has effected the phase compensation by the aid of condensers shunted across two of the ratio coils, a bifilar arrangement of the network being employed. More recently, this same idea has been developed in a very simple bridge described by Hartshorn[‡] and illustrated in diagrammatic lay-out by Fig. 142(*a*). In this the two four-terminal resistors to be compared are

$$z_3 = R + j\omega L_R \text{ and } z_4 = P + j\omega L_P\,;$$

they are joined in series to the a.c. supply by means of their current terminals and a link, which should have as low a resistance as possible. The leads must be close together so that the main circuit shall not set up any strong magnetic field. The ratio coils Q, S, α, β are shielded non-reactive resistances ; frequently, Q/S and α/β were given the value 100/1000. The resistances S and β are shunted by variable air condensers C_s, C_β each with a maximum value of 0·002 μF. The potential terminals of P and R are joined to the ratios by twisted flexible leads 2 or 3 metres in length. The ratios must be variable, and for this purpose small slide wires were included in the S and β branches. The detector is usually a 120 ohm telephone, and a current of telephonic frequency (800 cycles per second) is used, thus enabling adequate sensitivity to be obtained with a smaller current than would be necessary at lower frequencies. The inductance of four-terminal resistors is practically independent of frequency. To locate definitely the potentials of the components of the bridge with respect to earth the two resistances, X, Y, are connected as shown, earthed at their junction and adjusted so that $X/Y = P/R = Q/S = \alpha/\beta$ approximately. By this means the detector is brought to earth potential and " head effect " is eliminated. The shields of α, β, Q, and S are earthed.

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[*] C. Déguisne, "Die Bestimmung kleiner Phasenverschiebungen bei Neiderfrequenz," *Arch. f. Elekt.*, Vol. 5, pp. 375–382 (1917).

[†] H. Schering, "Die Doppelbrücke zum Messung des Phasenwinkels sehr kleiner Widerstände bei technischer Frequenz," *Elekt. Zeits.*, Vol. 38, pp. 421–423, 436–438 (1917). F. Fischer, "Messung der betriebsmässigen Ableitung von Fernsprechkabeln mit geerdetem und ungeerdetem Bleimantel," *Tel. u. Ferns. Tech.*, Vol. 10, pp. 137–140 (1921). K. Küpfmüller and P. Thomas, "Wechselstrombrücke zum Messen der Scheinwiderstände von Fernsprechkabeln," *Elekt. Zeits.*, Vol. 43, pp. 461–464 (1922). E. Biffi, "Il doppio ponte con corrente alternate," *L'Elettro.*, Vol. 10, pp. 508–512 (1923). W. Vogel, "Magnetische Messungen an Einleiter-Hochspannungskabeln," *Elekt. Zeits.*, Vol. 48, pp. 1361–1363 (1927). J. Krönert, "Thomsonbrücken für Wechselstrom," *Arch. f. tech. Mess.*, J. 921–5 (July), 1932).

[‡] L. Hartshorn, "The measurement of the inductances of four-terminal resistance standards," *Proc. Phys. Soc.*, Vol. 39, pp. 337–387 (1927). Also see "Anwendung der doppelten Thomson Brücke für Wechselstrom," *Schweiz. Elekt. Ver. Bull.*, Vol. 19, pp. 784–789 (1928).

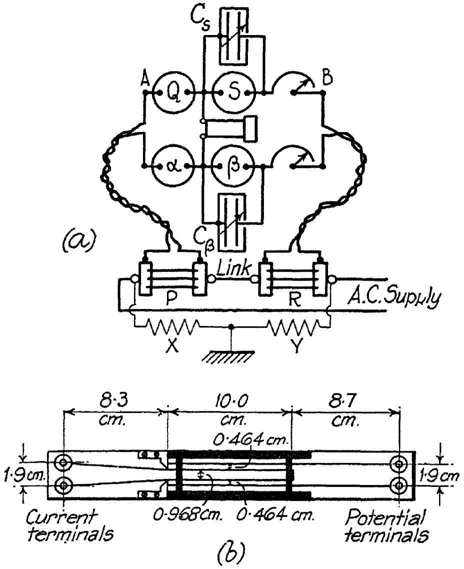

FIG. 142.—HARTSHORN'S ARRANGEMENT OF THE KELVIN DOUBLE BRIDGE FOR COMPARING FOUR-TERMINAL RESISTORS

The bridge is balanced in the following way. With the bridge as shown, balance is secured by adjustment of the resistance and capacitance of the S branch. Then with the link removed, balance is restored by adjusting the β branch. The link is then replaced and adjustment made in the setting of the S branch, the process being repeated if necessary until balance is undisturbed, whether the link is in place or not; this is the normal double adjustment of the Kelvin bridge. The supply is now transferred to the points A, B with the link removed and balance is restored by an adjustment of the S branch; this adjustment represents the effect of the self and mutual inductances of the potential leads connected to P and R. Hartshorn proves that if C_s is the setting of the simultaneous balance and C'_s is the final setting with A, B supplied, then very nearly

$$(L_P/P) - (L_R/R) = \tfrac{1}{2}S(C_s + C'_s) + \delta_{qs}$$

where $\delta_{qs} = (L_Q/Q) - (L_S/S)$ difference of residual time-constants of Q and S.

The bridge gives the difference between the time-constants of P and R. If one is a primary standard of calculable time-constant that of the other is determined. Hartshorn uses a four-terminal resistor of about 1 ohm with calculable residuals, shown in Fig. 142 (*b*). The resistance is a loop of 0·308 mm. diameter Eureka wire, provided with potential leads; all the wires are spaced out through holes in the ebonite mount. The residual inductance is 0·124 μH. Resistances of 1 ohm are tested by comparison with this standard in a bridge with equal branches, 100/100 or 1000/1000; resistances of 0·1 ohm by comparison with a 1 ohm standard in a bridge with 100/1000 ratio, 0·01 ohm resistances with a 0·1 ohm resistance in the same bridge, and so on down to 0·0005 ohm. Currents of 0·25 ampere give sufficient sensitivity for the higher resistances and 5 amperes for the lower resistances.

In a typical test, $P = 0{\cdot}1$ ohm, $R = 1{\cdot}0$ ohm nominally; $Q = 100$ ohms, $S = 1{,}000$ ohms; $\alpha = 50$ ohms, $\beta = 500$ ohms. At balance, $C_s = 1{,}330$ $\mu\mu$F., $C'_s = 1{,}342$ $\mu\mu$F. Also, $\delta_{qs} = (0{\cdot}6 - 1{\cdot}3)10^{-8}$ second. The time-constant of R was $1{\cdot}6 \times 10^{-8}$ second; hence, $L_P/P = 134{\cdot}5 \times 10^{-8}$ second.

27a. Astbury's and Arnold's Methods for Four-Terminal Resistors. Astbury[*] has described a simple bridge for measurement of the residual inductance of four-terminal resistors without the necessity of a calculable standard. The circuit is shown in Fig. 143 (*a*), Q, R, and S being fixed resistors, C a variable air condenser and r a variable resistor consisting of a Campbell constant inductance box and rheostat (see p. 125); X and Y are resistors set so that $X/Y \doteqdot R/S$ as in Hartshorn's method. All pairs of leads should be twisted together to avoid mutual inductance error. Following the usual definition, the impedance operator of the four-terminal resistor $(\rho + j\omega\lambda)$ is defined as the ratio of the p.d. at its voltage terminals v_1, v_2, to the current entering and leaving at the current terminals c_1, c_2; following the notation of p. 334 of

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[*] N. F. Astbury, "A simple method for measurements of residual inductance on potentiometers and four-terminal resistance coils," *Journal Sci. Insts.*, Vol. 8, pp. 221–223 (1931).

Instrument Transformers write this symbolically as $[v_1v_2/c_1c_2]$. With the telephone connection first on v_1 and then on v_2 let balance be secured by adjusting the impedance of c_2B; if z and z' are the operators for this arm when the double balance is obtained,

$$S\{P + [c_1v_1/c_1c_2]\} = R\{z + [v_1c_2/c_1c_2]\},$$

and

$$S\{P + [c_1v_2/c_1c_2]\} = R\{z' + [v_2c_2/c_1c_2]\}$$

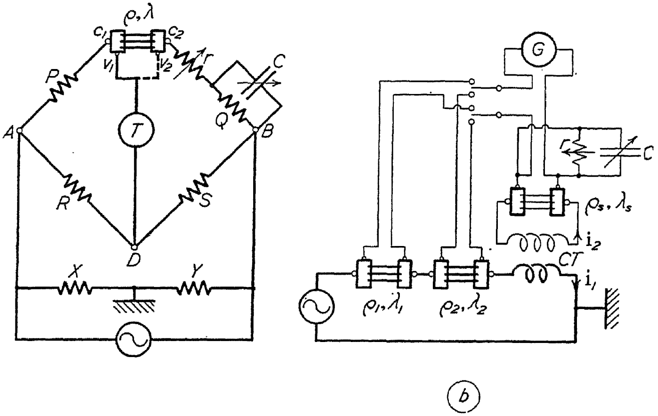

FIG. 143.—ASTBURY'S AND ARNOLD'S METHODS FOR FOUR-TERMINAL RESISTORS

Subtracting,

$$S\{[c_1v_2/c_1c_2] - [c_1v_1/c_1c_2]\} = R\{z' - z + [v_2c_2/c_1c_2] - [v_1c_2/c_1c_2]\},$$

or

$$S[v_1v_2/c_1c_2] = R\{z' - z - [v_1v_2/c_1c_2]\},$$

$$[v_1v_2/c_1c_2] = \frac{R}{R+S}(z' - z) \cdot$$

Neglecting second order quantities $z = r + Q(1 - j\omega CQ)$ and also $z' = r' + Q(1 - j\omega C'Q)$, which when substituted gives

$$\rho + j\omega\lambda = \frac{R}{R+S}[(r' - r) + j\omega Q^2(C - C')],$$

$$\rho = R(r' - r)/(R + S),$$

$$\lambda = Q^2R(C - C')/(R + S)$$

and

$$\lambda/\rho = Q^2(C - C')/(r' - r).$$

In a well-designed resistor the time-constant should be independent of frequency, but Silsbee has shown that skin-effect and other causes may make appreciable changes in the effective resistance and time-constant of resistors intended for large currents. In such cases testing with a small audio-frequency current, as in Hartshorn's or Astbury's methods is inappropriate, and measurements must be made with the rated current at power frequency. For this purpose the direct comparison method of Arnold,* Fig. 143 (*b*), is used. In this diagram (ρ, λ_1) and (ρ_2, λ_2) are two nominally equal resistors, one of which has a calculable residual inductance; (ρ_s, λ_s) is a third resistor operated from the secondary of a nickel-iron cored current transformer CT of nominal ratio K_{nc}, and such that ρ_s/K_{nc} is about 0·1 per cent larger than the value of ρ_1 or ρ_2; mutual inductance effects are minimized by the use of twinned leads. The drops across ρ_1 and ρ_2 in turn are opposed to that across ρ_s through a vibration galvanometer, balance being secured by adjustment of C and r in parallel with ρ_s; let the settings be C_1, r_1 and C_2, r_2 respectively. Remembering that the impedance of the C, r combination is high compared with that of ρ_s, λ_s, and neglecting squares and products of small quantities it is easy to show that

$$\rho_2/\rho_1 \doteqdot 1 + \rho_s[(1/r_1) - (1/r_2)] \text{ and } (\lambda_2/\rho_2) - (\lambda_1/\rho_1) = \rho_s(C_1 - C_2).$$

When ρ_1 and ρ_2 are not of the same nominal value ρ_s is a potential divider of special construction, the second balance being made on a fraction $q \doteqdot \rho_2/\rho_1$. The expression for the resistance ratio is then multiplied by q and the time-constant relation is unchanged.

NETWORKS CONTAINING RESISTANCE, SELF-INDUCTANCE, AND MUTUAL INDUCTANCE

28. Felici's Method for Comparison of Two Mutual Inductances. The most direct way to compare two mutual inductances is by means of the arrangement shown in Fig. 144 (*a*). This circuit was used by R. Felici,† in 1852, as a null method for demonstration of the laws of mutual inductance, and is mentioned in this connection by Maxwell.‡ Heaviside,§ in 1886, drew attention to the excellence of the arrangement as a mutual inductance balance, using a telephone and interrupted current. Finally, A. Campbell,‖ in 1908, adapted the circuit to alternating current and a vibration galvanometer.

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* A. H. M. Arnold, "The calibration of four-terminal resistance standards with alternating current at power frequencies," *Journal I.E.E.*, Vol. 69, pp. 1013–1018 (1931).

† R. Felici, "Mémoire sur l'induction électrodynamique," *Annales de Ch. et Ph.*, Vol. 34, 3rd series, pp. 64–77 (1852). "Nota sopra una osservazione del sig. A. De La Rive ad una esperienze fondamentali della teoria del l'induzione elettro-dinamica," *Nuovo Cimento*, Vol. 9, pp. 345–347 (1859).

‡ *Treatise*, 1st Edn., Vol. 2, Sec. 536, pp. 168–170 (1873).

§ *Electrical Papers*, Vol. 2, p. 110 (1892).

‖ A. Campbell, "On the use of variable mutual inductometers," *Proc. Phys. Soc.*, Vol. 21, pp. 74–75 (1910); *Phil. Mag.*, Vol. 15, 6th series, pp. 155–171 (1908).

Let M_2 be a mutual inductance standard which can be continuously adjusted; and let M_1 be the mutual inductance to be tested, its value being not greater than the maximum obtainable in M_2. Connect the primaries of the two inductances in series to a source of alternating current, and join the secondaries, with their windings in opposition, in series with a telephone or vibration galvanometer. Then, obviously, when no current flows through the detector,

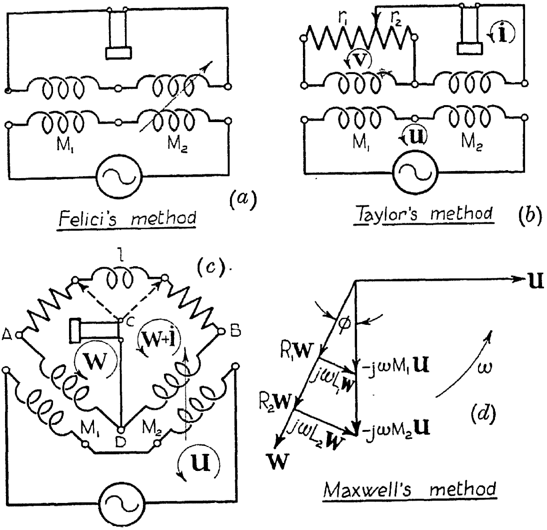

FIG. 144.—METHODS FOR THE COMPARISON OF TWO MUTUAL INDUCTANCES

$$M_1 = M_2$$

numerically, M_2 being adjusted until the indication of the detector is zero.

Experimental Example. In an experiment at 407·1 cycles per second with a Duddell vibration galvanometer as detector, M_1 was a fixed mutual inductance of 0·01 henry nominal value. M_2 was a Campbell mutual inductometer of 11 mH. range. The reading at balance was 984_8 microhenrys, so that $M_1 = 0{\cdot}00984_8$ henry.

This method has the advantage of quickness, one adjustment serving to secure balance; it is, moreover, not necessary to know the values either of the self-inductances or the resistances of the coils. The value of M_1 which can be measured is, however, limited by the range of the standard M_2.

At high frequencies, the method is sometimes complicated by the effects of self capacitance and eddy currents in one or both of the mutual inductances being compared. Campbell* has shown that self capacitance of the secondary coil of the unknown inductance may be represented by a condenser C_1 connected across it, and can be balanced by joining an adjustable condenser C_2 in parallel with the secondary of M_2. By successive adjustment of M_2 and C_2 balance is secured. If L_1, R_1, L_2, R_2 be the inductance and resistance of the secondary coils of the unknown and the inductometer, balance will occur when

$$M_1/M_2 = R_1C_1/R_2C_2 = (1 - L_1C_1\omega^2)/(1 - L_2C_2\omega^2)$$

or very nearly
$$M_1 \fallingdotseq M_2\left\{1 - \omega^2 C_2\left(\frac{L_1R_2}{R_1} - L_2\right)\right\}$$

Campbell also shows how to deal with the impurity effect due to eddy currents.

28a. Taylor's† Modification of Felici's Method. If the mutual inductance M_1 be greater than that of the standard M_2, a modification of the above method, shown in Fig. 144 (*b*), may be used. The standard, M_2, may now be fixed in value.

Let a resistance be connected across the coil M_1, the point of connection of the detector dividing the resistance into two parts, r_1 and r_2. Let z_1 be the impedance operator of the secondary of M_1 and z_2 of M_2, z_G being the operator for the detector; then the mesh equations are

$$(z_2 + z_G + r_2)\mathbf{i} - r_2\mathbf{v} = -j\omega M_2\mathbf{u},$$
$$-r_2\mathbf{i} + (z_1 + r_1 + r_2)\mathbf{v} = j\omega M_1\mathbf{u},$$

remembering that the secondaries are in opposition. If there is to be no current in the detector, then

$$\begin{vmatrix} -M_2 & -r_2 \\ M_1 & z_1 + r_1 + r_2 \end{vmatrix} = 0,$$

so that

$$M_1r_2 - M_2(z_1 + r_1 + r_2) = 0.$$

Now $z_1 = R_1 + j\omega L_1$, giving for the balance conditions

$$M_2L_1 = 0,$$
$$M_1r_2 = M_2(R_1 + r_1 + r_2).$$

The first condition implies that $L_1 = 0$; hence balance cannot be secured unless the self-inductance of the secondary coil of M_1 is very small. The second condition is

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* *Dictionary of Applied Physics*, Vol. 2, pp. 423-424 (1922). Also *Proc. Roy. Soc. A.*, Vol. 87, p. 397 (1912).

† A. H. Taylor, "On the comparison of mutual inductances," *Phys. Rev.*, Vol. 20, pp. 393 (1905).

$$M_1 = \frac{(R_1 + r_1 + r_2)}{} M_2.$$

Taylor's arrangement consists of a high resistance, say 5,000 to 100,000 ohms, depending on the value of R_1, connected across the secondary of M_1. Assuming L_1 to be negligible, and that R_1 is small compared to $r_1 + r_2$, balance can be very nearly secured when

$$M_1 = \frac{r_1 + r_2}{r_2} M_2,$$

which is Taylor's expression.

In a similar way, the method can be used to reduce the effect of a fixed standard mutual inductance of too great a value.

Experimental Example. The mutual inductance M_1 was that tested by Felici's method (p. 415), the frequency and detector remaining the same. M_2 was a 1·1 mH. Campbell mutual inductometer. The value of $r_1 + r_2$ was fixed at 2,000 ohms; as the resistance and inductance of the coil shunted by it was very small, a good balance was secured with $r_2 = 200$ ohms and $M_2 = 985$ μH., so that $M_1 = 0{\cdot}00985$ henry. In a second trial with $r_1 + r_2 = 4{,}000$ ohms, $r_2 = 400$ ohms, M_2 was again 985 μH. leading to the same value for M_1 as before.

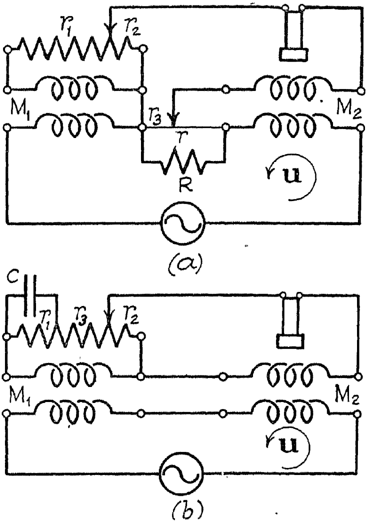

FIG. 145.—GEYGER'S METHODS FOR COMPARISON OF TWO FIXED UNEQUAL MUTUAL INDUCTANCES

Geyger's Methods. The simple potential divider arrangement used in Taylor's method is not sufficient for adjusting the detector current to zero; hence absolute balance is impossible without additional means to compensate for the effect of the inductance L_1 in the secondary of the mutual inductance to which the resistance load $r_1 + r_2$ is applied. Geyger* has described several methods for effecting this compensation and producing true balance, two of these being illustrated in Fig. 145.

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* W. Geyger, "Einfache Kompensationschaltungen zur Messung gegenseitiger Induktivitäten," *Arch. f. Elekt.*, Vol. 17, pp. 71–78 (1926). "Gegenseitige Induktivitäten. Messung nach Kompensationsmethode," *Arch. f. tech. Mess.*, V. 3522–2 (March, 1932).

Referring first to Fig. 145(*a*), no current will flow in the detector if the drop of voltage in r_2 is equal and opposite to the drop in r_3 plus the secondary voltage of M_2. Remembering that the mutuals have been arranged in opposition, the condition is

$$\frac{j\omega M_1 r_2 \mathbf{u}}{r_1 + r_2 + R_1 + j\omega L_1} = \left(\frac{r_3 R}{r + R} + j\omega M_2\right)\mathbf{u}$$

where r is the resistance of the slide wire in the primary circuit, suitably shunted by the resistance R. Separating the components gives as the two balance conditions

$$\omega^2 M_2 L_1 = \frac{r_3 R}{r + R}(R_1 + r_1 + r_2)$$

$$M_1 r_2 = M_2(R_1 + r_1 + r_2) + \frac{r_3 R}{r + R} L_1$$

To calculate M_1 from the second condition involves a knowledge of L_1; since the frequency is usually known, eliminate L_1 by the use of the first expression, giving

$$M_1 = \frac{(R_1 + r_1 + r_2)}{r_2}\left\{1 + \left[\frac{r_3 R}{\omega M_2(r + R)}\right]^2\right\} M_2$$

$$\doteqdot \frac{r_1 + r_2}{r_2}\left[1 + \left(\frac{r_3 R}{\omega M_2 r}\right)^2\right] M_2$$

if R_1 is neglected in comparison with $r_1 + r_2$ and R in comparison with r.

In Fig. 145 (*b*) the compensation is secured by the use of a condenser shunted across a portion of the potential divider. Then, for no current in the detector,

$$\frac{j\omega M_1 r_2 \mathbf{u}}{r_2 + r_3 + [r_1/(1 + j\omega C r_1)] + R_1 + j\omega L_1} = j\omega M_2 \mathbf{u}$$

which, on separation of the components, gives

$$M_1 r_2 = M_2(R_1 + r_1 + r_2 + r_3 - \omega^2 L_1 C r_1)$$

$$M_1 r_1 r_2 = M_2\left[r_1(r_2 + r_3) + \frac{L_1}{C} + r_1 R_1\right]$$

Eliminating L_1 gives

$$M_1 = \frac{M_2}{r_2}\left[R_1 + r_1 + r_2 + r_3 - \frac{\omega^2 C^2 r_1^3}{1 + \omega^2 C^2 r_1^2}\right]$$

Neglecting $\omega^2 C^2 r_1^2$ in comparison with unity and R_1 in comparison with $r_1 + r_2 + r_3$,

$$M_1 \doteqdot \frac{r_1 + r_2 + r_3}{r_2} M_2 - \frac{\omega^2 C^2 r_1^3 M_2}{r_2}$$

29. Maxwell's Method for Comparison of Two Mutual Inductances. The method described on page 414 is limited in its range to the maximum value of the available standard mutual inductance; those discussed on pages 416 and 417 effectively extend the range of the standard. Maxwell* has introduced a ballistic method for comparing two unequal mutual inductances; the network, as arranged for use with alternating current by A. Campbell,† in 1908, is shown in Fig. 144 (*c*).

The secondaries of the two mutual inductances are connected in series with resistances as shown, and, in addition, an adjustable self-inductance l is arranged so that it can be included in ACD or BCD at will. Care should be taken in arranging the network that all these inductive coils are placed well apart, so that there can be no interference between them. Suppose R_1 and L_1 are the total resistance and self-inductance of the branch DAC, and R_2, L_2 the corresponding values in the branch DBC; then, if z_G be the impedance operator for the galvanometer branch CD, the two mesh equations are

$$-z_G\mathbf{i} + (R_1 + j\omega L_1)\mathbf{w} = -j\omega M_1\mathbf{u},$$

$$(R_2 + j\omega L_2 + z_G)\mathbf{i} + (R_2 + j\omega L_2)\mathbf{w} = -j\omega M_2\mathbf{u};$$

so that for $\mathbf{i} = 0$

$$\begin{vmatrix} M_1 & R_1 + j\omega L_1 \\ M_2 & R_2 + j\omega L_2 \end{vmatrix} = 0.$$

Evaluating and comparing components,

$$\frac{M_1}{M_2} = \frac{R_1}{R_2} = \frac{L_1}{L_2}.$$

are the balance conditions. Hence, the rheostats and the mutual inductance standard must be adjusted to secure balance, while at the same time l is altered to keep the ratio $L_1/L_2 = M_1/M_2$.

When the current $\mathbf{i}$ is zero, the mesh equations given above admit of simple graphical interpretation, as shown in Fig. 144 (*d*). In this the vector diagram of currents and electromotive forces in the balanced network is drawn, the impedance drop round each mesh being equated to the electromotive force of mutual induction in the mesh. Two similar triangles are the result, a comparison of their corresponding sides giving at once the relations proved above.

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* *Treatise*, 1st Edn., Vol. 2, Sec. 755, p. 354 (1873). M. Brillouin, "Sur les méthodes de comparison des coefficients d'induction," *Comptes Rendus*, Vol. 93, pp. 1010–1014 (1881); "Comparison des coefficients d'induction," *Ann. de l'École Normale*, Vol. 11, pp. 339–424 (1882).

† A. Campbell, *loc. cit.*, pp. 73–74, "Inductance measurements," *Elecn.*, Vol. 60, pp. 626–627 (1908). Also *see* H. V. Carpenter, *Phys. Rev.*, Vol. 10, p. 52 (1900), for the adaptation of the conjugate to this method for the measurement of self-inductances.

Since it is immaterial which winding of a pair of coils be referred to as primary or secondary, it is clear that there are four possible groupings of the windings of M_1 and M_2. That arrangement should be chosen which makes L_1/L_2 most nearly equal to M_1/M_2. However, if the resistances R_1 and R_2 be made large compared with the reactances ωL_1 and ωL_2, then the angle ϕ will be small, and $R_1/R_2 \fallingdotseq M_1/M_2$. Hence, balance can be very nearly secured without making any provision for adjustment of the self-inductances, provided the resistances be made sufficiently large. In order that R_1 and R_2 shall not be unduly great, and thereby diminish the sensitivity, it is as well to take as the secondary coil of each pair that having the smaller self-inductance. A further advantage of increasing the resistances is that the effect of temperature change of resistance in the copper secondaries is swamped by the inclusion of resistance coils of manganin or other material of low temperature resistance coefficient, and greater permanence of the balance is thereby secured.

Experimental Example. The mutual inductance tested by Felici's and by Taylor's methods was compared with an 11 mH. Campbell mutual inductometer at 407·1 cycles per second by Maxwell's method, using a bridge of ratio about 2. The inductance l was included with the secondary (low inductance coil) of M_1, and was about 18 mH. in value to balance the inductance L_2 of the fixed winding of the Campbell standard, the latter being about 9 mH. The resistance boxes in the two meshes were set in the ratio of 2 to 1, the true value of R_1/R_2 being measured on a Wheatstone bridge after the a.c. tests were completed. Balance was found by adjustment of M_2 and l, the following results being obtained—

R_1 ohms.	R_2 ohms.	M_2 μH.	M_1 henry.
$211{\cdot}2_0$	$106{\cdot}0_2$	494_3	$0{\cdot}00984_7$
$411{\cdot}1_4$	$206{\cdot}0_4$	493_5	$0{\cdot}00984_7$
$610{\cdot}7_9$	$306{\cdot}0_0$	493_3	$0{\cdot}00984_6$
		Average	**$0{\cdot}00984_7$** henry

30. Campbell's Modification of Maxwell's Method. In the preceding method the coils M_1, M_2 were supposed to be separate pairs. A. Campbell* has shown, however, that the method can be applied to the rather more general case of a primary with two secondaries, such as would be encountered in the process of adjusting to equality the various sections of a stranded mutual inductance standard.

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* A. Campbell, *Proc. Phys. Soc., loc. cit.*, pp. 73–74; also *Elecn.*, Vol. 60, p. 626 (1908).

Let there be mutual inductance M_{12} between the secondaries AD and DB in Fig. 144 (*c*). Then the mesh equations are

$$(j\omega M_{12} - z_G)\mathrm{i} + [R_1 + j\omega(L_1 + M_{12})]\mathrm{w} = -j\omega M_1\mathrm{u},$$
$$(R_2 + j\omega L_2 + z_G)\mathrm{i} + [R_2 + j\omega(L_2 + M_{12})]\mathrm{w} = -j\omega M_2\mathrm{u},$$

and the balance determinant

$$\begin{vmatrix} M_1 & R_1 + j\omega(L_1 + M_{12}) \\ M_2 & R_2 + j\omega(L_2 + M_{12}) \end{vmatrix} = 0.$$

From this $$\frac{M_1}{M_2} = \frac{R_1}{R_2} = \frac{L_1 + M_{12}}{L_2 + M_{12}}$$

are the balance conditions.

In particular, as in the practical example cited above, let $M_1 = M_2$; then the above equations become independent of M_{12} and give

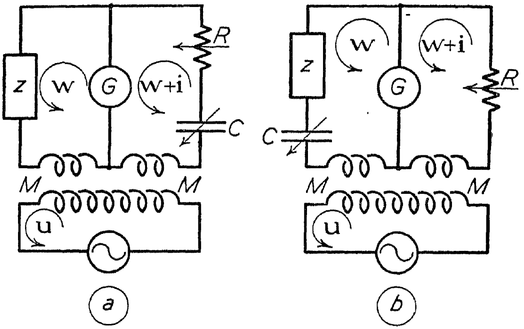

FIG. 146.—THE "HYBRID COIL" OR DIFFERENTIAL TRANSFORMER METHOD

$M_1/M_2 = R_1/R_2 = L_1/L_2 = 1$, as if M_{12} were not present. Hence, in adjusting the mutual inductances of two secondaries to equal values with respect to a common primary, the mutual inductance between the secondaries does not affect the balance of the network. Mr. Campbell's paper should be consulted for a discussion of several other practical applications of the principle of this method.

30a. The "Hybrid" Coil Method. The principle described in Section 30 has been applied in America* to a convenient and sensitive method for measuring capacitive or inductive impedances; its advantages deserve to be better known. Fig. 146 shows the connections, in which an essential feature is a three-winding transformer with two exactly similar secondaries having equal mutual inductances with respect to the common primary; this can be secured by winding the secondaries with twisted twin conductor. The arrangement resembles the "hybrid" coil used in telephone circuits, and is best toroidally wound.

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* A. T. Starr, "A note on impedance measurement," *W. Eng. and Exp. W.*, Vol. 9, pp. 615–617 (1932). An arrangement of this kind is made by the L.M. Ericsson Aktiebolaget.

With the arrangement of Fig. 146 (*a*), $z = P + jX$ is an unknown impedance, C and R are variable standards; then if m is the mutual inductance between the secondaries, z_s the impedance operator for one secondary and z_g for the galvanometer.

$$(j\omega m - z_g)\mathbf{i} + (z + z_s + j\omega m)\mathbf{w} = -j\omega M\mathbf{u}$$

$$\left(R + \frac{1}{j\omega C} + z_s + z_g\right)\mathbf{i} + \left(R + \frac{1}{j\omega C} + z_s + j\omega m\right)\mathbf{w} = -j\omega M\mathbf{u}$$

When $\mathbf{i} = 0$, $z = R + (1/j\omega C)$; that is

$$P = R \text{ and } X = -1/\omega C,$$

so that z is a capacitive impedance. For inductive impedances the arrangement of Fig. 146 (*b*) is used; it is easy to show in the same way that

$$P = R \text{ and } X = 1/\omega C$$

It will be seen that interchanging the source and galvanometer converts the method into the differential transformer* method introduced many years ago as a substitute for the much less convenient differential telephone; see p. 253.

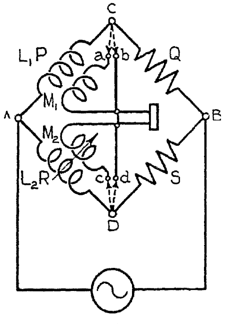

FIG. 147.—CAMPBELL'S METHOD FOR COMPARING TWO MUTUAL INDUCTANCES

31. Campbell's Method for Comparison of Two Mutual Inductances.

A further method for the comparison of two mutual inductances of any value has been introduced by A. Campbell,† and is illustrated in Fig. 147.

In this method the primary of the unknown mutual inductance and that of the standard are connected in the branches *AC* and *AD*.‡ The secondaries are arranged so that they can be connected at will in circuit with the detector. Let it be assumed that M_1 be greater than the largest value of the standard M_2.

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* For a shielded balanced differential transformer suitable for tests on two-core telephone cables, see J. Kühle, "Eine einfache Kompensationsschaltung zur Messung der Betriebswerte von Kapazität und Ableitung an Fernsprechkabeln," *Elekt. Zeits.*, Vol. 43, pp. 1205–1208 (1922). Also see W. A. Prowse, "Inductance measurements using a differential transformer," *Journal Sci. Insts.*, Vol. 13, pp. 219–221 (1936).

† A. Campbell, *Proc. Phys. Soc.*, *loc. cit.*, pp. 79–80. For a somewhat similar method, see H. Busch, *Phys. Zeits.*, Vol. 26, pp. 563–565 (1925).

‡ The winding which forms the branch *AD* should be the one which is not subdivided.

With the two-way switches arranged so that Cb and Dd are joined, let the self-inductances L_1, L_2 of the primaries be compared by Maxwell's method (*see* p. 309). The branch AC should contain an auxiliary adjustable self-inductance so that L_1 can be made greater than L_2, and also a rheostat so that P is greater than R. Let a balance be obtained by adjustment of Q and S and the rheostats or auxiliary inductance in AC, then

$$L_1/L_2 = P/R = Q/S. \qquad . \quad . \quad . \quad (a)$$

Now, without altering the branches $ACBDA$, connect Ca and Dc, so that the secondaries of the two mutual inductances are in series with the detector and inject voltages in opposition. Then let balance be restored by adjustment of M_2.

The condition for no current in the detector can easily be found from p. 71. Putting all mutual operators zero except $m_{15} = j\omega M_1$ and $m_{45} = j\omega M_2$, $\alpha = -j\omega M_2$, $\beta = -j\omega(M_1 + M_2)$, $\gamma = j\omega M_2$, $\delta = j\omega(M_1 + M_2)$. Remembering that the branch impedance operators are $z_1 = P + j\omega L_1$, $z_2 = Q$, $z_3 = S$, $z_4 = R + j\omega L_2$, the balance equation reduces to

$$[SP - QR + \omega^2(L_1M_2 - L_2M_1)] + j\omega[SL_1 - QL_2 - M_2(P + Q) \\ + M_1(S + R)] = 0.$$

Inserting the conditions given in Equation (a), the balance relations are,

$$M_1/M_2 = L_1/L_2 = (P + Q)/(R + S) = Q/S,$$

which gives M_1 in terms of M_2 and resistances Q and S, neither of which contains copper coils.

Experimental Example. In illustration of this procedure the following experiment was carried out at 407·1 cycles per second, using the mutual inductance tested by previous methods. The branch AC contained the low inductance coil of the mutual, an Ayrton-Perry variable standard of self-inductance and an adjustable resistance. Branch AD contained the fixed winding of a Campbell 11 mH. mutual inductometer, together with a resistance box. Q and S were resistances in the ratio of 2 to 1. Balance was found by adjustment of P, R, and the variable self inductance. The second balance was secured by adjustment of M_2.

Q ohms.	S ohms.	P ohms.	R ohms.	L mH.	M_2 μH.	M_1 henry.
50	25	$49{\cdot}4_3$	$24{\cdot}7_1$	18·1	492_3	$0{\cdot}00984_6$
100	50	$99{\cdot}4_3$	$49{\cdot}7_2$	18·1	492_4	$0{\cdot}00984_8$
200	100	$199{\cdot}4_3$	$99{\cdot}7_0$	18·1	492_2	$0{\cdot}00984_4$
					Average	**$0{\cdot}00984_6$** henry

32. Mutual Inductance Measured as Self-inductance. When two coils are given with their terminals completely accessible, the mutual inductance, M, between them can be found by a simple expedient. Let the coils be connected together in series or in parallel, the apparent self-inductance of the combination being measured by any suitable method. Then, re-arranging the connections so that the effect of mutual inductance is reversed, let a second observation of apparent self-inductance be taken. From these two measurements, M can now be found.

The simplest and most obvious way of arranging the coils is to join them in series, as shown in Fig. 148. When connected as in the left-hand diagram the apparent self-inductance between the terminals A and C will be

$$\lambda_1 = L_1 + L_2 + 2M\,;$$

and when the connections are changed so that M is reversed the apparent inductance is

$$\lambda_2 = L_1 + L_2 - 2M,$$

whence $$M = \tfrac{1}{4}(\lambda_1 - \lambda_2).$$

Experimental Example. The values of λ_1 and λ_2 were found at a frequency of 407·1 cycles per second for the 10·0 millihenry mutual tested by previous methods, the Heaviside-Campbell bridge of page 437 being used. The results were $\lambda_1 = 0{\cdot}17005_6$ henry and $\lambda_2 = 0{\cdot}13066_4$ henry, whence $M = 0{\cdot}00984_8$ henry, in agreement with values previously determined.

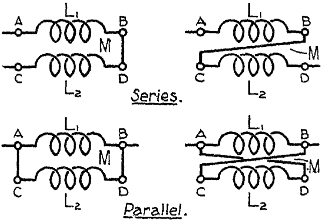

FIG. 148.—CONNECTIONS FOR THE MEASUREMENT OF MUTUAL INDUCTANCE AS SELF-INDUCTANCE

This method appears to have been known and used some time before it was described in the technical press. Dr. Alexander Russell,* in 1894, drew attention to it as a possible ballistic method; A. Trowbridge,† in 1904, re-introduced it as a new method and pointed out a simple modification. Assuming that λ_1 and λ_2

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* A. Russell, "Measuring coefficients of induction," *Elecn.*, Vol. 33, pp. 5–6 (1894).

† A. Trowbridge, "A method for the determination of coefficients of mutual induction," *Phys. Rev.*, Vol. 18, pp. 184–186 (1904).

are to be found by Maxwell's method, and that a suitable variable self-inductance standard is not available, Trowbridge suggests that λ_1 and λ_2 should be compared with a coil of inductance L, the value of which lies between λ_1 and λ_2. Then, if $\lambda_1/L = p$ and $\lambda_2/L = q$, the value of M is easily seen to be

$$M = \tfrac{1}{4}(p - q)L.$$

Dr. Russell, in his paper, suggested that the coils might be arranged in parallel, measurements being taken as before, with M successively positive and negative. Though the resulting relationships from which to determine M are simple when a ballistic bridge is used, the equations become unwieldy when applied to alternating currents. Fig. 148 shows the two arrangements, λ_3 and λ_4 being their apparent inductances; it is easy to show that*

$$\lambda_3, \lambda_4 \quad \frac{L_1R_2^2 + L_2R_1^2 \pm 2MR_1R_2 + (L_1L_2 - M^2)(L_1 + L_2 \mp 2M)\omega^2}{(R_1 + R_2)^2 + \omega^2(L_1 + L_2 \mp 2M)^2}$$

from which M is to be found. The method is inconvenient in practice.

Another process has been suggested by Chatterjee.† Measure the self-inductance L_1 on any convenient bridge. Then short-circuit the second coil and remeasure the effective inductance of the first. Then Maxwell has shown that the inductance will fall by an amount

$$\Delta L_1 = \omega^2M^2L_2/(R_2^2 + \omega^2L_2^2),$$

where L_2 is the inductance and R_2 the resistance of the second coil. Now repeat the measurements on the second coil, with the first coil open and closed respectively. The corresponding fall of inductance is

$$\Delta L_2 = \omega^2M^2L_1/(R_1^2 + \omega^2L_1^2),$$

where R_1 is the resistance of the first coil. Eliminating ω gives

$$M^2 = \frac{R_2^2L_1^2 - R_1^2L_2^2}{(R_2^2L_1/\Delta L_2) - (R_1^2L_2/\Delta L_1)}$$

A. Campbell has used a similar method for the direct measurement of large time-constants, without the necessity of independent determination of inductance and resistance. Set up any convenient bridge for the measurement of self-inductance, preferably one in which inductive coupling effects are not likely to cause error, and measure any convenient coil. Let its resistance be r and its inductance l, and let the resistance and inductance of the coil of which the time-constant is required be R, L. Short circuit the terminals of the unknown coil R, L and bring it into proximity with the coil r, l so that there is mutual inductance M between them. The bridge will become unbalanced; let balance be restored, thus measuring the effective resistance ρ and inductance λ of the combination, then

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* *See*, for example, Alexander Russell, *A Treatise on the Theory of Alternating Currents*, Vol. 1, 1st edition, p. 170 (1904).

† L. M. Chatterjee, "Über eine Methode zur Bestimmung des Koeffizienten der als Selsbstinduktion gemessenen Gegeninduktion," *Phys. Zeits.*, Vol. 34, pp. 711–712 (1933).

$$\rho = r + \frac{\omega^2 M^2 R}{R^2 + \omega^2 L^2} \text{ and } \lambda = l - \frac{\omega^2 M^2 L}{R^2 + \omega^2 L^2},$$

so that $L/R = (l - \lambda)/(\rho - r)$,

giving the time-constant directly in terms of the changes in balance settings.

33. Hartshorn's Method for Impure Mutual Inductances.

All the methods so far described assume that the mutual inductances concerned are perfect, i.e. the secondary voltage and primary current in them are exactly in quadrature. While this is true of most inductances at low frequencies, it is certainly not so at the higher telephonic frequencies, as has been pointed out on page 153. In consequence of the effects of self- and inter-capacitance of the windings, eddy currents, and leakage, the secondary voltage and primary current are no longer in exact quadrature but are separated in phase by an angle somewhat less than 90 degrees; such a mutual inductance is described as impure. None of the preceding methods make provision for the effects of impurity in the mutual inductances under comparison.

Again, it is further shown on page 155 that, in order to make the potentials of the primary and secondary coils quite definite and thereby to stabilize their mutual capacitance, it is necessary to have a common point on the two windings, especially when the mutual inductance is used at high frequencies. Except quite fortuitously in one of Geyger's methods, Fig. 145 (*a*), the tests described are made without a common point. Consequently, if tests are made by any of the above comparison methods on large mutual inductances at high frequencies it will be found that only minimum sound can be obtained in the bridge telephone because the impurity terms are not balanced. Moreover, the sound minimum is liable to change, since the relative potentials of the windings are not accurately defined.

To overcome these defects Hartshorn* has described methods in which common points are used, and the impurity is allowed for. If one of the mutuals is a standard of known impurity, then the impurity of the other may be found. Fig. 149 (*a*) shows the circuit for comparing two unequal mutuals, M_1 and M_2 having impurities σ_1 and σ_2. The resistances R_1 and R_2 are non-reactive ratio coils; L is a variable self-inductance, R_4 a variable resistance, and r_3 a slide wire resistance. The telephone T_2 is first switched off and the primary circuit is balanced as an inductance bridge by adjustment of L and R_4 until no sound is heard in T_1. Then at balance $R_1\mathbf{i}_1 = R_2\mathbf{i}_2$. The telephone T_2 is now connected and variations of r_3 and M_2, the standard inductometer, are made until T_2 is silent. Then, since there is again no current in the secondary circuit $\mathbf{i}_1$ and $\mathbf{i}_2$ are unchanged. Noting that $\mathbf{i}_1$ is in the positive-cyclic direction while $\mathbf{i}_2$ is in the negative direction, and that the mutual inductances are of the same sign, the total applied voltage in the secondary circuit is easily seen to be

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* L. Hartshorn, "A precision method for the comparison of unequal mutual inductances at telephonic frequencies," *Journal Sci. Insts.*, Vol. 2, pp. 145–151 (1925).

$$(j\omega M_2 - \sigma_2)\mathbf{i}_2 - r_3\mathbf{i}_2 - (j\omega M_1 - \sigma_1)\mathbf{i}_1 = 0$$

Making use of the first balance,

$$R_1(r_3 + \sigma_2 - j\omega M_2) - R_2(\sigma_1 - j\omega M_1) = 0$$

from which $$M_1 = M_2 R_1/R_2$$

$$\sigma_1 = R_1(\sigma_2 + r_3)/R_2$$

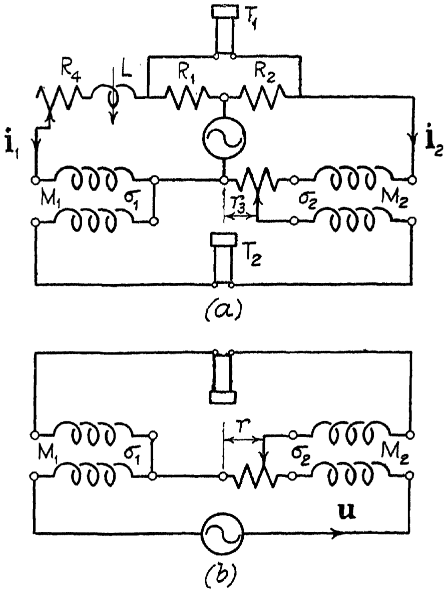

FIG. 149.—HARTSHORN'S METHODS FOR THE COMPARISON OF IMPURE MUTUAL INDUCTANCES

When testing standard mutuals the effect of residual inductance l_1, l_2 in R_1, R_2 must be taken into account. Substituting $R_1 + j\omega l_1$ and $R_2 + j\omega l_2$ for R_1 and R_2 above makes

$$M_1R_2 - M_2R_1 + l_1(r_3 + \sigma_2) - l_2\sigma_1 = 0,$$

$$(\sigma_2 + r_3)R_1 - R_2\sigma_1 + \omega^2(l_1M_2 - l_2M_1) = 0.$$

If the product of small quantities be neglected, then, very nearly,

$$M_1R_2 \doteqdot + M_2R_1$$

as before. Using this approximate relation in the second condition,

$$\frac{\sigma_1}{\omega M_1} = \frac{\sigma_2}{\omega M_2} + \frac{r_3}{\omega M_3} + \frac{\omega l_1}{R_1} - \frac{\omega l_2}{R_2}$$

If δ_1, δ_2 be the phase defects of the two mutuals, α_1 and α_2 the phase angles of the ratio arms, then

$$\delta_1 = \delta_2 + \frac{r_3}{\omega M_2} + \alpha_1 - \alpha_2$$

hence residual inductance makes no error if the ratio coils have equal time-constants, i.e. if $\alpha_1 = \alpha_2$.

When the two mutuals to be compared are of the same nominal value the circuit of Fig. 149 (*b*) is used. The two mutuals are arranged to act in opposition round the secondary circuit; then for no sound in the telephone

$$r\mathbf{u} + (j\omega M_1 - \sigma_1)\mathbf{u} + (j\omega M_2 - \sigma_2)\mathbf{u} = 0$$

or

$$M_1 + M_2 = 0$$

$$\sigma_1 + \sigma_2 - r = 0.$$

In either case it may be necessary to reverse the potential points on the slide wire resistance; the sign of r_3 and r in the balance equations must then be changed.

34. Maxwell's Method for Comparison of Self and Mutual Inductance. Maxwell* has described a bridge in which the mutual inductance between two coils is compared with the self-inductance of one of them by a ballistic method. Wien,† in 1891, published his experiments with the method when used with an alternating current, Fig. 150 (*a*) showing the network adopted.

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* *Treatise*, 1st Edn., Vol. 2, Sec. 756, p. 356 (1873).

† Max Wien, "Messung der Inductionsconstanten mit dem 'optische Telephon,'" *Ann. der Phys.*, Vol. 44, pp. 689–712 (1891). W. Geyger, "Kompensationsschaltungen mit einer gegenseitigen Induktion," *Arch. f. tech. Mess.*, J. 921–6 (Jan., 1936) classifies a number of bridges containing a single mutual inductance.

Making use of Heaviside's theory of the generalized Wheatstone network,* on pages 71 or 76, put all mutual inductance operators equal to zero, except $m_{16} = j\omega M$. Then $\alpha = 0$, $\beta = 0$, $\gamma = -j\omega M$, $\delta = 0$, and the balance equation becomes

$$S(P + j\omega L) - QR + j\omega M(Q + S) = 0.$$

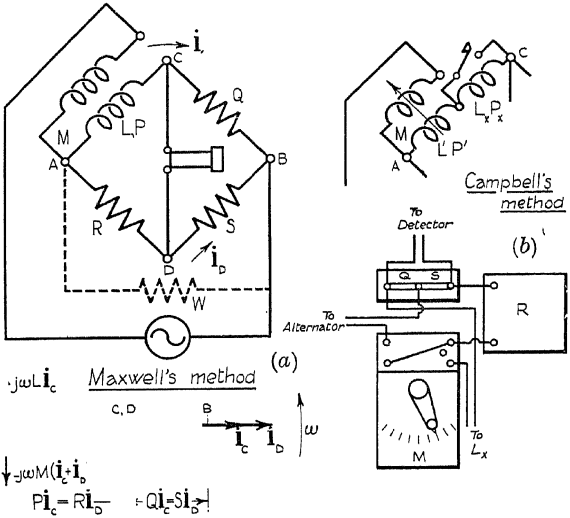

FIG. 150.—MAXWELL'S METHOD FOR COMPARING THE MUTUAL INDUCTANCE BETWEEN A PAIR OF COILS WITH THE SELF-INDUCTANCE OF ONE OF THEM. CAMPBELL'S MODIFICATION FOR THE MEASUREMENT OF LARGE INDUCTANCES

Separating components, the two balance conditions are

$$SP = QR$$

$$L = -\left(1 + \frac{Q}{S}\right)M. \qquad \qquad (a)$$

From this it is seen that M must be arranged to be negative (since L is positive), that is, to oppose the self-inductance; and that balance is only possible if $L > M$.

The vector diagram for the network is drawn in Fig. 150 (a).

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* The shortest method is adopted here. This network has, however, been worked out by two other methods on pp. 68 and 71, to which the student is referred.

The vector **e** is the potential difference between A and B. Since Q, R, and S are pure resistances and the points C, D are at the same potential, the currents $\mathbf{i}_C$ and $\mathbf{i}_D$ must be in phase with **e**. The inductive drop $j\omega L\mathbf{i}_C$ must then be exactly balanced by the induced electromotive force $-j\omega M(\mathbf{i}_C + \mathbf{i}_D)$. From the geometry of the diagram, the balance conditions follow at once.

Assuming L/M to be fixed, as, for example, in the case of the windings of a transformer, balance must be secured by the successive adjustment of Q/S and P/R until the balance equations are satisfied. These two adjustments obviously interfere with one another, so that the process of balancing becomes tedious. In order to avoid this trouble, Maxwell* has described a simple modification, adapted to alternating current by H. Rowland,† in 1898. Referring to Fig. 150 (a), let a resistance W be connected across the branch points AB; it is then easy to prove that there will be no current in the detector when

$$SP = QR,$$

and

$$= -\left\{1 + \frac{Q}{S} + \frac{Q(S+R)}{WS}\right\} M.$$

Balance is now attained by fixing Q/S and adjusting R and W successively. These adjustments do not interfere, so that balance is quickly convergent. As before, L must be greater than M, the latter being negative (*see* also page 556).

Experimental Example. To illustrate these methods, the following experiment was performed. M was a 111 mH. Campbell mutual inductometer, L being the inductance of its fixed winding. Q and S were equal ratio coils, R being an adjustable resistance. Balance was secured at 407·1 cycles per second by adjustment of R and M, using a Duddell vibration galvanometer.

Q ohms.	S ohms.	$R = P$ ohms.	M μH.	L henry.
10	10	$39{\cdot}1_6$	5029_0	$0{\cdot}10058_0$
100	100	$39{\cdot}1_6$	5029_0	$0{\cdot}10058_0$
			Average	**$0{\cdot}10058_0$** henry

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* *Loc. cit.*, p. 357,

† H. Rowland, "Electrical measurement with alternating currents," *Phil. Mag.*, 5th series, Vol. 45, pp. 66–85 (1898); *Amer. J. Sc.*, 4th series, Vol. 4, pp. 429–448 (1897).

In a second trial M was fixed, balance being found by adjustment of R and of the auxiliary resistance W.

Q ohms.	S ohms.	$R = P$ ohms.	W ohms.	M μH.	L henry.
10	10	$39{\cdot}2_2$	$95{\cdot}8_0$	40,000	$0{\cdot}10055_2$
10	10	$39{\cdot}2_5$	$36{\cdot}3_9$	30,000	$0{\cdot}10060_2$
				Average	**$0{\cdot}10057_7$** henry

In order to make use of Maxwell's method when L is less than M, M. Brillouin,* in 1882, and C. H. Lees,† in 1916, have suggested modifications to the ballistic bridge. Unfortunately, these bridges can only be balanced for aggregate zero quantity in the galvanometer, and the modifications are not applicable to alternating current. In the case of an alternating current bridge in which L is less than M, balance can be attained by loading the branch AC with known self-inductances until the total inductance of the branch exceeds the value of M. Balance is then found in the usual way.

If a variable standard of mutual inductance is available, A. Campbell‡ has shown how Maxwell's method can be used to measure self-inductances of a value greater than twice the highest reading of the standard. The fixed coils of the standard are included in the branch AC (Fig. 150 (b)) in series with the coil to be tested, L_x, P_x. The ratio Q/S is fixed and the coil L_x short circuited, balance being secured by adjustment of M and the rheostat R. Let M_o, R_o be the readings, then from Equation (a), page 429,

$$P' = \frac{Q}{S} R_o \text{ and } L' = -\left(1 + \frac{Q}{S}\right) M$$

Now unshort the coil and re-balance by adjustment of M and R to values M_1 and R_1, then

$$P' + P_x = \frac{Q}{S} R_1 \text{ and } L' + L_x = -\left(1 + \frac{Q}{S}\right) M_1.$$

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* M. Brillouin, "Comparison des coefficients d'induction," *Ann. de l'École Normale*, Vol. 11, pp. 339–424 (1882).

† C. H. Lees, "On a general bridge method for comparing the mutual inductance between two coils with the self-inductance of one of them," *Proc. Phys. Soc.*, Vol. 28, pp. 89–93 (1916).

‡ A. Campbell, "Inductance measurements," *Elecn.*, Vol. 60, pp. 626–627 (1908).

Subtracting these two relations gives numerically

$$P_x = \frac{Q}{S}(R_1 - R_o),$$

and
$$L_x = \left(1 + \frac{Q}{S}\right)(M_1 - M_o).$$

In practice, Q/S is made equal to 9, 99, etc., so that L_x becomes $10\,(M_1 - M_o)$, $100\,(M_1 - M_o)$, etc. The method is very convenient but, for precise work, suffers from the usual disadvantages attending the use of unequal ratio branches. If Q be made equal to S, then numerically

$$P_x = R_1 - R_o$$
$$L_x = 2(M_1 - M_o)$$

Experimental Example. M was a 111 mH. Campbell mutual inductometer, Q and S coils in a Paul ratio box (Fig. 49), R an adjustable resistance. The frequency was 407·1 cycles per second, the detector being a Duddell vibration galvanometer. The observed values were $Q = 900$ ohms, $S = 100$ ohms, $M_0 = 1006_2$ μH., $R_0 = 4{\cdot}3_6$ ohms; $M_1 = 7064_5$ μH., $R_1 = 14{\cdot}9_3$ ohms. Hence, $P_x = 9(14{\cdot}9_3 - 4{\cdot}3_6) = 95{\cdot}1_3$ ohms, and $L_x = 10(7064_5 - 1006_2)10^{-6} = 0{\cdot}6058_3$ henry.

Maxwell's method has been adapted by Kuriyama* to measure impedances which carry large currents at high voltages, by the artifice of using instrument transformers between the h.v. circuit and the bridge as in Fig. 151. The conjugate position for source and galvanometer is used. When balance has been secured,

$$S\mathbf{i} - R\mathbf{i}_s + j\omega M\mathbf{i}_s = 0$$

If K_c, K_v are the ratios and β, γ the phase-angles of the current and voltage transformers, then it is shown in *Instrument Transformers* (p. xxiv) that

$$\mathbf{i}_s = -\frac{1}{K_c}\varepsilon^{j\beta}\,\mathbf{i}_p \text{ and } \mathbf{v}_s = -\frac{1}{K_v}\cdot\varepsilon^{j\gamma}\,\mathbf{v}_p$$

Neglecting the impedance of the current transformer primary in comparison with $z = P + j\omega L$

$$\mathbf{i}_p = \mathbf{v}_p/z \text{ and } \mathbf{i} = \mathbf{v}_s/(Q + S).$$

Substituting and reducing

$$\frac{S}{Q+S}(P + j\omega L) = \frac{K_v}{K_c}(R - j\omega M)\varepsilon^{j(\beta-\gamma)}$$
$$\doteqdot \frac{K_v}{K_c}(R - j\omega M)[1 + j(\beta - \gamma)]$$

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* M. Kuriyama, "On the a.c. bridge with instrument transformers and characteristics of each of its elements," *Journal I.E.E., Japan*, Vol. 53, pp. 1–3 (English abstract), pp. 1–12 (Japanese paper) (1933).

So that

$$P \doteq \frac{K_v}{K_c} \cdot \frac{Q+S}{S} [R + \omega M(\beta - \gamma)] \doteq \frac{K_v}{K_c} \cdot \frac{Q+S}{S} \cdot R$$

$$L \doteq -\frac{K_v}{K_c} \cdot \frac{Q+S}{S} \left[M - \frac{R}{\omega}(\beta - \gamma) \right] \doteq -\frac{K_v}{K_c} \cdot \frac{Q+S}{S} \cdot M$$

Kuriyama has used the method for tests on machine windings and transformers, both single- and three-phase, to measure mutual impedance between power and communication circuits, etc.

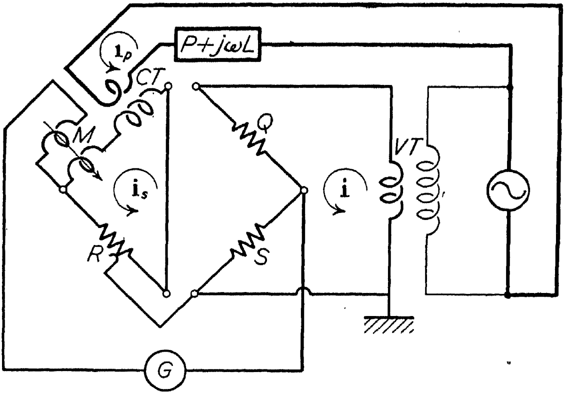

FIG. 151.—KURIYAMA'S METHOD FOR LARGE CURRENTS AND VOLTAGES

35. Campbell's Method for Measuring Self-inductance. In using Campbell's modification of Maxwell's method to measure self-inductance (Fig. 150 (*b*)), the result is given in terms of the difference of readings on the inductometer. The initial balance value M_0 accounts for the self inductance L' of the inductometer winding in the branch *AC*, and may be fairly considerable in value. In order that the range of the bridge be not unduly restricted by M_0 being too large, and, further, to guard against inaccuracy in the difference $M_1 - M_0$ when M_1 is not very different from M_0, Campbell* has introduced a "balancing coil" into the branch *AD* (Fig. 152). The object of this is to reduce the value of M_0 to be zero, or, at the worst, very small.

* A. Campbell, "On the use of variable mutual inductances," *Proc. Phys. Soc.*, Vol. 21, pp. 75–76 (1910).

On page 71 put $m_{16} = j\omega M$, so that $\alpha = 0$, $\beta = 0$, $\gamma = -j\omega M$, $\delta = 0$. Since $z_1 = P + j\omega L_1$, $z_2 = Q$, $z_3 = S$, $z_4 = R + j\omega L_2$, balance occurs when

$$S(P + j\omega L_1) - Q(R + j\omega L_2) + \omega M(S + Q) = 0\,;$$

from which

$$SP = QR,$$

$$L_1 - L_2\frac{Q}{S} = -\left(1 + \frac{Q}{S}\right) M. \quad . \qquad (b)$$

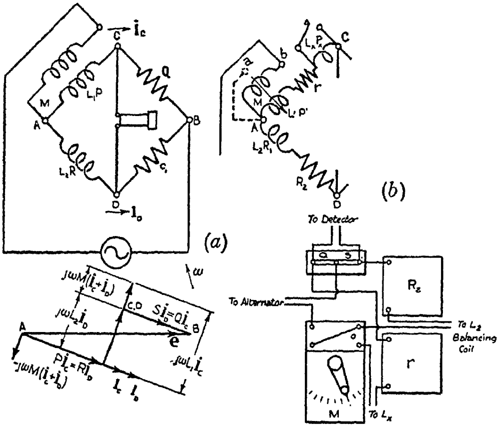

FIG. 152.—CAMPBELL'S METHOD FOR MEASURING SELF-INDUCTANCE, USING A "BALANCING COIL"

The vector diagram is drawn in Fig. 152 (*a*) and is self-explanatory.

Fig. 152 (*b*) shows the arrangement of the network for the measurement of a large inductance. The coil to be measured, L_x, and a constant inductance rheostat r are included in the branch AC in series with the fixed winding of the inductometer, L'. In the branch AD, L_2 is a balancing coil having an inductance equal to $L'S/Q$; R_2 is a resistance such that the total resistance of AD is about equal to S/Q times that of AC. Let L_x be short-circuited, balance being obtained by adjustment of M to a value M_0 and r to a value r_0 ; M_0 is now a very small quantity, accounting for the inductance of the leads to the coil L_x and for any slight differences between the inductances of AC and AD. Then, from Equation (b),

$$S(P' + r_o) = QR$$

and $$l + L' - \frac{Q}{S} \quad \frac{S}{Q} L' = -\left(1 + \frac{Q}{S}\right) M_o = l$$

if l represents the inductance excess of AC over AD.

Inserting the coil L_x, the reading of r must be *reduced* to a value r_1 while M is increased to M_1. Then

$$S(P' + P_x + r_1) = QR$$

and $$L_x + l = -\left(1 + \frac{Q}{S}\right) M_1.$$

Subtracting these relations from the former, gives numerically

$$P_x = r_o - r_1$$

$$L_x = \left(1 + \frac{Q}{S}\right) (M_1 - M_o) \qquad (c)$$

In practice, the ratio Q/S is made equal to 9, 99, etc., giving $L_x = 10$ or 100 times the difference of readings of the inductometer. The balancing coil has a value $L_2 = 1/9$ or $1/99$ of L'.

With the new type of inductometer described on page 151 the use of external balancing coils is unnecessary, since the fixed winding of the inductometer is tapped to give the desired inductance ratio. In Fig. 152 (b) $L_2 + L'$ is the entire fixed winding upon which M acts, in which L_2/L' has the value 1/9 or 1/99 to suit a bridge of ratio 10 or 100.

Experimental Example. Using the apparatus described on page 432, a balancing coil about 1/9 of L' was inserted in AD, together with a resistance box; r was now an adjustable resistance. At 407·1 cycles per second the balancing values were $Q = 900$ ohms, $S = 100$ ohms, $M_0 = -5$ μH., $r_0 = 239{\cdot}4_9$ ohms; $M = 6058_0$ μH., $r = 144{\cdot}2_0$ ohms. Hence, $P_x = 239{\cdot}4_9 - 144{\cdot}2_0 = 95{\cdot}2_9$ ohms, and $L_x = 10\,(6058_0 + 5)10^{-6}$ $= 0{\cdot}6058_5$ henry.

36. Residual Errors in Campbell's Method. In using the bridge of Fig. 152, large errors in effective resistance measurements may arise on account of the small residual reactances of the unequal ratio branches Q, S. The general nature of the effect is similar to that discussed by Giebe in connection with Maxwell's method for comparison of two self-inductances (*see* p. 315), and the theory has been given by Campbell* in a similar manner.

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* A. Campbell, "On the use of mutual inductometers," *Proc. Phys. Soc.*, Vol. 22, pp. 207–219 (1910).

Following the method on p. 315, put $Q + j\omega\lambda$ for Q and $S + j\omega\mu$ for S in the analysis on p. 434 preceding Equation (*b*). The balance equations are then

$$SP - QR = \omega^2[(L_1 - M)\mu - (L_2 + M)\lambda] \quad . \quad . \quad (d)$$
$$SL_1 - QL_2 = (Q + S)M - P\mu + R\lambda$$

when M is taken as negative, as is necessary to secure balance.

(i) If $Q = S$, then $(P - R)Q = \omega^2[(L_1 - M)\mu - (L_2 + M)\lambda]$,

and $$(L_1 - L_2 - 2M)Q = R\lambda - P\mu.$$

If, in addition, $\lambda = \mu$, $(P - R)Q = \omega^2\lambda(L_1 - L_2 - 2M)$

and $$-(P - R)\lambda = Q(L_1 - L_2 - 2M),$$

so that either $Q^2 = -\omega^2\lambda^2$, which is absurd, or

$$L_1 - L_2 = 2M$$

and $$P = R,$$

which is in agreement with Equation (*b*), p. 434, when $Q = S$. Thus, if the ratio branches have equal resistances and also equal residuals, no error will be introduced into the bridge. This can be checked by balancing the bridge, and then interchanging the ratios. They should be adjusted until the interchange does not upset the balance.

(ii) If $Q \neq S$, consider the bridge shown in Fig. 152 (*b*). Suppose the balancing coil L_2 to be adjusted so that with L_x short-circuited, balance occurs when $M = M_0$ and $r = r_0$, M_0 being a very small reading. Let $M = M_1$ and $r = r_1$ when L_x is introduced and balance is again secured. Putting these conditions in Equation (*d*) and applying the method of differences gives

$$(r_1 - r_0 + P_x)S = \omega^2[L_x\mu - (M_1 - M_0)(\lambda + \mu)]$$
$$SL_x = (Q + S)(M_1 - M_0) - (r_1 - r_0 + P_x)\mu.$$

Now λ and μ are usually very small compared with L_x and M_1, so that, approximately,

$$P_x \fallingdotseq r_0 - r_1 + \omega^2\left(\frac{Q}{S}\mu - \lambda\right)\frac{(M_1 - M_0)}{S}$$
$$L_x \fallingdotseq \left(1 + \frac{Q}{S}\right)(M_1 - M_0).$$

From these equations, L_x and P_x are found in terms of changes of r and M and the residuals λ, μ. The effect of the latter is negligible in L_x, but may be considerable in P_x. For example, suppose $Q = 99$ ohms, $S = 1$ ohm, $r_0 - r_1 = 20$ ohms, $M_1 - M_0 = 1$ millihenry, $\lambda = 10$ microhenrys, $\mu = 1$ microhenry; then at 1,000 cycles per second,

$$P_x \fallingdotseq 20 + 3{\cdot}6 \fallingdotseq 23{\cdot}6 \text{ ohms}$$
$$L_x \fallingdotseq 0{\cdot}1 - 3{\cdot}6 \times 10^{-6} \text{ henry}.$$

Hence the error in P_x due to neglect of residual effects is about 18 per cent, whereas in L_x the error would only be 3·6 in 100,000.

(iii) If $Q \neq S$, but in addition, $Q\mu = S\lambda$; i.e. if the residuals of Q and S are in the same ratio as their resistances, then the balance equations reduce to

$$P_x = r_o - r_1$$
$$L_x = \left(1 + \frac{Q}{S}\right)(M_1 - M_o)$$

which is identical with Equation (c), p. 435. Hence, if the residuals of the ratio branches are in the same ratio as their resistances, errors due to residual effects are entirely eliminated. This result can be very nearly attained by the use of a ratio box made up of Duddell-Mather gauze,* in which capacitance is negligible, the resistance and residual inductance being each proportional to the length of gauze used. The same end can also be attained by the following artifice. Let a coil be constructed of carefully stranded wire, and its self capacitance be measured. Then, from the formulae given on page 135, calculate the effective inductance and resistance of the coil at a given frequency, making use of the measured value of self capacitance and of its inductance and resistance at low frequency. Now put the coil in the bridge and supply current at the frequency for which the calculations are made. Then, if the measured values do not agree with the calculated values, add inductance to Q or S until agreement is obtained. The residual effects are then eliminated and another coil will be measured correctly with the same Q/S and frequency.

37. The Heaviside-Campbell Equal Ratio Bridge.

O. Heaviside† has given the general theory of a large number of induction balances, including, in particular, certain important bridges now to be described in which equal ratio branches are used. A. Campbell‡ has shown that the adoption of the method of differences described above renders these bridges of great value for the measurement of small self-inductances.

A convenient practical arrangement is shown in Fig. 153 (*a*). The branch AC now contains the inductometer fixed coil L', and the coil to be tested, L_x. The branch AD contains a balancing coil equal to L' and a constant inductance rheostat r. By taking balances with L_x in and out of the bridge, the balance conditions give numerically

$$P_x = r_1 - r_o$$
$$L_x = 2(M_1 - M_o),$$

when the branches CB, DB are equal.

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* *See* p. 91.

† O. Heaviside, *Electrical Papers*, Vol. 2, pp. 33–38, pp. 106–115, pp. 284–286 (1892).

‡ A. Campbell, "On the use of variable mutual inductances," *Proc. Phys. Soc.*, Vol. 21, pp. 76–78 (1910). "On the use of mutual inductometers" *Proc. Phys. Soc.*, Vol. 22, pp. 207–219 (1910).

The balancing coil in AD in the equal ratio bridge introduces a loss of sensitivity and range ; a simple modification of the inductometer renders the use of a balancing coil unnecessary and permits of increased sensitivity. Let the primary of the inductometer be composed of two equal coils, (marked L'' in Fig. 153 (*b*)), one being put in the branch AC and the other in AD. Mutual induction then acts on each of these branches from the secondary coil, and there will also be mutual inductance between the two halves of the primary winding.

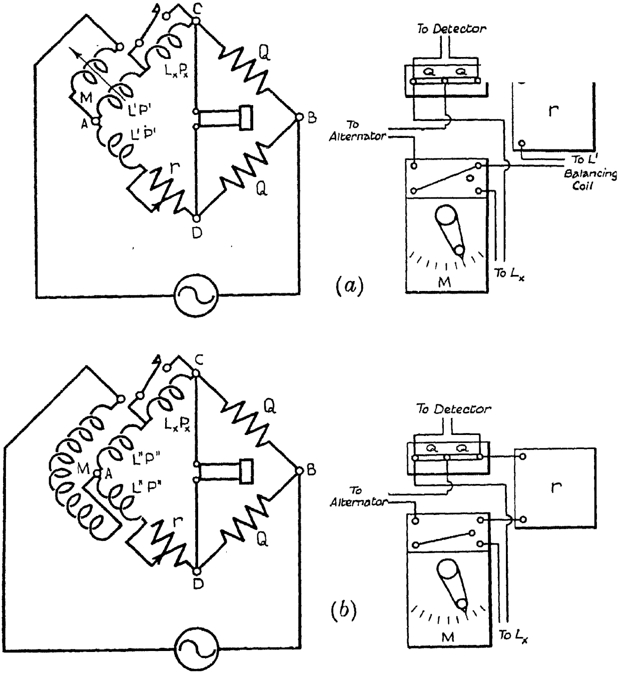

FIG. 153.—THE HEAVISIDE-CAMPBELL EQUAL RATIO BRIDGE FOR THE MEASUREMENT OF SMALL INDUCTANCES

First suppose that L_1, P are the constants of AC, Q of CB, S of BD, and L_2, R of AD, as in Fig. 154. To accord with the convention of p. 70 suppose two separate mutual inductors M_{16}, M_{46}, each consisting of parallel similarly wound pairs of coils with a common point at A, are used, and that there is mutual inductance M_{14} between the windings inserted in AC and CD. Then, on p. 71, put $z_1 = P + j\omega L_1$, $z_2 = Q$, $z_3 = S$, $z_4 = R + j\omega L_2$, $m_{16} = j\omega M_{16}$, $m_{14} = j\omega M_{14}$, $m_{46} = j\omega M_{46}$,

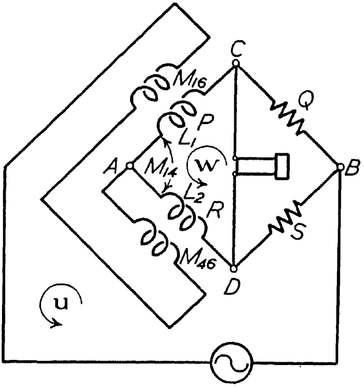

FIG. 154.—ILLUSTRATING THEORY OF GENERAL HEAVISIDE BRIDGE

the remaining mutual operators being zero. This results in $\alpha = 0$, $\beta = 0$, $\gamma = j\omega(M_{14} - M_{16} + M_{46})$, $\delta = 2j\omega M_{14}$, and therefore,

$$S(P + j\omega L_1) - Q(R + j\omega L_2) - j\omega(M_{14} - M_{16} + M_{46})(Q + S) + 2j\omega SM_{14} = 0.$$

Separating components gives

$$SP = QR$$

$$L_1 - L_2\frac{Q}{S} = -(M_{16} - M_{46})\left(1 + \frac{Q}{S}\right) + M_{14}\left(\frac{Q}{S} - 1\right).$$

In the particular case of $Q = S$, $P = R$ and the term containing M_{14} vanishes; then

$$L_1 - L_2 = -2(M_{16} - M_{46}).$$

If the mutual inductors are symmetrically constructed and similar windings are included in AC and AD respectively, the inductances of these will balance very nearly or can be annulled by a small amount of mutual inductance. When the test coil is inserted, L_1 is necessarily greater than L_2 by the amount of the added inductance in AC, and this is balanced out by an appropriate adjustment of the mutual inductances. It is obviously wasteful to have M_{16} and M_{46} acting in opposite senses round the mesh ADC. In the Campbell inductor the two equal primaries act on a common secondary in such a way that the mutual induction e.m.f.'s in ADC are additive, as shown diagrammatically in Fig. 153 (*b*). For this bridge, therefore,

$$L_1 - L_2 = -2(M_{16} + M_{46}) = -2M$$

where M is the total mutual inductance between the secondary and its primaries; the minus sign indicates, since $L_1 - L_2$ is essentially positive, that the mutual inductance must oppose the self-inductance to obtain balance, as is physically obvious. This condition involves reversal of the connections to the coil in the source lead if the similarly wound, parallel coil convention is retained in this diagram.

If the bridge has equal ratio branches, as in Fig. 153 (*b*), the above discussion shows that the effect of M_{14} is zero. Also $M_{16} + M_{46}$ is the total mutual inductance between primary and secondary, i.e. the reading of the standard. Then, applying the method of differences, remembering that the two primary coils are identical, and that the mutual inductance opposes the self-inductance, we have numerically,

$$P_x = r_1 - r_0$$
$$L_x = 2(M_1 - M_0).$$

The equal ratio method is subject to the following errors—

Residual Effects. The effects of the leads joining L_x to the bridge, of slight inequalities in the two primary coils, and of the inductance of r, are eliminated by the method of differences. The initial reading M_0 may be negative if it should so happen that AD has a greater residual inductance than AC. Slight residuals in the branches Q, Q can be eliminated by repeating observations with these branches reversed and taking the mean. Since r is of constant inductance, residuals in AC and AD are cancelled by the procedure described.

Impurity. Owing to the effects of self and inter-capacitances, and of eddy currents, the secondary induced electromotive force in the inductometer will not be truly in quadrature with the primary current; the mutual inductance operator becomes $j\omega M - \sigma$, σ being called the "impurity." S. Butterworth* has shown how this enters into the balance of an equal ratio bridge. Consider Fig. 152 (*b*) and suppose $Q = S$; with the source connected to a and L_x removed balance by adjustment of r and L_2. Introduce L_x, connect the source to b, and re-balance by adjustment of r and M. Then $L_x = 2M$, and $P_x = 2\sigma + r_0 - r_1$. Transference of the ratios removes their residuals as described above.

Thus, impurity has no effect on the inductance measurement, but may considerably influence a measurement of effective resistance. For example, in a 10 millihenry standard at 2,000 cycles per second, $\sigma \doteqdot 0{\cdot}1$ ohm; hence neglect of σ may lead to an error of 2 per cent in the measurement of effective resistance of a 20 millihenry inductance having a resistance of 10 ohms.

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* S. Butterworth, "Capacity and eddy current effects in inductometers," *Proc. Phys. Soc.*, Vol. 33, pp. 312–354 (1921). *See* also page 153.

Experimental Examples. The following experiment was made at 407·1 cycles per second using a 111 mH. Campbell mutual inductometer for M. With the arrangement of apparatus shown in Fig. 153 (*a*), the equal ratios were 100 ohms each. The balancing coil was an Ayrton-Perry variable self-inductance. With $M = 0$, and L_x short circuited, the variable self-inductance and r were adjusted to give balance, r_0 being then $0{\cdot}4_7$ ohm. With the self-inductance of the mutual inductometer fixed winding thus balanced, L_x was inserted, balance being restored by adjustment of M to 2033_5 μH., and r to $5{\cdot}6_6$ ohms. Hence, $P_x = 5{\cdot}6_6 - 0{\cdot}4_7 = 5{\cdot}1_9$ ohm and $L_x = 2 \times 2033_5 \times 10^{-3} = 40{\cdot}67_0$ mH.

Removing the balancing coil and re-arranging the connections to the mutual inductometer, as shown in Fig. 153 (*b*), the test was repeated with the following results—

Q ohms.	M_0 μH.	r_0 ohms.	M μH.	r_1 ohms.	L_x mH.	P_x ohms.
10	5	$0{\cdot}0_3$	2034_1	$5{\cdot}2_2$	$40{\cdot}67_2$	$5{\cdot}1_9$
90	5	$0{\cdot}0_3$	2034_0	$5{\cdot}2_2$	$40{\cdot}67_0$	$5{\cdot}1_9$
100	5	$0{\cdot}0_3$	2034_2	$5{\cdot}2_2$	$40{\cdot}67_4$	$5{\cdot}1_9$
				Average	**$40{\cdot}67_2$** mH.	**$5{\cdot}1_9$** ohms

To show the applicability of the method for the measurement of small inductances, a 1·1 mH. Campbell mutual inductance was used to measure at 407·1 cycles per second a small coil tested by Butterworth's method (*see* p. 388). Using the arrangement of Fig. 153 (*b*), $Q = 10$ ohms, $r_0 = 0{\cdot}0_3$ ohm, $M_0 = 0{\cdot}0_0$ μH., $r_1 = 0{\cdot}6_8$ ohm, $M_1 = 25{\cdot}7_0$ μH., were found to give balance. Hence, $P_x = 0{\cdot}6_8 - 0{\cdot}0_3 = 0{\cdot}65$ ohm, and $L_x = 2(25{\cdot}7_0 - 0{\cdot}0_0) = 51{\cdot}4_0$ μH.

Modification for Measurement of Residuals. Young* has described a form of Heaviside equal ratio bridge for the measurement of residual inductances (whether positive or negative) in resistors from 0·01 ohm to 10 megohms at frequencies up to 100 kilocycles per second. The mutual inductor has fixed toroidal balanced windings on a non-magnetic core, half the primary being included in AC and half in AD. The mid-point of the secondary is joined to A, and its two halves are connected in parallel to the source, each in series with a resistor by means of which the currents in them can be varied in any desired proportion. This has the effect of putting different inductances into the two bridge arms to the extent of 10 μH. by steps of 0·001 μH.; the two resistors are operated by a common control in the form of three Feussner decade dials. The resistor under test is joined in AC, and a standard of calculable reactance is included in AD.

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* C. H. Young, "A bridge for measuring small phase angle," *Bell Lab. Rec.*, Vol. 13, pp. 277–280 (1935).

Although the Heaviside-Campbell equal ratio bridge is primarily intended for the measurement of small self-inductances, Dye* has shown that it can readily be adapted to measure very large inductances, such as iron-cored choking coils or intervalve transformers, and is one of the best methods for this purpose. Referring to Fig. 106 on page 306, it has been shown that the effective inductance of a coil shunted by a non-reactive resistance can be made as small as desired by proper choice of the shunt: moreover, if the shunt be small enough the effective resistance of the combination becomes less than the actual resistance of the coil. Now the primary winding of an intervalve transformer, for example, may have an inductance of many henrys and a resistance of some thousands of ohms; hence by suitably shunting the winding, the effective inductance and resistance of the combination may be brought within the range of an equal ratio bridge containing low impedance standards of mutual inductance and resistance.

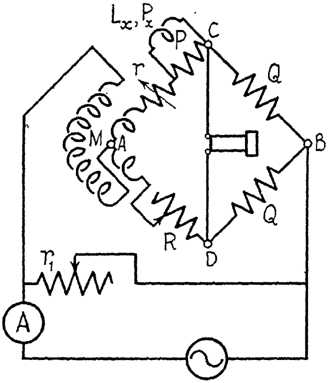

FIG. 155.—DYE'S MODIFICATION OF THE HEAVISIDE-CAMPBELL BRIDGE FOR THE MEASUREMENT OF LARGE INDUCTIVE IMPEDANCES

The arrangement of the bridge is shown in Fig. 155. The ratios Q, Q are 1,000 ohms each; r is a resistance of about 10 ohms adjustable to 0·01 ohm. For normal intervalve transformers P is 1,000 ohms and M will lie within the range of a 10 mH. standard; with poor transformers it may be necessary to reduce P to 500 ohms in order that M shall not exceed 10 mH. L_x, P_x are the inductance and resistance of the coil to be tested. The point B is directly earthed to stabilize the potential of the network and reduce earth capacitance effects; if desired, a Wagner earthing device would be superior to this direct earthing but is found in practice to be hardly necessary.

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* D. W. Dye, "The performance and properties of telephonic frequency intervalve transformers," *Exp. W. and W. Eng.*, Vol. 1, pp. 691–698 (1924); Vol. 2, pp. 12–21, 74–84 (1925). Also see A. Campbell, "A versatile inductometer bridge," *Journal Sci. Insts.*, Vol. 4, pp. 305–311 (1927).

The procedure is as follows. By means of the shunt r_1 and the thermo-ammeter A the p.d. across P can be adjusted to a desired value; the whole bridge has a resistance of about 1,000 ohms, so that the current in CB, which is very nearly that in P at balance, can be calculated by the use of a simple shunt multiplier. The values of M and r are then observed first with the coil disconnected from P (M_o, r_o) and then after shunting it across P (M, r). Let L', P' be the effective inductance and resistance of the shunted coil; then it is clear that

$$L' = 2(M - M_o) = 2\Delta M \text{ and } P - P' = r - r_o = \Delta r$$

Now $$P' + j\omega L' = P(P_x + j\omega L_x)/(P + P_x + j\omega L_x)$$

from which it is easy to deduce that

$$L_x = \frac{2P^2\Delta M}{(\Delta r)^2 + \omega^2(2\Delta M)^2}$$

$$P_x = \frac{P^2\Delta r}{(\Delta r)^2 + \omega^2(2\Delta M)^2} - P$$

Dr. Dye gives in his paper a very complete study of the properties of intervalve transformers, using this method; the original should be referred to for full details.

38. Campbell's Methods for the Measurement of the Self-inductance of Four Terminal Resistors. Closely related to the above methods are certain bridges suggested by Campbell* for the measurement of the small inductance of a low resistor or shunt. A shunt is a four-terminal resistor, i.e. it has two current terminals and two potential terminals, and cannot be directly measured in a simple bridge owing to this fact. On page 409 adaptations of the Kelvin bridge for the measurement of low value four-terminal impedances have been described; we shall now consider further methods making use of the Mz pair discussed on page 308. Referring to Fig. 156 (*a*) the four-terminal resistor r having self-inductance l is joined as shown with the low-reading inductometer M and a second mutual inductor M_1. The secondary of M_1 is closed through a slide-wire, the portion tapped having resistance S and residual inductance λ; R and L are the resistance and inductance of the secondary coil, together with the part of the slide-wire above the contact. It is easy to show that at balance.

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* A. Campbell, "On the measurement of small inductances and on power losses in condensers," *Proc. Phys. Soc.*, Vol. 29, pp. 347–349 (1917).

$$M_1S = r(L + \lambda) + (R + S)(l - M),$$

$$(L + \lambda)(l - M)\omega^2 - \lambda M_1\omega^2 = r(R + S)$$

If L/R be large and λ/S small, while λM_1 is negligible in comparison with $L(l - M)$, the second condition becomes

$$L(l - M)\omega^2 = r(R + S).$$

Since the voltage injected by M must oppose the reactance drop ωl in the resistor and a component of drop down the slide wire in quadrature with the source current, M is necessarily negative (opposing self-inductance) and greater than l. Numerically

$$L(l + M)\omega^2 = r(R + S),$$

from which l is found independently of M_1 or λ. In practice it is not easy to fulfil the essential conditions except at high frequencies, say 800 cycles per second; as it is well known that the inductance and resistance of a properly designed shunt are practically independent of frequency this limitation is not serious. In an actual example tested at this frequency, $r = 0{\cdot}01$ ohm, $R + S = 5{\cdot}99$ ohm, $S = 0{\cdot}274$ ohm, $L = 0{\cdot}01$ henry, $\lambda < 1$ μH., $M_1 = 370$ μH.; balance was obtained when $M = 0{\cdot}177$ μH., hence $l = 0{\cdot}063$ μH.

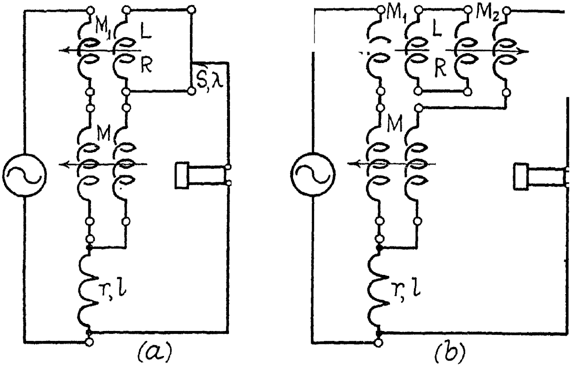

FIG. 156.—CAMPBELL'S METHODS FOR FOUR-TERMINAL RESISTANCES

A more widely applicable method is shown in Fig. 156 (*b*), where M_1 and M_2 are adjustable mutual inductances, the value of which need not necessarily be known. The secondary of the former is joined to the primary of the latter to form a loop of inductance L and resistance R. The coils M_1, M_2 should be at some distance from each other and from the low reading inductometer M, the only direct mutual inductance between the alternator and the detector circuits being *via* the latter. Balance is obtained by adjustment of M and M_1 or M_2, when

$$(M - l)/r = L/R$$

and $$Rr = [M_1 M_2 - (M - l)L]\omega^2.$$

From these,

$$l = M - (Lr/R)$$

so that, if r is known, l is found without the need to know M_1 and M_2.

In carrying out the test, M must be positive and greater than l and also $M_1 M_2 > L^2 r/R$.

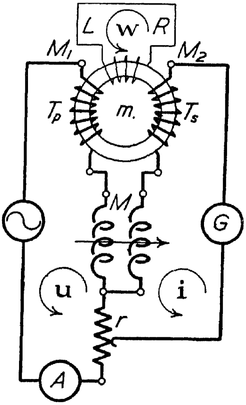

Fig. 157.—Campbell's Method for Measuring Iron Losses

With a slight modification, the network of Fig. 156 (*b*) can be used for the measurement of iron losses in a laminated ring specimen at low field strengths, Campbell's[*] arrangement being shown in Fig. 157. In this diagram M is a variable mutual inductor and r an inductionless resistor with a sliding contact. The iron ring has superposed uniformly wound primary and secondary coils with T_p and T_s turns respectively, m being the mutual inductance between them. The iron losses are fictitiously represented by assuming the core to be free from loss and to have a uniformly distributed tertiary coil upon it forming a closed loop of resistance R and inductance L. The loss in the resistance is equivalent to that in the core, and the reaction of the tertiary upon the flux to that of the eddy currents in the plates. Balance is secured by adjustment of M and r, and very high sensitiveness is obtainable, even on quite small specimens.

Remembering that T_p and T_s are superposed and assuming the windings of M to be parallel coils, it is easily seen that when the detector current is zero the mesh equations are

$$-j\omega M_2 \mathbf{w} = [r + j\omega(m - M)]\mathbf{u},$$

and $$(R + j\omega L)\mathbf{w} = j\omega M_1 \mathbf{u},$$

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[*] A. Campbell, "On the magnetic properties of silicon iron (Stalloy) in alternating magnetic fields of low value," *Proc. Phys. Soc.*, Vol. 32, pp. 232–242 (1920). Also *see idem.*, Vol. 22, pp. 214–217 (1910).

where M_1 and M_2 are the mutual inductances between the tertiary and the primary and secondary respectively; note that $M_1/M_2 = T_p/T_s$. From these equations the balance conditions are

$$(m-M)/r = -L/R,$$
$$Rr = \omega^2(M_1M_2+L(m-M)]=\omega^2[M_1M_2-(L^2r/R)]$$

The second mesh equation gives

$$\mathbf{w} = j\omega M_1\mathbf{u}/(R+j\omega L)$$

so that
$$I_w = \omega M_1 I_u/\sqrt{(R^2+\omega^2L^2)},$$

where I_w and I_u are the r.m.s. values of $\mathbf{w}$ and $\mathbf{u}$. The iron losses are

$$I_w{}^2R = \omega^2M_1{}^2RI_u{}^2/(R^2+\omega^2L^2).$$

From the second balance condition

$$r(R^2+\omega^2L^2) = \omega^2M_1M_2R,$$

so that
$$\text{Iron loss} = T_p r I_u{}^2/T_s \text{ watts.}$$

When the eddy current reaction is small L/R is negligible and

$$M \doteqdot m = 4\pi 10^{-9}T_pT_s\,\mu A_i/l_i \text{ henrys}$$

where A_i is the iron section and l_i the mean perimeter of the ring, whence the permeability μ can be found, corresponding with known magnetizing forces. It is necessary to supply a sine current—by tuning or otherwise—and to exclude current harmonics from the detector by means of a choking coil or condenser.

39. Campbell's Long Range Frequency Meter. This important instrument,* made by the Cambridge Instrument Co., is a further application of the Mz pair, as shown in Fig. 158. In this circuit, M_1, M_2 are two mutual inductances, r is a fixed resistance. The secondaries of M_1 and M_2 form a closed loop of resistance R and self-inductance L, a portion s being tapped off as shown. The current circulating in the loop will be $-j\omega M_1\mathbf{u}/(R+j\omega L)$ which will set up in the detector circuit an e.m.f. $-j\omega M_2\,j\omega M_1\mathbf{u}/(R+j\omega L)$; this, together with the drop in s, must equal the drop in r if the telephone is to be silent. Thus

$$(s-j\omega M_2)j\omega M_1\mathbf{u}/(R+j\omega L) = r\mathbf{u}$$

or
$$j\omega M_1(s-j\omega M_2) = r(R+j\omega L).$$

---

* A Campbell, "On the determination of resistance in terms of mutual inductance," *Proc. Roy. Soc. A.*, Vol. 107, pp. 310–312 (1925). "A direct-reading frequency meter of long range," *Proc. Phys. Soc.*, Vol. 38, pp. 97–100 (1925); "A new frequency meter of long range," *Journal Sci. Insts.*, Vol. 2, pp. 381–384 (1925).

Separating components, balance occurs if

$$sM_1 = rL$$
$$\omega^2 M_1 M_2 = rR$$

If M_1, r and L are fixed, then s can be chosen to satisfy the first condition; balance may then be obtained at any frequency by adjustment of M_2 only, giving $\omega^2 = Rr/M_1M_2$

$$\text{or } f = a/\sqrt{M_2} \text{ where } a = \sqrt{Rr}/2\pi\sqrt{M_1}.$$

By changing R alone, or by altering r and s in the same ratio, any desired value can be given to the multiplier a and a number of ranges easily provided. A further great advantage is that the frequency is measured by a single adjustment, that of M_2, which is perfectly smooth and continuous.

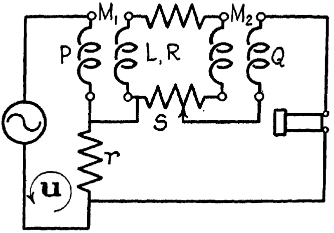

FIG. 158.—CAMPBELL'S FREQUENCY METER

The inductometer M_2 consists of a pair of flat bobbins, mounted horizontally, to carry the primary turns, the secondary being carried by a third flat bobbin moving between the other two, as in the inductometer in Fig. 58, page 148. It is assumed in the theory that there is no direct mutual inductance between P and Q; hence the bobbins carrying the windings of M_1 are mounted in the vertical plane, but are provided with a small amount of tilting motion for the following purpose. Suppose r to have residual inductance l and that m represents the direct mutual induction of P or Q; then balance occurs when

$$sM_1 = rL + R(l - m)$$
$$\omega^2[M_1M_2 - L(l - m)] = rR$$

Thus, whether l is negligible or not, it can be cancelled by an equal small value of m, which can be produced by tilting the bobbin of M_1 slightly out of the vertical. This adjustment may further be employed to compensate for the effect of stray fields due to leads or external circuits, so that sharp balance can always be secured.

Five ranges are usually provided, with multipliers of 0·3, 0·5, 1, 2, and 3, with a total range of 180 to 4,000 cycles per second; lower ranges down to 18 cycles per second can easily be included. An actual accuracy of 1 in 1,000 is obtainable, and temperature errors are negligible except perhaps at the lowest ranges, where R is largely composed of copper; even then the temperature correction is small. The power consumption is slight.

When the network is used for the determination of the ohm in absolute measure the following procedure is adopted: M_1, R, r, and L are fixed, so that the first balance condition is satisfied by adjustment of s alone. For a given frequency the second condition is satisfied by adjustment of M_2. The two mutual inductances are compared with the calculable standard (Fig. 53 (*d*)) and are thus expressed in terms of length measurements. The frequency is set by comparison with a tuning fork, itself checked by a phonic wheel method. The product rR is thus expressed in terms of (length/time)^2, and the resistors can be found in terms of the International ohm by d.c. bridge measurement. Using home-made apparatus, Campbell (1925) found the ratio True ohm/Int. ohm to be $1{\cdot}00054 \pm 0{\cdot}0001$, in close agreement with the accepted figure. In his test, $M_1 = 14$ mH., $M_2 = 10$ mH., $R = 30$ ohms, $r = 2$ ohms, $L = 3{\cdot}5$ mH., $S = 0{\cdot}5$ ohm; $f = 100$ cycles per second. The current was about 30 mA. taken from a triode generator and the detector a vibration galvanometer. The method has been adopted at the National Physical Laboratory (*Report for* 1931, p. 112) with considerable refinements, and has given excellent results; full details are given in the reference cited on p. 518.

40. Churcher and Dannatt's Method for Measuring the Losses in Large Oil Switches. The measurement of the losses in large oil switches and similar apparatus is one of considerable practical importance. The high current, low volt-drop and low power-factor render it very difficult to obtain adequate accuracy by wattmeter methods unless a reflecting wattmeter can be used. This is usually impracticable, since the tests are to be done on the works test-bed; Churcher and Dannatt* have shown that a bridge method utilizing portable apparatus is ideal for this purpose, the connections being shown in Fig. 159. In this diagram r is a slide-wire of 1 ohm total resistance; M is an astatic mutual inductor of about 5 millihenrys range, one winding being able to carry about 2·5 ampere; CT is a calibrated current transformer, the ratio K_c and phase angle β being determined by a precision method with the test circuit as a burden. The potential difference at the terminals of the switch is balanced against the secondary e.m.f. of the mutual inductor and the drop in a portion of the slide-wire; bifilar potential leads are taken from the switch to the bridge and balance is secured by varying M and r, using a shunted portable vibration galvanometer. The shunt blunts the resonance peak, which is an advantage when possible considerable variations in frequency, with consequent loss of sensitivity in a sharply tuned instrument, may occur ; adequate sensitivity is secured even when shunted to 1/15 of normal.

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* B. G. Churcher and C. Dannatt, "The measurement of the energy loss in large oil switches," *World Power*, Vol. 4, pp. 314–319 (1925). B. G. Churcher, *Elecn.*, Vol. 101, pp. 518–520, 545–547 (1928).

Let L and R be the effective inductance and resistance of the complete switch ; then, as is obvious from the vector diagram, balance occurs if

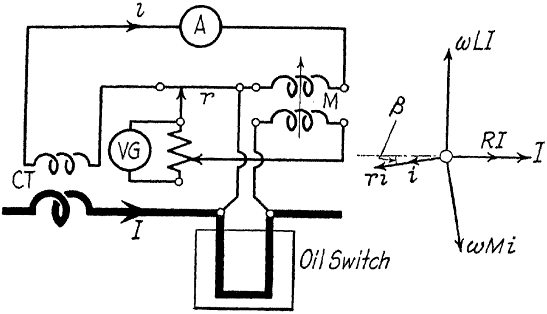

FIG. 159.—CHURCHER AND DANNATT'S METHOD FOR MEASURING LOSSES IN LARGE OIL SWITCHES

$$\omega LI - (\omega Mi\cos\beta + ri\sin\beta) = 0$$
$$RI - (ri\cos\beta - \omega Mi\sin\beta) = 0$$

Remembering that $I = K_c i$, these relations give

$$L = \frac{1}{K_c}\left(M\cos\beta + \frac{r}{\omega}\sin\beta\right)$$
$$R = \frac{1}{K_c}(r\cos\beta - \omega M\sin\beta)$$

The power lost in the switch is, therefore,

$$I^2R = (K_c i)^2R = i^2K_c(r\cos\beta - \omega M\sin\beta) \doteqdot i^2K_c r$$

and the impedance drop is

$$IZ = I(R^2 + \omega^2L^2)^{\frac{1}{2}} = i(r^2 + \omega^2M^2)^{\frac{1}{2}}.$$

The current i may be read on the ammeter shown and does not exceed 3 amperes. The original paper gives the result of a test on a 5,000 ampere, 11,000 volt, 50 cycle switch; ωM was about 5 times r, so that the power-factor is about 0·2.

Possible sources of error and their cure are: (i) Impurity in M, which is negligible at low frequencies. (ii) Stray field errors, which may be avoided by astatic design of M and judicious lay-out of the apparatus. (iii) Residual inductance in r, usually quite negligible in a low resistance slide wire. (iv) Errors due to capacitive coupling between the primary and secondary of CT, and errors due to wave-form are also negligible.

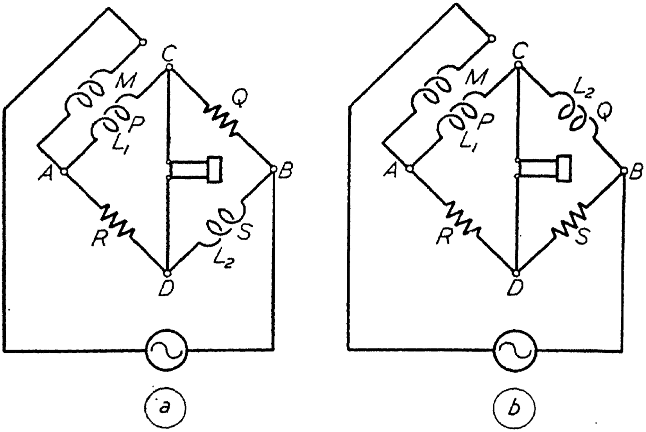

FIG. 160.—BUTTERWORTH'S AND CAMPBELL'S FREQUENCY BRIDGES

41. Butterworth's Method for Frequency Measurement. Butterworth* has suggested a modification of Maxwell's method (Fig. 150), which is useful for the measurement of frequency and also as a wave-filter. The circuit is shown in Fig. 160 (*a*). Writing $z_1 = P + j\omega L_1$, $z_2 = Q$, $z_3 = S + j\omega L_2$, $z_4 = R$, and $m_{16} = j\omega M$ in the appropriate equation on p. 76 gives for the balance condition

$$-\omega^2 L_2(M + L_1) = QR - SP$$

and

$$L_1 S + L_2 P = -M(S + Q).$$

Hence M must be negative, giving the numerical relationships,

$$\omega^2 L_2(M - L_1) = QR - SP,$$

and

$$L_1 S + L_2 P = M(S + Q).$$

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* S. Butterworth, *Proc. Phys. Soc.*, Vol. 24, p. 86 (1912).

In practice, it is convenient to make $L_1 = L_2$ and to fix M, P, Q, and S. The second condition is then a constant and the first can be satisfied by adjustments of R alone; the range of frequency is infinite, and M should be greater than L_1.

Experimental Example. Butterworth's method was used to calibrate a triode oscillator used for a.c. bridge measurements and illustrated in Fig. 82, page 220. The frequency could be varied by changing the anode condenser. In the bridge, M was an 11 mH. Campbell mutual inductometer, the fixed winding of which had an inductance L_1 of 958_2 μH. With $P = 12$ ohms, $Q = 9$ ohms, $S = 10{\cdot}6_9$ ohms, $L_2 = 955_0$ μH., balance was secured by adjustment of R and small changes of M, with the following results (a Duddell vibration galvanometer was used)—

Anode Condenser μF.	R ohms.	M μH.	Frequency cycles/second.
0·3	$29{\cdot}4_5$	1087_8	529·1
0·4	$25{\cdot}8_5$	1088_7	459·6
0·5	$23{\cdot}3_9$	1089_7	407·3

Dunand* has recently re-introduced Butterworth's frequency bridge with an important simplification. Fixing $L_1 = 2M = 2L_2$ and making $R = 0$ take $P = S$; then the balance conditions become simply $\omega L_2 = S$ and $Q = 2S$. Balance is secured by simultaneous variation of P, S and Q, a single switch maintaining $P = S = Q/2$. Again, by choosing $L_2 = 10^{-n}$ henrys, where n is an integer ω becomes direct reading; the bridge is, therefore, a linear pulsatance bridge.

Another modification, due to Campbell, is shown in Fig. 160 (*b*). Putting $z_1 = P + j\omega L_1$, $z_2 = Q + j\omega L_2$, $z_3 = S$, $z_4 = R$, $m_{16} = j\omega M$ on p. 76 gives for balance

$$\omega^2 ML_2 = SP - QR$$
$$RL_2 - SL_1 = M(Q + S)$$

In practice it is best to make $L_1 = L_2$ and to fix M, Q, R, and S. This makes the second condition constant, and the first is satisfied by adjustment of P only, no matter what the frequency may be.

42. The Hughes Balance.

This bridge is of considerable historical interest, since it was employed by Professor Hughes† in some of the earliest measurements of eddy-current effects in thin and thick wires of different materials. He, however, interpreted his results by a totally incorrect theory of the bridge, his astonishing conclusions leading the late Lord Rayleigh, Oliver Heaviside, and others to investigate the problem and to deduce the correct balance conditions for the network.*

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* Dunand, "Procédés de mesure dans la téléphonie à longue distance et le fréquencemètre," *Bull. Soc, Franç. des Elecns.*, Vol. 7, 4th series, pp. 202–208 (1928).

† D. E. Hughes, "Induction balance and experimental researches therewith," *Proc. Phys. Soc.*, Vol. 3, pp. 81–89 (1880). "The self-induction of an electric current in relation to the nature and form of its conductors," *Journal, S.T.E.*, Vol. 15, pp. 6–25 (1886).

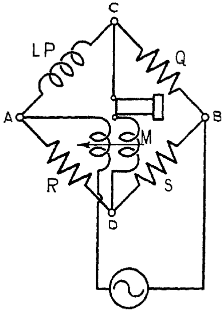

FIG. 161.—THE HUGHES BALANCE

In the original arrangement, interrupted current and a telephone were employed, H. Rowland,† in 1898, applying alternating current and an electrodynamometer to the method. A. Campbell,‡ in 1907, replaced the latter instrument by the superior vibration galvanometer. The network is shown in Fig. 161, from which it is seen that mutual inductance between the detector and source branches is compared with a self-inductance and resistances. To find the balance conditions, on page 71 put $z_1 = P + j\omega L$, $z_2 = Q$, $z_3 = S$, $z_4 = R$, and all mutual operators equal to zero except $m_{56} = j\omega M$. Then, $\alpha = -j\omega M$. $\beta = 0$, $\gamma = j\omega M$, $\delta = 0$, so that

$$S(P + j\omega L) - QR - j\omega M(P + R + j\omega L) - j\omega M(Q + S) = 0$$

Separation of the components gives

$$\omega^2 ML = QR - SP$$

$$SL = M(P + Q + R + S) \text{ for balance.}$$

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* O. J. Lodge, "On intermittent currents and the theory of the induction balance," *Proc. Phys. Soc.*, Vol. 3, pp. 187–212 (1880). Lord Rayleigh, *Journal, S.T.E.*, Vol. 15, pp. 28–40, 54–55 (1886); also "Notes on Electricity and Magnetism; II. The self-induction and resistance of compound conductors," *Phil. Mag.*, 5th series, Vol. 22, pp. 469–500 (1886). O. Heaviside, "On the use of the bridge as an induction balance," *Elecn.*, Vol. 16, pp. 489-491 (1886); *Electrical Papers*, Vol. 2, pp. 33–38 (1892). The student interested in the historical development of electrical science will find the perusal of this classic dispute of considerable interest.

† H. Rowland, "Electrical measurement by alternating currents," *Phil. Mag.*, 5th series, Vol. 45, pp. 66–85 (1898). *See* also L. Graetz, "Eine neue Methode zur Messung von selbstpotential und Inductionscoefficienten," *Ann. der Phys.*, Vol. 50, pp. 766–771 (1893).

‡ A. Campbell, "On the measurement of mutual inductance by the aid of a vibration galvanometer," *Proc. Phys. Soc.*, Vol. 20, pp. 626–638 (1907).

A. Campbell has shown that the method is useful as a means of measuring frequency.* For this purpose M is a variable mutual inductance; the resistance S is a portion of a slide wire, B being the contact thereon. The resistance Q consists of the remainder of the slide wire and a non-inductive resistance box; by this means $Q + S$ can be kept constant although Q/S is varied. The resistance R is a fixed non-inductive resistance. The resistance of P will usually require a correction for temperature, since it is wholly or partly composed of a copper coil; this correction is more important at low frequencies. Two different methods can be adopted in practice—

If P *be constant and known,* then $P + Q + R + S = a$ is constant, since $Q + S = b$ is constant. If L be fixed and balance is secured by varying M and the position of the slider B, eliminate M from the above equations. Then

$$\omega^2 = \frac{a}{L^2}\left[\frac{R(b-S)}{S} - P\right],$$

so that the frequency can be found in terms of a single variable S. The slide wire can thus be marked directly in frequency values. The range of the bridge can be extended to n times the frequency, either by reducing L to L/n, or by changing P and Q so that $(P + R)/(Q + S) = 1/n$.

If P *be not exactly known,* it will be necessary either to apply a correction to the above formula to allow for the change of P with temperature; or it may be simpler, if the standard inductometer M is accurately known, to eliminate P from the equations. The resulting equation for ω^2 is then

$$\omega^4 = \frac{(R + S)\ (Q + S)}{ML} - \frac{S^2}{M^2}$$

Campbell† gives the following values as suitable for a range of frequency from 10 up to 120 cycles per second; $L = 0{\cdot}1$ henry, $P = 25$ ohms; $R = 5$ ohms; $Q + S = 4$ ohms; M from 1·7 to 0·28 millihenry, and S from 0·6 to 0·1 ohm.

Experimental Example. The Hughes balance was used in a frequency calibration of the triode oscillator tested previously by Butterworth's method (p. 451). The branch AC contained a coil for which $L = 10$ mH. and $P = 25$ ohms. $Q + S$ was a slide wire of $4{\cdot}08_1$ ohms resistance, and $R = 5$ ohms. Using the first method, $b = 4{\cdot}08_1$ ohms and $a = 34{\cdot}08_1$ ohms. M was a 1·1 mH. Campbell inductometer, balance being found by adjustment of M and S. For the two highest frequencies the detector was a 150 ohm telephone ; for the remaining observations a Duddell vibration galvanometer was employed.

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* C. E. Hay has also recommended the method for the measurement of inductance and effective resistance of telephonic loading coils; see *Journal P.O.E.E.*, Vol. 5, p. 451 (1912–13).

† *Dictionary of Applied Physics*, Vol. 2, p. 434.

Anode Condenser μF.	S ohms.	M μH.	Frequency cycles/second.
0·1	$0{\cdot}16_0$	$46{\cdot}_9$	929·6
0·2	$0{\cdot}26_0$	$76{\cdot}_3$	647·5
0·3	$0{\cdot}32_7$	$96{\cdot}_0$	529·4
0·4	$0{\cdot}37_5$	$110{\cdot}_0$	459·2
0·5	$0{\cdot}41_5$	$121{\cdot}_7$	406·9

Déguisne* has shown that the Hughes balance is a convenient method for measuring self-inductance. Taking $Q = S$ the balance conditions become $SL = M(P + R + 2S)$ and $\omega^2 ML = S(R - P)$, from the first of which L can be calculated if P be known. To find L and P the following process is suggested. First let $S^2 = 2\omega^2 M^2$; then it is easy to show that

$$L = 4M(R + S)/3S \text{ and } P = (R - 2S)/3 \quad .$$

Alternatively, if $S^2 = 3\omega^2 M^2$, then

$$L = 3M(R + S)/2S \text{ and } P = (R - S)/2$$

In either case M is set to the desired value and balances effected by variation of R.

43. Kurokawa's Frequency Bridge. Kurokawa† has described the bridge shown in Fig. 162 for the measurement of frequency over a wide range. The branches AC, BD contain the primary and secondary windings respectively of a fixed mutual inductance, each winding being in series with an adjustable resistance ; the branch CB is made of zero resistance, while AD contains a variable resistance R. Retaining Q for the present, in the general expressions on page 71, put $z_1 = P + j\omega L_1$, $z_2 = Q$, $z_3 = S + j\omega L_2$, $z_4 = R$, $m_{13} = j\omega M$; then $\alpha = 0$, $\beta = j\omega M$, $\gamma = j\omega M$, $\delta = j\omega M$, and

$$(P + j\omega L_1)(S + j\omega L_2) - QR - j\omega M(Q + S + j\omega L_2 + j\omega M) \\ + j\omega M(S + j\omega L_2) - j\omega MR = 0$$

* C. Déguisne, "Brückenmessungen mit dem 'Phasenschlitten,'" *Arch. Elekt.*, Vol. 14, pp. 487–490 (1925).

† K. Kurokawa, "A new frequency bridge," *Journal Elect. Soc. Waseda*, Vol. 8, pp. 251–253 (1927).

Separating components,

$$SP - QR = \omega^2(L_1L_2 - M^2)$$

$$M(Q + R) = SL_1 + PL_2$$

are the balance conditions. Now make $Q = 0$ and $P = S$, then

$$S^2 = \omega^2(L_1L_2 - M^2)$$

$$MR = S(L_1 + L_2)$$

or $\omega = S/(L_1L_2 - M^2)^{\frac{1}{2}}$, and $R = S(L_1 + L_2)/M$

In the apparatus designed by Kurokawa $L_1 = L_2 = 16{\cdot}2435$ mH., $M = 0{\cdot}2L_1 = 3{\cdot}2487$ mH.; then $R = 10S$ and $f = 10S = R$. Hence the frequency is given directly by the reading of R. A single handle simultaneously adjusts P, S and R, so that $P = S = R/10$ is always maintained. A range of frequency from a few cycles per second up to 5,000 or more can be obtained, the upper limit being fixed by self-capacitance in the inductances. The bridge is very useful and is free from any serious source of error.

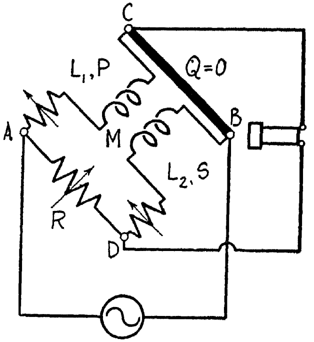

FIG. 162.—KUROKAWA'S FREQUENCY BRIDGE

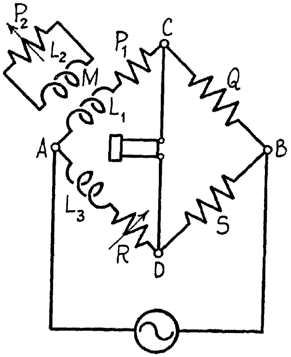

FIG. 163.—SASE AND MUTÔ'S FREQUENCY BRIDGE

44. Sase and Mutô's Frequency Bridge. A further type of direct-reading frequency bridge has been described by Sase and Mutô,* and is shown in Fig. 163. This is really a modified form of the Maxwell bridge for comparison of two self-inductances, the modification consisting in coupling to the coil L_1P_1 a closed loop L_2P_2 by mutual inductance M. Now Maxwell has shown that the effect of a closed secondary circuit is to increase the apparent resistance and to reduce the apparent inductance of the primary. Hence the effective operator of the branch AC is $P + j\omega L$ where

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* M. Sase and C. Mutô, "On the new frequency meter," *Journal Elec. Soc. Waseda*, Vol. 8, pp. 179–196 (1927). I have to thank these writers for their kindness in providing an English translation of their Japanese paper. The meter is made by the Yokokawa Electric Meter Works, of Tokyo.

$$P = P_1 + \frac{\omega^2 M^2 P_2}{P_2^2 + \omega^2 L_2^2} \text{ and } L = L_1 - \frac{\omega^2 M^2 L_2}{P_2^2 + \omega^2 L_2^2}$$

balance will be secured, therefore, if

$$L/L_3 = P/R = Q/S$$

In this particular bridge $Q = S$ and the construction is arranged so that $L_3 = L_1 - \frac{1}{2}L_2$, $R = P_1 + \frac{1}{2}P_2$ and $L_2 = M$. It is then easy to show that the two balance conditions reduce to one, namely, $\omega^2 = P_2^2/L_2^2$ or $f = P_2/2\pi L_2$. The actual instrument has $Q = S = 50$ ohms, $L_2 = M = 500/2\pi = 77{\cdot}79$ mH., $L_1 = 90$ mH, $L_3 = 51{\cdot}10$ mH., and $P_1 = 25$ ohms; with these values $f = 2P_2$. The relation $R = P_1 + \frac{1}{2}P_2 = 25 + \frac{1}{2}P_2$ is arranged by a set of five Feussner type resistance dial switches giving respectively 4 × 1,000, 9 × 100, 9 × 10, 9 × 1 and 9 × 0·1 cycles per second. The instrument is direct reading and has but one adjustment; it consumes little power; and it has a range from about 30 to 5,000 cycles per second with a precision of measurement to 0·1 cycle.

Pegler* uses the same principle applied to the Owen bridge of Fig. 140 (*b*), p. 404. The branch AC contains the coil of inductance L_1 and resistance P_1 in series with a variable resistor p; the coil is coupled by mutual inductance M to the closed loop L_2, P_2 as in Fig. 163. The remaining branches are exactly as in Fig. 140 (*b*), the condensers C_1, C_2 being fixed, R also fixed, and Q variable. If the operator for the coil in the bridge is $P' + j\omega L'$, from above

$$P' - P_1 = \omega^2 M^2 P_2/(P_2^2 + \omega^2 L_2^2) \text{ and } L_1 - L' = \omega^2 M^2 L_2/(P_2^2 + \omega^2 L_2^2).$$

It is easy to show that P' is a maximum for $P_2 = \omega L_2$ when

$$(P' - P_1)_{max} = \omega M^2/2L_2 = \text{constant} \times \text{frequency}.$$
$$(L_1 - L')_{max} = M^2/2L_2 = \text{a constant}.$$

The balance conditions of the bridge are (p. 405)

$$L_1 = C_2 R Q \text{ and } C_1(P_1 + p) = C_2 R$$

when the loop is opened, and

$$L' = C_2 R Q' \text{ and } C_1(P' + p') = C_2 R$$

when it is closed. Subtracting,

$$L_1 - L' = C_2 R(Q - Q') \text{ and } P' - P_1 = -(p' - p)$$

If P_2 is set until $P' - P_1$ is a maximum, then $p' - p$ is a maximum also, and is proportional to the frequency. At the same time the corresponding $L_1 - L'$ is constant, i.e. $Q - Q'$ is constant independently of the frequency. In Pegler's bridge $R = 500$ ohms, $C_1 = C_2 = 0{\cdot}2$ μF., the two coils have about 500 turns with mean diameter of 20 cm., p and Q are adjustable in 0·1 ohm steps and P_2 to the nearest ohm. A Wagner earthing device is easily applied.

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* G. D. Pegler, "A bridge for the determination of the frequency of an alternating current in the audio frequency range," *Proc. Phys. Soc.*, Vol. 46, pp. 783–789 (1934).

NETWORKS CONTAINING RESISTANCE, SELF-INDUCTANCE, MUTUAL INDUCTANCE, AND CAPACITANCE

45. Modified Carey Foster's Method. This method is adapted from a ballistic bridge, introduced by G. Carey Foster,* in 1887, for the comparison of a condenser with a mutual inductance. The ballistic bridge in its original form has a quantity or aggregate balance, but by a simple modification, continuous balance is obtained and the bridge can be used with alternating current. This modification, originally introduced by A. Heydweiller,† in 1894, consists in the inclusion of a resistance S in series with the condenser C, as shown in Fig. 164 (*a*).

Assuming first that the resistance of the branch AC is not zero, substitute on page 71 the operators $z_1 = P$, $z_2 = Q$, $z_3 = S - \frac{j}{\omega C}$, $z_4 = R + j\omega L$, and $m_{46} = j\omega M$; R is the total resistance of the branch AD. Then $\alpha = 0$, $\beta = 0$, $\gamma = j\omega M$, $\delta = 0$, so that

$$P\left(S - \frac{j}{\omega C}\right) - Q(R + j\omega L) - j\omega M\left(Q + S - \frac{j}{\omega C}\right) = 0.$$

Separating the components, gives

$$-M = C(QR - SP)$$

$$L = -M\left(1 + \frac{S}{Q}\right) - \frac{P}{\omega^2 CQ}$$

when balance is secured. By the simple expedient of making $P = 0$, the balance can be made independent of frequency, as shown by Heydweiller. Putting $P = 0$, the balance conditions reduce to

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* G. Carey Foster, "Note on a method of determining coefficients of mutual induction," *Proc. Phys. Soc.*, Vol. 8, pp. 137–146 (1887). Also *see* A. Ròiti, "Misure assolute di alcuni condensatori," *Mem. Acc. Tor.*, 2nd series, Vol. 38, pp. 57–77 (1888).

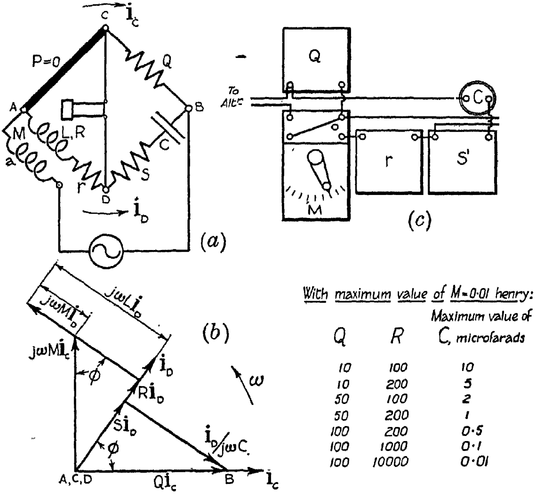
† A. Heydweiller, "Ueber die Bestimmung von Inductionscoefficienten mit dem Telephon," *Ann. der Phys.*, Vol. 53, pp. 499–504 (1894). The modification has also been independently introduced by H. Rowland, "Electrical measurement by alternating currents," *Phil. Mag.*, 5th series, Vol. 45, pp. 66–85 (1898), using an electrodynamometer; A. Campbell, "On the measurement of mutual inductance by the aid of a vibration galvanometer," *Proc. Phys. Soc.*, Vol. 20, pp. 626–638 (1907), and also "Inductance measurements," *Elecn.*, Vol. 60, pp. 626–627 (1908).

$$-M = CQR,$$

$$L = -M\left(1 + \frac{S}{Q}\right).$$

From the first condition M must be negative to secure balance, i.e. M must oppose L; then numerically

$$M = CQR \text{ and } L = M\left(1 + \frac{S}{Q}\right).$$



With maximum value of M=0·01 henry:

Q	R	Maximum value of C, microfarads
10	100	10
10	200	5
50	100	2
50	200	1
100	200	0·5
100	1000	0·1
100	10000	0·01

FIG. 164.—MODIFIED CAREY FOSTER'S METHOD FOR COMPARING CAPACITANCE WITH MUTUAL INDUCTANCE

Fig. 164 (*b*) shows the vector diagram for the bridge. The vector AB is the applied potential difference across the points A and B; and, since $P = 0$, AB must also equal the drop of potential $Q\mathbf{i}_C$ in the branch CB. Since C and D are at the same potential, the sum of the potential drops $S\mathbf{i}_D$ and $\mathbf{i}_D/j\omega C$ must balance $Q\mathbf{i}_C$. Since the drop of potential down the branch AC is zero, the total e.m.f. of mutual induction in AD, namely, $j\omega M(\mathbf{i}_C + \mathbf{i}_D)$, must exactly balance the impedance drop $(R + j\omega L)\mathbf{i}_D$. From the geometry of the two similar triangles the above equations may be verified at once.

In practical working, the second balance condition implies that L be greater than M. Hence, if necessary, an auxiliary self-inductance should be included in the branch AD until $L \geqq M$. The resistance Q, being connected across the source—neglecting the impedance drop in the coil a—should be capable of carrying a fair current. It is preferably an oil-cooled standard, (so that temperature changes can be allowed for), of low residual inductance.

When the method is used to measure a mutual inductance in terms of a standard condenser, Q is fixed and L arranged to be equal to or greater than M. Balance is secured by successive adjustment of R or C and S.

Chatterjee* has used the Carey Foster bridge to find the coupling coefficient of a mutual inductor or other air-cored transformer. In Fig. 165, M, Q, and C are fixed, r and S the variable adjustments. The inductance l is added, when L_1 and L_2 are each less than M, to make the total inductance of AD greater than M. R is the total resistance of AD as before. Joining 1 to 2 and 3 to 4 puts L_1 into the bridge, balance being given by S_1 and r_1; then

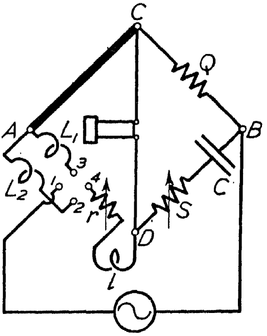


FIG. 165.—CHATTERJEE'S METHOD FOR THE COUPLING COEFFICIENT OF TWO COILS

$$M = CQR \text{ and } L_1 + l = M\left(1 + \frac{S_1}{Q}\right)$$

Joining 1 to 3 and 2 to 4 puts L_2 into the bridge and balance is restored by settings S_2 and r_2. Since M, C, and Q are constant R is unchanged; then

$$M = CQR \text{ and } L_2 + l = M\left(1 + \frac{S_2}{Q}\right)$$

From these four relations it is easy to show that

$$k = \frac{M}{\sqrt{(L_1 L_2)}} = \frac{Q}{\sqrt{\left[(Q + S_1)(Q + S_2) - \frac{l}{CR}\left(S_1 + S_2 + 2Q - \frac{l}{CR}\right)\right]}}$$

If L_1 and L_2 are each $> M$, l may be omitted and

$$k = Q/\sqrt{[(Q + S_1)(Q + S_2)]}$$

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* L. M. Chatterjee, "Über eine Brückenmethode zur Bestimmung des Koppelungskoeffizienten eines Transformators mit Luftkern," *Zeits. f. Phys.*, Vol. 89, pp. 601–604 (1934).

A. Campbell has shown (*loc. cit.*) that Carey Foster's method is one of the best for the measurement of the capacitance and equivalent series resistance of a condenser, using a standard mutual inductometer for M. Fig. 164 (c) shows a convenient arrangement of apparatus for this test. The resistance Q is set at a definite fixed value. The branch AD contains the fixed winding of the inductometer, a resistance box r and, if necessary, an auxiliary inductance to ensure that the inductance L of the branch exceeds M. The value of r is chosen so that QR is a convenient multiplier, e.g. some power of 10, R being the total resistance of AD. A convenient set of values for Q and R is shown in the diagram. The branch DB contains the condenser, the capacitance C and series resistance ρ of which are to be found, together with an adjustable resistance box S'. Balance is secured by successive adjustment of S' and M; then, since $S = S' + \rho$, the above balance condition gives

$$C = M/QR \text{ and } \rho = Q\left(\frac{L}{M} - 1\right) - S'.$$

In accurate work, the capacitance of the leads to C should be allowed for by repeating the test with the leads disconnected from the terminals of the condenser, and deducting the resulting capacitance from the first balance.

The values of Q and R should be measured after the test, and L should be accurately known. S' should not be too large, i.e. if L is much greater than M, a smaller value of Q should be chosen. A Campbell inductometer reading up to 10 millihenrys is convenient for M, and gives a range from a small fraction of a microfarad up to 10 microfarads.

Experimental Example. Carey Foster's method was used to test a paper condenser at 407·1 cycles per second, the detector being a Duddell vibration galvanometer. M was an 11 mH. Campbell inductometer, the following values being found at balance. $Q = 100$ ohms, $R = 200$ ohms, $L = 1658_2\ \mu$H., $S' = 100{\cdot}4_4$ ohms, $M = 8008{\cdot}_0\ \mu$H. With the leads removed from C, $M_0 = 0{\cdot}_4\ \mu$H., and $C = 8007{\cdot}_6/20{,}000$ $= 0{\cdot}4003_8\ \mu$F., and $\rho = 100\left(\frac{1658_2}{800_8} - 1\right) - 100{\cdot}4_4 = 5{\cdot}20$ ohms.

In a second experiment on a good mica condenser, $Q = 100$ ohms, $R = 200$ ohms, $L = 1922_2\ \mu$H., $S' = 186{\cdot}9_5$ ohms, $M = 6693{\cdot}0\ \mu$H., and $M_0 = 1 \mu$H. Thus, $C = 6692{\cdot}_0/20{,}000 = 0{\cdot}3346_0\ \mu$F and also

$$100\left(\frac{1922_2}{669_2} - 1\right) - 186{\cdot}9_5 = 0{\cdot}2_9 \text{ ohm.}$$

46. Experimental Troubles in the Modified Carey Foster Method. In practical work, especially in measurements of high precision on small condensers, or on condensers with low power-factors, certain important sources of error enter into the method.

(i) *Residual Errors and Impurity Effects.* At high frequencies residual inductance* in Q and S', self capacitance in the windings of the mutual inductance and the effects of impurity become important. Indeed, when measuring small air condensers by this method, the measurements serve rather as a drastic test of the imperfections of the apparatus than as a means of testing the condenser. Butterworth† has shown how these various sources of error may be allowed for, and the reader is advised to consult the original paper for full details of the procedure. The corrections are particularly important in power-factor measurements, especially at the upper audio frequencies.

Since the effect of residual inductance in Q and S' may become important, even at frequencies of moderate value in tests on low power factors, it is useful to see how the balance conditions become altered by the residuals. In the analysis above, p. 457, put $Q + j\omega\lambda$ for Q, and $S + j\omega\mu$ for S then

$$\frac{1}{C} = \frac{QR}{M}\left[1 - \frac{\omega^2}{QR}\ \ \lambda L - (\lambda + \mu)M\Big\}\right],$$

and $S' + \rho = Q\left(\frac{L}{M} - 1\right) + \frac{\lambda R}{M}$,

are the balance conditions when $P = 0$ and M is opposing L. The correction in the value of $1/C$ is, except at very high frequencies, usually small. Hence

$$C \doteq \frac{M}{QR}\left[1 + \frac{\omega^2}{QR}\Big\{\lambda L - (\lambda + \mu)M\Big\}\right],$$

and $$\rho = Q\left(\frac{L}{M} - 1\right) - S' + \frac{\lambda R}{M}.$$

In tests on large condensers the correction to be applied to C is usually negligible, except at the upper telephonic frequencies. The correction in the expression for ρ may become important in accurate tests of the power-factor of good condensers. When small condensers are tested, it is generally necessary to apply corrections for residuals both to the capacitance and to the loss resistance. In measurements at high frequencies made upon good condensers (e.g. a well-made mica standard or an air condenser), ρ calculated from the above formula is often an apparently negative quantity. This absurd result is the consequence of neglecting the effects of impurity in the mutual inductance; Butterworth has shown in the paper cited how this quantity can be corrected for in such cases.

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* *See* A. Campbell, *Proc. Roy. Soc., A*, Vol. 87, pp. 402–406 (1912).

† S. Butterworth, *Proc. Phys. Soc.*, Vol. 33, p. 313, pp. 334–337 (1921).

In the experiments on page 460, if the value of λ be $0{\cdot}5\ \mu$H., the correction in the first case is $0{\cdot}0_{12}$ ohm, or 0·2 per cent, so that $\rho = 5{\cdot}2_1$ ohms. In the second test the correction is $0{\cdot}0_{15}$ ohm, so that ρ for the mica condenser is $0{\cdot}3_{05}$ ohm; this is an important correction, being about 5 per cent.

(ii) *Earth Capacitances.* With small condensers trouble is experienced due to earth capacitances. It is sometimes a good plan to earth the terminal of the inductometer which is joined to the source. At the higher audio frequencies, when telephones are used to detect balance, it becomes very difficult to secure sharp balance, unless it can be ensured that the points C and D at balance are at the potential of the observer. To a first approximation this can be attained by earthing the point D instead of the source terminal of the inductometer. Both these devices only provide rough allowance for earth capacitance effects. To remove such troubles entirely Dr. Dye has shown how to find a proper earth point which is free from this defect. His device is described on page 543.

47. Campbell's Precision Condenser Bridges. The Carey Foster method with Heidweiller's modification is one of the most convenient and accurate methods for the measurement of condenser capacitance and power factor. The reference standards, being mutual inductance and resistance, have high accuracy and permanence. By using a variable mutual inductance the capacitance can be read directly, but unfortunately this is not the case for the power factor. To overcome this defect Campbell* has designed two modifications of the Carey Foster method which are direct reading both for capacitance and power factor.

Referring to Fig. 164 on page 458, the branch DB contains the condenser to be tested, without any added resistance, C being its capacitance and ρ its series loss-resistance. The branch AC, which in the

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* A. Campbell, "Two precision condenser bridges," *Proc. Phys. Soc.*, Vol. 43, pp. 564–568 (1931); *British Patents*, No. 317,642 (22nd Aug., 1929), No. 350,789 (18th June, 1931).

Carey Foster method is left blank, is given a resistance P in Campbell's first modification. The resistances P, Q, and R are variable, and the mutual inductance is fixed. In the balance conditions given on page 457 put $S = \rho$; then it is not difficult to show that, when M is negative,

$$C = M/(QR - P\rho),$$

and

$$\cos\varphi = \omega C\rho = \frac{P}{\omega M} + \frac{\omega(L - M)}{R - (P\rho/Q)}$$

where $\cos\varphi$ is the power-factor of the condenser. The term $P\rho$ can be made very small in comparison with QR; also as a rule L and M are made equal.

With these approximations, which can be made very exact,

$$C \doteqdot M/QR \text{ and } \cos\varphi \doteqdot P/\omega M.$$

The value of M is made an integral power of 10 (e.g. 1 mH.) and Q is a conductance box reading $1/Q$ directly in millimhos, while R can be arranged in steps of 10, 100, 1,000, 10,000 ohms to give various ranges for C. For a given value of ω, $\cos\varphi$ is proportional to P; this can be a slide wire graduated to read $\cos\varphi$ directly for the given frequency. The branch point C is then the sliding contact on the wire, the latter having so low a resistance that the portion in the Q arm can generally be neglected. Balance is secured by varying P and Q after choosing R to give a suitable range. Power-factors outside the range of P can be found by altering L by a known amount, so that $L - M \neq 0$ and calculating by means of the more accurate formula. Instruments with two values of M have been made with a range from $1\mu\mu$F. to 10μF., reading power-factors directly at 50 and 800 cycles per second from 0·0001 up to 0·01.

In the second system, actually adopted in the instrument constructed by the Cambridge Instrument Co., an ordinary Carey Foster-Heydweiller network is employed in combination with an M-correcter loop described on page 161, the purpose of which is artificially to adjust the impurity of the mutual inductor and thereby to balance the loss resistance of the condenser. The circuit is shown in Fig. 166 (a), the loop of resistance r and self-inductance l being coupled to the windings of M by mutual inductances m and μ. Using the result of page 161, or otherwise, it is not difficult to show that the balance conditions are numerically as follows, if M is negative and m of opposite sign to μ.

$$\frac{Ml - m\mu}{CQ} = Rl + (L - M)r - \frac{Mr\rho}{Q}$$

$$\frac{M}{CQR} = 1 - \frac{\omega^2}{Rr}\left[\left(L - M - \frac{M\rho}{Q}\right) l - m(m - \mu) + \frac{m\mu\rho}{Q}\right].$$

Making $M = L$ and r much greater than ωl, these give

$$\cos\varphi \doteqdot \omega C\rho \doteqdot \omega m\mu/Mr,$$

$$M/C \doteqdot \left[R + \frac{\omega^2 m(m - \mu)}{r}\right] Q - \frac{\omega^2 m\mu\rho}{r}.$$

By a suitable choice of m, μ, and ρ, the last term can be made negligible in comparison with QR, giving

$$M/C \doteqdot \left[R + \frac{\omega^2 m(m - \mu)}{r}\right] Q.$$

In the actual instrument m and μ are varied together and are always nearly equal, so that to a high degree of approximation

$$C \doteqdot M/QR.$$

As in the other system, M and R are fixed, and C is obtained directly from a conductance box reading $1/Q$. To make the power-factor scale read directly, the frequency is observed and r set proportional to it, say $\omega = kr$, then

$$\cos \varphi = km\mu/M = \text{constant} \times m\mu.$$

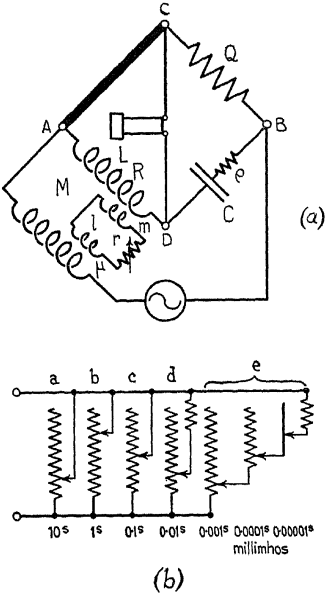

FIG. 166.—CAMPBELL'S PRECISION CONDENSER BRIDGE

The variation of $m\mu$ is carried out by a small inductometer, the fixed coils of which are in series with the primary and secondary of M and the moving coil is in the loop circuit; the scale is marked proportional to $m\mu$ and reads $\cos \varphi$ directly. With $M = 1$ mH. a range of 10μF. down to $100\mu\mu$F. is obtained, though much smaller values can be measured by the method of differences. Over a range of 50 to 2,000 cycles per sec. power-factors from 0·01 down to 0·0001 can be read. At the low frequencies a vibration galvanometer is a suitable detector, the telephone being used at high frequencies. With condensers below $0{\cdot}1\mu$F. an amplifier in the detector circuit is desirable; a simple amplifier and moderate powered source enables measurements of capacitance to be made over the whole range to about 1 in 10,000 and power-factor to 0·0001.

The construction of the conductance box is worthy of notice and is shown in Fig. 166 (*b*). It consists of three-step switch decades, *a*, *b*, *c*, in parallel, the step resistances being

∞, 100, 50, $33\frac{1}{3}$, 25, 20, $16\frac{2}{3}$, $14\frac{2}{7}$, $12\frac{1}{2}$, $11\frac{1}{9}$, 10 ohms,
∞, 1000, 500, $333\frac{1}{3}$, 250, 200, $166\frac{2}{3}$, $142\frac{6}{7}$, 125, $111\frac{1}{9}$, 100 ohms,
∞, 10000, 5000, $3333\frac{1}{3}$, 2500, 2000, $1666\frac{2}{3}$, $1428\frac{4}{7}$, 1250, $1111\frac{1}{9}$, 1000 ohms,

giving ten steps each of 10, 1, and 0·1 millimhos, or a range of 0 to 111 millimhos by steps of 0·1 millimho. To provide continuous adjustment down to zero in this manner would be practically impossible, but in practice a 1 : 100 range is found adequate; even so the arrangement of fine steps on this principle would involve resistances up to megohms, which would be costly and inconvenient. To avoid this the dial *d* gives ten steps of

0·50, 0·51, 0·52, . . . 0·60 millimho (2,000 ohms down to 1666·7 ohms),

while *e* consists of two step-switches and a slide wire all in series giving 0·50000, 0·50001, . . . 0·51000 millimho (2,000 ohms down to 1960·8 ohms). Dials *d* and *e* always keep a minimum value of 1 millimho in circuit, which is allowed for by marking decade *b* 1, 2, 3 . . . millimho instead of 0, 1, 2, 3, etc. In *e* the small conductance variations are not strictly independent, but the approximation is very close.

Bekku, Doté and Nakamura* have published papers dealing with a bridge similar to Campbell's and with related methods for the special purpose of capacitance tests at high voltages. The applications include the determination of the impedance diagram (frequency locus) of testing transformers to show the normal resonating modes; the measurement of inter-line or line-earth transmission constants of long overhead power lines; and the measurement of the mutual impedance between such power lines and communication circuits.

48. Campbell's Methods for Mutual Inductance and Frequency. A. Campbell† has given a method which serves to check a mutual inductance standard against a known condenser. The network is arranged as in Fig. 167 (*a*), a condenser of capacitance C being put in parallel with the resistance S. This diagram should be compared with Figs. 130 and 152 (*a*).

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* S. Bekku, M. Doté and Y. Nakamura, "A new a.c. bridge and its application," *Elect. Lab. Min. Comm. Tokyo. Res.*, No. 244, pp. 1–64, Jan. (1929). "On the measurement of line constants of overhead transmission lines with the new a.c. bridge," *ibid.*, No. 285, pp. 1–42, May (1930). H. Rokkaku, M. Doté and Y. Nakamura, "On the field measurement of line constants of an overhead transmission line with a new a.c. bridge," *ibid.*, No. 310, pp. 1–45 (1931). All papers are in Japanese with brief English summaries.

† A. Campbell, *Proc. Phys. Soc.*, Vol. 21, pp. 78–79 (1910).

In the analysis on page 434, put $S/(1 + j\omega CS)$ in place of S; then the balance equation is

$$\frac{S(P + j\omega L_1)}{(1 + j\omega CS)} - Q(R + j\omega L_2) + j\omega M\left(Q + \frac{S}{(1 + j\omega CS)}\right) = 0\,;$$

from which

$$(SP - QR) = \omega^2 QSC(M - L_2),$$

and $S(L_1 + M) - Q(L_2 - M) = QRSC.$

When the bridge is used with equal ratio branches, $Q = S$, and $L_2 = L_1 = L$, then

$$P - R = \omega^2 QC(M - L)$$

and

$$2M = QRC.$$

From this, M is found without the need for measurement of ω, which must, however, be steady to satisfy the first condition.

Again, in Fig. 167 (*b*) insert a condenser of capacitance C in series with the coil L_1, P. This diagram is a mutual inductance analogue of the resonance bridge of Fig. 137. Campbell* has shown that this bridge is a very useful wave-filter or frequency bridge for low frequencies. With the condenser out of the bridge and M set to zero, balance is secured as an ordinary Maxwell inductance bridge, so that

$$SP = QR \text{ and } L_1S = L_2R$$

The condenser is now inserted and balance restored by adjustment of M; then

$$\omega^2 MC = S/(Q + S)$$

For example, with $Q/S = 250$, $M = 10$ mH., $C = 1\ \mu$F. makes the network suitable for $f = 100$ cycles per second. The condenser may be imperfect, its series loss resistance being included in P. If the frequency be known the bridge may be used for measurements on imperfect condensers and is then best set up in the equal ratio form.

Haworth† has used a similar modification of the Heaviside-Campbell bridge of Fig. 153, by means of which the effective capacitance and series resistance of an imperfect condenser may be measured.

In the bridge illustrated in Fig. 153 (*b*) on page 438, let the coil L_x, P_x be replaced by an imperfect condenser C, ρ. Now the impedance operator for a condenser is $-j/\omega C = -j\omega/\omega^2 C$, so that a condenser may be thought of as possessing a negative inductance of amount $1/\omega^2 C$. Hence, on page 440, write $1/\omega^2 C$ for L_x, giving for balance

$$\rho = r_1 - r_0,$$
$$C = 1/2(M_1 - M_0)\omega^2,$$

numerically, r_1, r_0 and M_1, M_0 being the readings of the rheostat and inductometer with the condenser in and out of the bridge.

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* A. Campbell, "On wave-form sifters for alternating currents," *Proc. Phys. Soc.*, Vol. 24, pp. 107–111, 158–159 (1912). Also see S. Chiba, *Journal I.E.E., Japan*, No. 405, pp. 294–300 (1922).

† H. F. Haworth, "The measurement of electrolyte resistance using alternating currents," *Trans. F. Soc.*, Vol. 16, pp. 365–391 (1921). The second balance condition is incorrectly quoted.

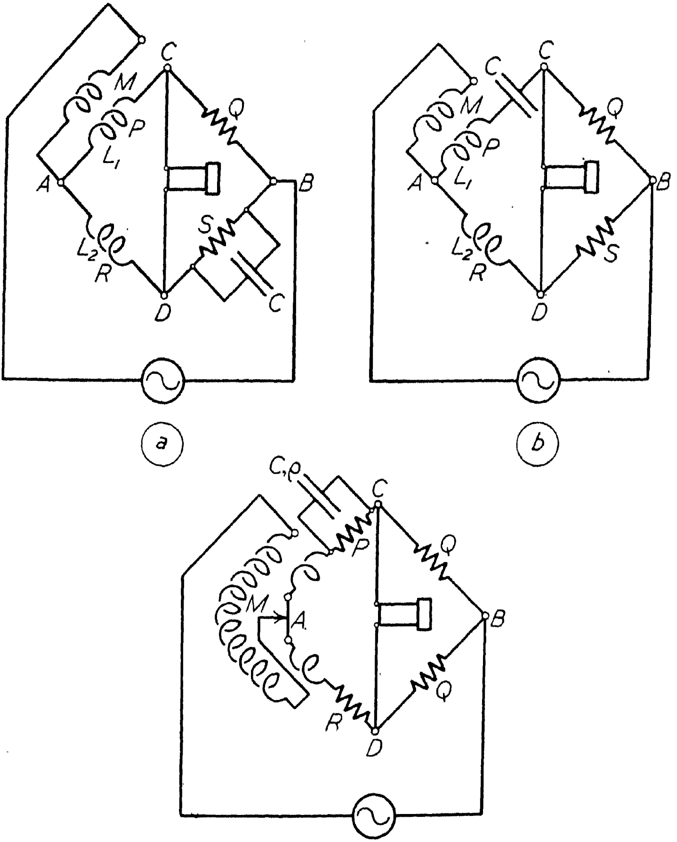

Fig. 167.—Campbell's Capacitance-Mutual Inductance Bridges

49. Campbell's Wide Range Capacitance Bridge. Campbell* has described a modification, Fig. 167 (*c*), of the Heaviside equal ratio bridge of Fig. 153 (*b*) by means of which quick measurements of capacitances from 1 $\mu\mu$F. to 30 μF. may be made, the power-factor being given with fair accuracy. The arrangement resembles Dye's method of Fig. 155, the condenser of capacitance C and series loss-resistance ρ replacing the coil L_xP_x as a shunt across the resistance P, while the rheostat r is removed from the place shown and replaced as described below. The resistances P and R are equal and may be given the values $\sqrt{10}$, 10, $\sqrt{1{,}000}$, 100, $\sqrt{100{,}000}$, 1,000, and $\sqrt{10^7}$ ohms by a single switch. The two halves of the inductometer secondary are separated by a slide wire, the point A being the slider thereon; this rheostat is to balance any differences up to $\pm$ 0·6 ohms in the P and R branches, and can be shunted to increase the accuracy of reading tenfold. A small auxiliary zero-setting inductometer of $\pm$ 2 μH. is provided.

To make a measurement, suitable equal values of P and R are chosen, the inductometer set to zero and the condenser removed from the bridge; balance is secured by adjustment of the auxiliary inductometer and the sliding contact on the rheostat. The condenser is now shunted across P and balance restored by adjusting the slide wire rheostat and the main inductometer. Then the inductometer reading multiplied by a factor gives the value of C and the change in the rheostat setting enables the power-factor of the condenser to be calculated.

Let P', L' be the effective resistance and inductance of the combination C, ρ in parallel with P; then

$$P' + j\omega L' = P\left(\rho + \frac{1}{j\omega C}\right)\Big/\left(P + \rho + \frac{1}{j\omega C}\right)$$

$$= P\,(1 + j\omega C\rho)\,[1 - (P + \rho)j\omega C]/[1 + (P + \rho)^2C^2\omega^2]$$

from which

$$P - P' = \frac{P^2(P + \rho)C^2w^2}{1 + (P + \rho)^2C^2\omega^2}$$

$$L' = \frac{-P^2C}{1 + (P + \rho)^2C^2\omega^2}$$

Now let $P - P' = r$ be the change in the rheostat reading and $L = -L'$ be the self-inductance read from the scale of the inductometer and equal to twice the mutual inductance. Then

$$C = \frac{L}{P^2}\left(1 + \frac{r^2}{\omega^2L^2}\right)$$

$$\cos\phi = \omega C\rho = \frac{r}{\omega L}\left(1 - \frac{r}{P}\right) - \frac{\omega L}{P}$$

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* A. Campbell, "A capacitance bridge of wide range and a new inductometer," *Proc. Phys. Soc.*, Vol. 39, pp. 145–149 (1927). "A versatile inductometer bridge," *Journal Sci. Insts.*, Vol. 4, pp. 305–311 (1927).

By the use of a high frequency for small condensers, say 800 cycles or by using a frequency of 100 or so for large condensers (> 0·3 μF.) the frequency correction may be made small; then

$$C \doteqdot L/P^2 \text{ and } \cos\phi \doteqdot (r/\omega L) - (\omega L/P)$$

The values of P specified above give factors of 1/10, 1/100, etc. The inductometer is marked to read L directly and has a range of - 10 to + 105 microhenrys in the variable part, extended by two fixed steps to 305 μH. The range of capacitance is from about 1 $\mu\mu$F. to 30 μF. and can be read with accuracy; the power-factor, depending on the difference between two quantities, is not so well determined, especially for good condensers. The whole arrangement is built in a self-contained form by the Cambridge Instrument Co.

50. Hay's and Davies's Methods. C. E. Hay* has suggested a modified form of the Hughes balance to measure the equivalent capacitance and shunt resistance of an imperfect condenser by means of a mutual inductometer. In Fig. 161, let the coil $P + j\omega L$ be replaced by $P/(1 + j\omega CP)$; then from page 452 the balance equation is

$$\frac{SP}{(1 + j\omega CP)} - QR - j\omega M\left\{Q + R + S + \frac{P}{(1 + j\omega CP)}\right\} = 0,$$

from which

$$\begin{aligned} \omega^2 M(Q + R + S)CP &= QR - SP \\ -QRCP &= M(P + Q + R + S) \end{aligned}$$

are the balance conditions.

From the second it should be noted that M must be negative, since C is essentially positive. Inserting this condition and solving for C and P gives the capacitance and shunt loss resistance,

$$C = \frac{M(Q + S)\ (R + S)}{Q^2R^2 + \omega^2M^2(Q + R + S)^2}$$

and

$$\frac{Q^2R^2 + \omega^2M^2(Q + R + S)^2}{QRS - \omega^2M^2(Q + R + S)}$$

The equivalent series loss- resistance bridge has been used by Davies† as a convenient method for measuring frequency; see Fig. 168. In the analysis on p. 452 replace $P + j\omega L$ by $P + (1/j\omega C)$, where $P = P_1 + \rho$, then

$$S[P + (1/j\omega C)] - QR - j\omega M[P + Q + R + S + (1/j\omega C)] = 0,$$

so that

$$-M/C = QR - SP \text{ and } -\omega^2MC = S/(P + Q + R + S)$$

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* C E. Hay, "Alternate current measurements, with special reference to cables, loading coils, and the construction of non-reactive resistances," *Journal, P.O.E.E.*, Vol. 5, pp. 451–454 (1913); also *Professional Papers*, No. 53, pp. 25–26 and 44–45.

† R. M. Davies, "A simple frequency bridge," *Journal Sci. Insts.*, Vol. 10, pp. 274–276 (1933).

The second condition shows that M must be negative. If M and C are fixed, the bridge may be balanced for a given frequency by adjusting S and either Q or R. Let SP be made small in comparison with QR; this is easily done by making $P_1 = 0$, when $P = \rho$ is rarely more than a few tenths of an ohm. For example, at $f = 600$ cycles per second, with $M = 10$ mH., $C = \frac{1}{3}$ μF., $M/C = 3 \times 10^4$ and if $R = 100$ ohms, $Q = 300$ ohms, S will be about 15 ohms. With $\rho = 0{\cdot}2$ ohm, $S\rho$ is 3 which is negligible in comparison with $QR = 3 \times 10^4$. With this simplification, numerically

$$M/C \doteqdot QR$$

which can be set once for all, subject to minor adjustments to give sharp balance. The second condition,

$$\omega^2 MC \doteqdot S/(Q + R + S)$$

numerically, can be satisfied by adjustment of S alone.

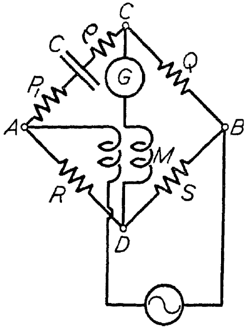

FIG. 168.—DAVIES'S FREQUENCY BRIDGE

51. Campbell's Frequency Bridge. The circuit for this method* is shown in Fig. 169 (*a*). Suppose the condenser to be perfect and let the self-inductance and resistance of the coil of the variable mutual inductance in the detector circuit be L_2, R_2. Then the equation for **i** in terms of **u** is

$$\left\{R_2 + z_G + j\left(\omega L_2 - \frac{1}{\omega C}\right)\right\}\mathbf{i} = -j\left(\omega M + \frac{1}{\omega C}\right)\mathbf{u},$$

where $z_G = R_G + jX_G$ is the operator for the detector. Hence, $\mathbf{i} = 0$ when $\omega^2 = -1/MC$. Since ω^2 is essentially positive, it follows from this equation that the primary and secondary coils of the inductometer must be in opposition, so that M is negative. Assuming this adjustment to be made, the balance condition is numerically

$$\omega^2 = 1/MC$$

The method serves to measure at ordinary frequencies fairly large capacitances in terms of mutual inductance and frequency. It is, however, chiefly used in the inverse sense for the measurement of frequency in terms of M and C. The method is most suitable for the determination of high frequencies, say, 1000 cycles per second upwards, since *MC* is inconveniently great unless ω is large. For low frequency modifications see page 557.

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* A. Campbell, " On the use of variable mutual inductances," *Proc. Phys. Soc.*, Vol. 21, pp. 80–82 (1910). *See* also p. 74.

When used as a frequency bridge, it is possible to obtain sharp balance only if certain important conditions are observed, as follows.

The Frequency must be Constant and the Wave Form Pure.

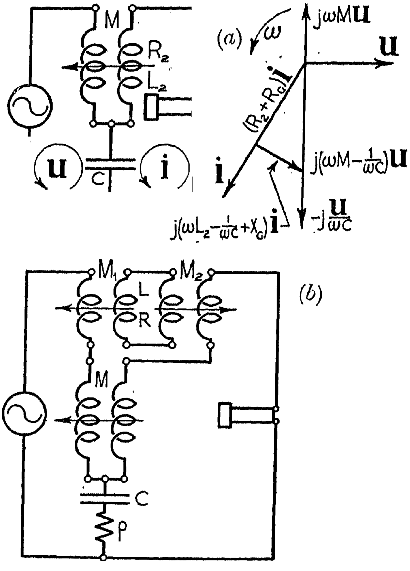

FIG. 169.—CAMPBELL'S FREQUENCY BRIDGE

The need for constancy of frequency is obvious. The wave form must be pure when, as is usual at the higher audio frequencies, telephones are used to detect balance, since two or more coexistent frequencies cannot be balanced by the same value of *MC*. This property is sometimes employed when the bridge is used as a wave filter; see page 231.

The Condenser must be as Perfect as Possible. In order to see the necessity for this condition, suppose the condenser to have losses represented by a resistance ρ in series with it.

Then, for $-j/\omega C$ write $\rho-(j/\omega C)$ in the above analysis. Balance will occur when $\rho-j\left(\omega M+\frac{1}{\omega C}\right)=0$; i.e. $\rho=0$, necessitating a perfect condenser, and $-\ \omega^2 = 1/MC$. In practice, it will be found that a mica condenser is usually sufficiently good to allow of reasonably sharp balance. It is even possible to attain fair success with a good paper condenser, but, as a rule, a minimum indication is all that can be secured with the average condenser.

To obtain a sharp balance, it is necessary, therefore, to compensate for the imperfection of the condenser. Campbell* has suggested several devices which enable this to be carried out, one of the best being shown in Fig. 169 (*b*). In this, M is the inductometer; C, ρ the imperfect condenser; M_1, M_2 are auxiliary mutual inductances, preferable variable, the secondary of one being linked to the primary of the other to form a loop, L, R. M, M_1, M_2 should be at a distance from one another so that they do not have any mutual influence. Balance is obtained when M and M_1 or M_2 are adjusted so that

$$-1/\omega^2 C = M-\rho L/R,$$

and
$$\rho R=\omega^2\left[M_1M_2-L\left(M+\frac{1}{\omega^2 C}\right)\right].$$

Solving for ρ and $\omega^2 C$ gives numerically, M being negative,

$$\rho=\omega^2 RM_1M_2/(R^2+\omega^2L^2)$$

and
$$1/\omega^2 C=M+[\omega^2 LM_1M_2/(R^2+\omega^2L^2)].$$

If ω be known, ρ and C can be determined; conversely, if the method is intended for frequency measurements, let L/R be small, then $\omega^2 \risingdotseq 1/MC$. The auxiliary circuit need not, therefore, be known, but by its aid sharp balance may be obtained. When using telephones, trouble due to harmonics will be largely overcome if the primary of the inductometer has a fairly high inductance.

Experimental Examples. The simple Campbell bridge (Fig. 169 (*a*)) was used to test the frequency calibration of a triode oscillator previously tested by other methods (*see* pp. 451 and 454). In the first two observations a 150 ohm telephone was used; in the remainder the detector was a Duddell vibration galvanometer. As paper condensers, having considerable losses, were employed, only a minimum indication could be secured. M was a 111 mH. Campbell mutual inductometer.

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* A. Campbell, "On the measurement of small inductances and on power losses in condensers," *Proc. Phys. Soc.*, Vol. 29, pp. 350–353 (1917). For another use of the network, *see* p. 307 and Fig. 94(*b*).

Anode Condenser μF.	C μF.	M μH.	Frequency cycles/second.
0·1	$1{\cdot}016_1$	30,030	911·1
0·2	$1{\cdot}016_1$	59,790	645·7
0·3	$1{\cdot}016_1$	89,650	527·3
0·4	$2{\cdot}023_4$	59,690	458·0
0·5	$2{\cdot}023_4$	75,610	406·9

In a second test, with the anode condenser set at 0·5 μF., C was $2{\cdot}103_8$ μF. and $M = 72720$ μH. gave a minimum indication; hence $f = 406{\cdot}9_0$ cycles per second as found above. The balance was then made sharp by the use of the arrangement shown in Fig. 169 (b). M_2 was an 11 mH. Campbell mutual inductance, M_1 a fixed value mutual of 9840 μH. L contained the fixed coil of M_2 and the low inductance coil of M_1, totalling 10,757 μH. A resistance box made R up to 193 ohms. Then, by adjusting M and M_2 successively, true balance was obtained when $M = 7257_5$ μH. and $M_2 = 2500$ μH. Inserting these values in the balance equations and solving the resulting quadratic in ω^2 gives $f = 407{\cdot}0_4$ cycles per second. Using the approximate formula gives $407{\cdot}3_1$ cycles per second, showing the smallness of the correction due to M_1 and M_2, these serving to give exact balance with little influence on the calculated value of frequency. It should also be noted that the simple Campbell arrangement, giving $406{\cdot}9_0$, as against the true $407{\cdot}0_4$, is only in error by about 3 parts in 10,000.

The Mutual Inductometer should be Free from Impurity. At high frequencies, the electromotive force induced in the secondary of a mutual inductance is slightly out of quadrature with the primary current (*see* p. 153), the mutual inductance being said to be impure.

Since the Campbell frequency bridge is peculiarly adapted for use at high frequencies, the effects of impurity become of importance. Butterworth,* in an elaborate paper, has shown how the Campbell bridge must be modified in order that impurity may be compensated, the reader interested in precise measurements at the higher acoustic frequencies being referred to this paper for a detailed discussion.

Referring to Fig. 169 (a), let the condenser have a series loss resistance ρ, and let the mutual have an impurity σ.

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* S. Butterworth, *loc. cit.*, p. 337 (1921).

Then Butterworth's modification of the Campbell bridge consists in connecting a small condenser of capacitance C' across the upper open ends of the mutual inductometer and also including a small adjustable resistance S in series with the condenser C (*see* also Fig. 192, p. 556). Balance can then be found by independent successive adjustments of S and M. When the condenser loss and the mutual impurity are thus compensated, if R_1, L_1 are the resistance and inductance of the inductometer winding joined to the alternator, Butterworth shows that very nearly,

$$\omega^2 MC = 1 - \omega^2 CC'[R_1 R_2 - \omega^2 (L_1 - M)(L_2 - M)],$$

$$\text{and} \quad S + \rho + \sigma = \omega^2 C'[R_1(L_2 - M) + R_2(L_1 - M)],$$

from the first of which ω can be calculated (*see* p. 557).

Experimental Example. In a test on a triode oscillator by the simple bridge of Fig. 169 (*a*), mica condensers were used for C, totalling $1{\cdot}451_3$ μF. Minimum indication occurred when $M = 10555_5$ μH., so that the frequency is approximately $406{\cdot}6_3$. A small mica condenser of $0{\cdot}0110_6$ μF. was then connected across the open ends of the mutual inductance, and a small resistance S was joined in series with C. Adjusting M and S, true balance occurred when $M = 10550_5$ μH., and $S = 0{\cdot}02$ ohm; then since $R_1 = 56{\cdot}6$ ohms, $L_1 = 244{,}200$ μH., $R_2 = 39{\cdot}4$ ohms, $L_2 = 100{,}570$ μH., $C' = 0{\cdot}0110_6$ μF., the above equation gives $f = 406{\cdot}0_1$ cycles per second.

Campbell's method is very quick in practice. Even when uncompensated for condenser losses and impurity the minimum point can be located within a few parts in 10,000 when using a good condenser. Compensated by Butterworth's method to give a true balance, it forms an extremely sensitive means of detecting very slight changes of frequency. By connecting an adjustable air condenser in parallel with C, variations in f as small as a few parts in a million can be measured by the small adjustments of this condenser necessary to restore balance.

Applications. Nukiyama and Kobayashi* have used the method as a means of measuring the self-capacitance of a coil. Let the primary coil of the mutual inductance have resistance R_1, self-inductance L_1, and self-capacitance C_o, the latter represented as a condenser shunted across the coil. Then at balance $\omega = 1/\sqrt{MC + L_1C_o}$ and $\omega C_oR_1 = 0$. The first condition may be satisfied by adjusting M or C; the second cannot be satisfied unless $R_1 = 0$, so that perfect balance is not possible. These workers show that if M be fixed and tests be made at two frequencies, first with the coil shunted with a known small capacitance and then when unshunted, sufficient data are obtained to enable C_o to be calculated. Thus writing $\omega = 1/\sqrt{M'C}$ where $M' = M + (C_oL_1/C)$ balance at two frequencies by adjusting C to values C_1, C_2; then, with known frequency, $M_1' = M + (C_oL_1/C_1)$, $M_2' = M + (C_oL_1/C_2)$. Now shunt the coil with a known capacitance C_{01} and again test at two frequencies, the balancing values of C being C_3 and C_4; then

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* H. Nukiyama and K. Kobayashi, "On the measurement of the natural frequency of an inductance coil at audio frequency," *Phil. Mag.*, 6th series, Vol. 48, pp. 962–971 (1924).

$$M_1'' = M + [(C_o + C_{01})L_1/C_3],\ M_2'' = M + [(C_o + C_{01})L_1/C_4]$$

Eliminating L_1 and M,

$$C_o = C_{01} \div \left[\frac{M_1'' - M_2''}{M_1' - M_2'}\,\frac{\left(\dfrac{1}{C_1} - \dfrac{1}{C_2}\right)}{\left(\dfrac{1}{C_3} - \dfrac{1}{C_4}\right)} - 1\right]$$

They also describe a modification for securing sharp balance.

It has been pointed out above that Campbell's frequency bridge becomes inconvenient at low frequencies on account of the large values then assumed by MC. Chiba[*] has described a number of modifications of the simple bridge intended to adapt it to low frequency measurements; some of these are considered as examples in circuit transformations in Appendix I.

The Campbell frequency bridge has been applied with modifications to measurements at radio frequencies. T. L. Eckersley[†] has suggested its use for the measurement of the losses in large condensers. W. Jackson[‡] has made a thorough investigation of the bridge when used for the measurement of the effective resistance of a coil, following a suggestion due to E. B. Moullin.

52. Kennelly[§] and Velander's Frequency Bridge. On page 470 Campbell's method of measuring frequency has been described, and it has been shown, moreover, that perfect balance is impossible unless the condenser be quite free from losses. Now, in practice, condensers with solid dielectrics are never free from imperfection, so that some device is necessary in order that the imperfection may be compensated. Campbell's bridge, even when corrected for condenser losses by one of the methods described above is unsuitable for low frequency measurements, below 250 cycles per second for example, owing to the large values of M and C necessarily involved. To overcome both these difficulties, Kennelly and Velander have devised a wide-range frequency bridge shown in Fig. 170.

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[*] S. Chiba, "Modifications of Campbell's arrangement for measuring telephone frequency," *Journal I.E.E., Japan*, No. 405, pp. 294–300 (1922).

[†] T. L. Eckersley, *Journal I.E.E.*, Vol. 61, p. 940 (1923).

[‡] W. Jackson, "High-frequency resistance measurement by the use of a variable mutual inductance," *Journal I.E.E.*, Vol. 68, pp. 296–304 (1930).

[§] A. E. Kennelly and E. Velander, "A rectangular component two-dimensional alternating current potentiometer," *Journal F. Inst.*, Vol. 188, pp. 1–26 (1919). E. Velander, "A frequency bridge," *Journal Amer. I.E.E.*, Vol. 40, pp. 835–839 (1921).

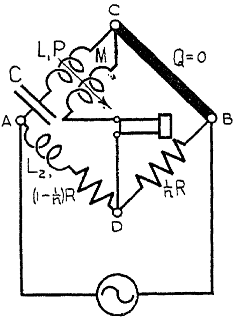

Fig. 170.—Kennelly and Velander's Frequency Bridge

The bridge contains the following elements: in the branch AC the condenser is connected in series with the fixed coils of the inductometer, the secondary winding of which is put into the detector circuit. A small variable self-inductance L_2 is included in the branch AD. Then, if R be the total resistance of the branches ADB (inclusive of the resistance of L_2), the branch DB consists of the nth part of R.

To find when no current flows in the detector, put

$$z_1 = P + j\left(\omega L_1 - \frac{1}{\omega C}\right),\ z_2 = 0,\ z_3 = R/n,\ z_4 = \left(1 - \frac{1}{n}\right)R + j\omega L_2,$$

and $m_{15} = j\omega M$ on page 71. Then $\alpha = 0$, $\beta = -j\omega M$, $\gamma = 0$, $\delta = j\omega M$, and the balance equation is

$$\frac{R}{n}\left\{P + j\left(\omega L_1 - \frac{1}{\omega C}\right)\right\} + j\omega M\frac{R}{n} + j\omega M\left\{\left(1 - \frac{1}{n}\right)R + j\omega L_2\right\} = 0.$$

Separating components gives

$$\omega = \frac{1}{\sqrt{C(L_1 + nM)}}$$

$$L_2 = CRP\left(1 + \frac{L_1}{nM}\right)$$

for balance.

From this it is seen that the resistance P, which includes the equivalent series resistance of the condenser, does not enter into the expression for ω, so that an imperfect condenser may be used. Moreover, the effect of the potential divider arrangement of the resistance R is to multiply the range of the inductometer n times. Hence, by a suitable choice of n, low frequencies can be measured by means of a condenser and an inductometer of reasonably small value.

The most convenient practical arrangement of the bridge is to use a fixed condenser C and to fix the multiplier n. The branch AC should contain a small rheostat, P', say. Then balance by varying M and P' or L_2. Provided that the primary of the inductometer has a low resistance, very precise adjustment of L_2 will not be necessary.

For sensitivity the balance detector should have a low impedance, about equal to that of the secondary of the inductometer together with R/n. A low impedance vibration galvanometer or telephone is most suitable, though high impedance detectors can be used if connected to the bridge through a transformer (*see* p. 232).

It is very convenient to make up the capacitance C from two equal condensers which can be connected in series or parallel as desired. Since their capacitance in series will be one-quarter of the value when in parallel, the range of the bridge can be doubled, without altering any other constituent of the bridge. Moreover, it is not necessary that the two condensers be very accurately adjusted. Velander has shown that a difference of 2 per cent between the two condensers will only produce an error of 5 in 100,000 in the doubling of the frequency range.

Velander has described a portable, self-contained frequency bridge having a range from 400 to 3,200 cycles per second. The resistance R/n is fixed at 100 ohms, R being 500, 2,000, or 8,000 ohms. This gives $n = 5$, 20, and 80, corresponding roughly to frequencies in the ratio 4 : 2 : 1. The inductometer could be varied up to 10 millihenrys ; and C was made of two mica condensers of 0·4 microfarad each, so that the working capacitance is either 0·2 or 0·8 microfarad. The whole apparatus is fitted up in a box with terminals for attachment to the source and to the telephones. A system of dial switches provides for the selection of a suitable multiplier n, for the insertion of the necessary compensating inductance L_2, and for the adjustment of M. A four-point plug enables the condensers to be changed from the series to the parallel position.

At frequencies below 500 a vibration galvanometer will give high sensitivity; for acoustic frequencies a telephone is convenient. Since, by suitable choice of n, small coils can be used and impurity effects reduced thereby, the method appears to be applicable at high frequencies above the telephonic range. A crystal detector and galvanometer is then suitable, and the point C may be earthed to avoid earth capacitance troubles (Fig. 170).

Experimental Example. The triode oscillator tested by other methods (*see* pp. 451, 454 and 472) was calibrated by Kennelly and Velander's bridge. M was an 11 mH. Campbell mutual inductometer, its fixed winding forming L_1. C was a mica condenser of $0{\cdot}334_6$ μF. P was about 6 ohms. R/n was a decade resistance box set at 100 ohms, the total value of R being given the values tabulated. L_2 was an Ayrton-Perry variable inductance.

Anode Condenser μF.	R ohms.	M μH.	L_2 mH.	Frequency cycles/second.
0·1	1510·7	532_8	$5{\cdot}0_0$	918·3
0·2	3510·7	488_2	$10{\cdot}9_5$	647·0
0·3	8510·7	306_0	$24{\cdot}1_0$	530·1
0·4	8510·7	411_0	$24{\cdot}1_0$	459·3
0·5	8510·7	524_0	$23{\cdot}3_0$	407·1

The detector was a telephone for the two highest frequencies and a tuned Duddell vibration galvanometer for the remainder.

53. The High Voltage Bridges of Dawes and Hoover and of Geyger. A bridge for measuring dielectric losses in cables has been given by Dawes and Hoover,* and is illustrated in Fig. 171 (*a*). In this diagram Q is a fixed resistance of about 50 to 100 ohms when pieces of cable about 10 ft. long are to be tested; S is a variable resistance of about 1,000 ohms. The drop of voltage in Q and S is about 1 volt, so that if B is earthed the points C and D are only slightly above earth potential and earth capacitance effects at these points are inappreciable; A is the high voltage terminal and may be several kilovolts above earth. The dielectric under test is represented by C_1; P is the resistance of the branch AC and L_1 the inductance of the primary of M. The branch AD contains a standard air condenser C_2 with a guarded low voltage electrode; the guard is earthed and is, therefore, very nearly at the potential of the low voltage plate. The effective series loss resistance of this condenser is R. Balance is secured by adjusting M and S.

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* C. L. Dawes and P. L. Hoover, "Ionization in paper insulated cables," *Trans. Amer. I.E.E.*, Vol. 45, pp. 141–159 (1926). C. L. Dawes, H. H. Reichard, and P. H. Humphries, *ibid.*, Vol. 48, pp. 382–395 (1929). C. L. Dawes, P. L. Hoover, and H. H. Reichard, "Some problems in dielectric loss measurements," *ibid.*, Vol. 48, pp. 1271–1280 (1929). E. W. Davis and W. N. Eddy, "Some problems in high voltage cable development," *ibid.*, Vol. 48, pp. 373–379 (1929) adapts the method for three-phase use. C. L. Dawes and A. F. Daniel, "High voltage bridge for measurements of cables with grounded sheaths," *ibid.*, Vol. 51, pp. 198–201 (1932).

Writing $z_1 = P + j\left(\omega L_1 - \frac{1}{\omega C_1}\right)$, $z_2 = Q$, $z_3 = S$, $z_4 = R - \frac{j}{\omega C_2}$, and $m_{15} = j\omega M$ in the expressions on page 71, $\alpha = 0$, $\beta = -j\omega M$, $\gamma = 0$, $\delta = j\omega M$, so that

$$S\left[P + j\left(\omega L_1 - \frac{1}{\omega C_1}\right)\right] - Q\left(R - \frac{j}{\omega C_2}\right) + j\omega SM + j\omega M\left(R - \frac{j}{\omega C_2}\right) = 0$$

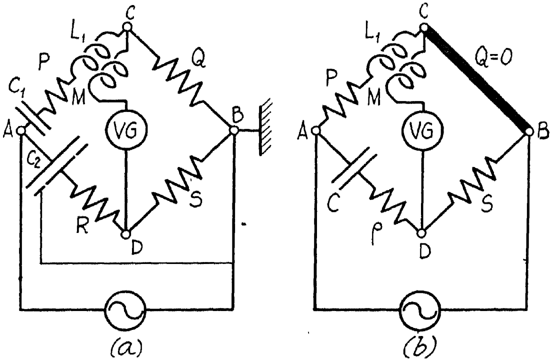

FIG. 171.—(*a*) DAWES AND HOOVER'S BRIDGE ; (*b*) GEYGER'S BRIDGE

Separating the components,

$$P = \frac{1}{S}\left(QR - \frac{M}{C_2}\right)$$

$$\frac{1}{C_1} \quad \frac{}{S} \cdot \frac{}{C_2} + \omega \left[L_1 + M\left(1 + \frac{R}{S}\right)\right]$$

are the balance conditions in their general form. Now for a well-designed air condenser R is nearly zero ; also ωL_1 may be neglected in comparison with the relatively enormous $1/\omega C_2$. If ρ be the loss resistance of C_1, P' being the resistance of L_1 together with the leads, then $P = P' + \rho$. To high approximation, therefore,

$$C_1 \doteqdot \frac{S}{Q} C_2 \text{ and } \rho \doteqdot \frac{M}{SC_2} - P' \doteqdot \frac{M}{SC_2}$$

so that $\cos\phi \doteqdot \omega C_1 \rho \doteqdot \omega M/Q$

The authors have made an extensive investigation of the properties of impregnated paper insulated cables up to 70 kilovolts ; full details of procedure and results are given in their papers.

Geyger* has introduced what seems to be a modification of this bridge, shown in Fig. 171 (*b*), for measurement of capacitance and dielectric losses in cables and condensers. Referring to the diagram, Geyger's bridge is derived from Dawes and Hoover's by putting $1/C_1 = 0$, $C_2 = C$, $R = \rho$, and $Q = 0$, where C is the capacitance and ρ the series loss resistance of the condenser under test. Making these changes in the general expressions gives

$$P = -M/CS \text{ and } L_1 + M\left(1 + \frac{\rho}{S}\right) = 0.$$

Since P, C and S are positive, balance is only attainable if M is reversed. Changing the sign of M, the balance conditions become numerically

$$C = M/SP \text{ and } \rho = \left(\frac{L_1}{M} - 1\right) S, \text{ giving } \cos\phi \risingdotseq \omega C\rho \risingdotseq \omega\,(L_1 - M)/P.$$

It will be seen on comparison with Fig. 164 that the bridge is nothing more than the Carey Foster method of p. 458 with the source and detector interchanged. It follows, therefore, that the Dawes and Hoover bridge is a generalized conjugate Carey Foster method.

Geyger gives a comprehensive series of tests made by the method on condensers ranging from Leyden jars to pieces of cable.

54. Laurent's and Serner's Bridges. Practically all a.c. bridges measure the two perpendicular components of an impedance, namely, the reactance and resistance. The bridges now to be noticed are almost unique in measuring the magnitude and phase-angle of an impedance. Referring to Fig. 172 (*a*) one of Serner's† bridges is shown, $S + jX$ being the unknown impedance of magnitude $Z = \sqrt{(S^2 + X^2)}$ and phase-angle $\phi = \arctan X/S$. The resistor Q is fixed; R, P, C, and M are variable. It will be seen from Fig. 171 that the bridge is a further modification of the conjugate Carey Foster network. The procedure is first to replace Z by a resistor, S' say, of about equal magnitude, to set M to zero and balance the resulting resonance bridge by adjustments of C and P after setting $R = S'$. Then $\omega^2 LC = 1$ and $P = Q$; the current $\mathbf{i}_c$ is now in phase with the p.d. across AB. Replace Z in the branch BD, and restore balance by M and R. Then in the appropriate

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* W. Geyger, "Eine einfache Kompensationsschaltung zur Messung der Kapazität und des dielektrischen Verlustwinkels von Kondensatoren und Kabeln," *Arch. f. Elekt.*, Vol. 12, pp. 370–375 (1923); Vol. 21, pp. 529–534 (1929). For another use of the conjugate Carey Foster bridge, *see* A. W. Smith, "The measurement of the power-factor and capacitance of a condenser by comparison with a mutual inductance," *Rev. Sci. Insts.*, Vol. 4, pp. 280–284 (1933).

† A. Serner, "A new bridge for the direct measurement of impedance," *W. Eng.*, Vol. 14, pp. 59–62 (1937).

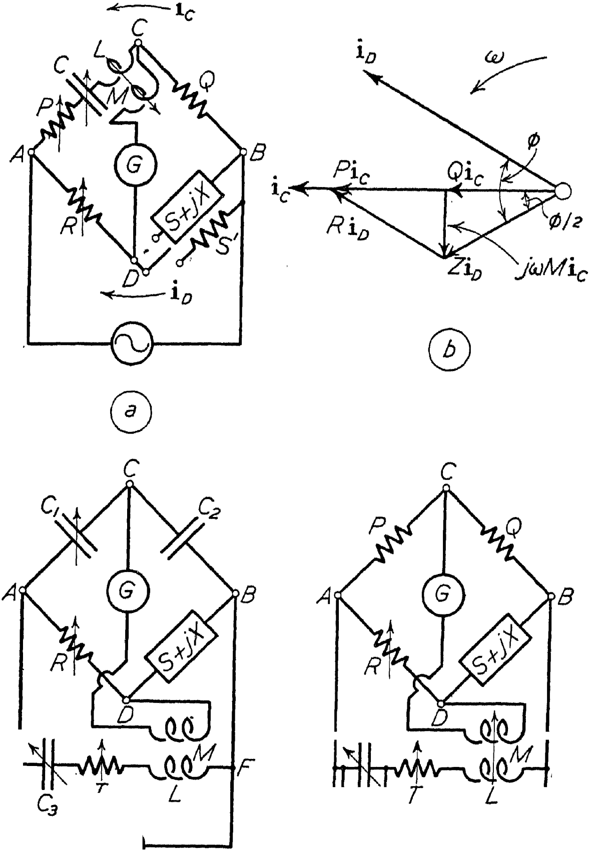

FIG. 172.—SERNER'S AND LAURENT'S BRIDGES FOR MEASURING Z AND ϕ DIRECTLY

equation on p. 75 put $z_1 = P + j\left(\omega L - \frac{1}{\omega C}\right)$, $z_2 = Q$ $z_3 = S + jX$, $z_4 = R$, $m_{15} = j\omega M$; then for balance

$$\left[P + j\left(\omega L - \frac{1}{\omega C}\right)\right](S + jX) - QR + j\omega M(R + S + jX) = 0$$

Inserting the original balance conditions makes

$$(Q + j\omega M)(S + jX) - R(Q - j\omega M) = 0$$

which gives

$$\omega MX = Q(S - R),$$

and

$$-QX = \omega M(S + R),$$

which on multiplication give

$$-X^2 = S^2 - R^2$$

or

$$Z = \sqrt{(S^2 + X^2)} = R.$$

Again $\tan\phi = X/S$ and also

$$\begin{aligned}\tan\frac{\phi}{2} &= [\sqrt{(1 + \tan^2\phi)} - 1]/\tan\phi \\ &= (Z - S)/X = (R - S)/X \\ &= -(S - R)/X = -\omega M/Q.\end{aligned}$$

Thus by fixing Q at the maximum available value of ωM, angles up to 90° can be measured.

The vector diagram is readily drawn, as in Fig. 172 (*b*). Since branch AC is tuned, the voltage across AB is $(P + Q)\mathbf{i}_C = 2Q\mathbf{i}_C$ since $P = Q$. Assuming the unknown to be an inductive impedance, $\mathbf{i}_D$ lags behind the p.d. $Z\mathbf{i}_D$ by the angle ϕ. But since the p.d. across CD is zero, the drop $Z\mathbf{i}_D$ must be equal to the vector sum of $Q\mathbf{i}_C$ and $j\omega M\mathbf{i}_C$, and leads on $\mathbf{i}_C$ by arctan $\omega M/Q$, i.e. by $\phi/2$.

In Fig. 172 (*a*) since $P = Q$ and the arm AC is tuned the point C is midway in potential between A and B, and the current in the primary of M is in phase with the p.d. across AB. These two conditions are separated in the second of Serner's bridges, Fig. 172 (*c*). C_1 and C_2 are equal condensers such that their reactances approximately equal the value of Z; for example, with $Z = 10{,}000$ ohms at 796 cycles per second ($\omega = 5{,}000$), 0·05 μF. is suitable. M is a fixed mutual inductor of about 0·2 H., its secondary being in series with the galvanometer and its primary in series with a condenser C_3 and a resistance T across the source. C_3 is set to resonate with

L (about 0·2 H.) at a given frequency (0·16 μF. at $\omega = 5{,}000$). Balance is secured by regulation of R and T; since T replaces $P + Q$, i.e. $2Q$ above

$$Z = R \text{ and } \phi = 2 \arctan 2\omega M/T;$$

T includes the loss resistance of C_3 and the resistance of L.

Serner's second bridge was anticipated in a bridge invented by Laurent* in 1924, shown in Fig. 172 (d). In this P and Q are equal resistances. The resonance adjustment of the primary of M is effected by a shunted condenser; M is variable. Balance is secured by regulation of M and R; if T is the total resistance of EF then

$$Z = R \text{ and } \phi = 2 \arctan kM,$$

if T is set so that $T = 2\omega/k$, where k is a constant. In the self-contained apparatus made by the Svenska Radioaktiebolaget (L. M. Ericsson), R is a 4-decade resistor reading to 11,110 by 1 ohm steps; M is a toroidal inductor with a 4-decade secondary, the switches being marked to read tan $\phi/2$ directly at a given frequency. The proper value of T for a series of frequencies from 400 to 3,600 is provided by a set of three dial switches.

For tests on impedances with superposed direct current, Serner's arrangement has the advantage that C_1 and C_2 automatically block the direct current from the branches ACB and the detector, so that it flows entirely in the test impedance. Also for large values of Z it is easier to obtain residual-free ratio arms by using condensers rather than resistors.

Frequency Bridge. By a slight modification Serner's bridge of Fig. 172 (c) becomes a bridge introduced many years ago by Schering and Engelhardt† for frequency measurement. In this diagram imagine C_1 replaced by a variable resistor P; C_2 is unchanged (fixed 1 μF.); $Z = S$, known fixed resistor; R another fixed resistor; C_3 is omitted. Balance is secured by adjustment of P and T. If ωL is small compared with T it is easy to show that the balance conditions are very nearly,

$$M/T = C_2PS/(S + R) \text{ and } \omega^2 MC_2P/T = R/(S + R).$$

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* T. Laurent, "Über das Nebensprechen und andere damit zusammenhängende Erscheinungen," *E.N.T.*, Vol. 5, pp. 179–213 (1928). "On impedance and impedance measurements as well as a description of the impedance measuring set manufactured by Svenska Radioaktiebolaget," *L.M. Ericsson Rev.*, Nos. 7–9, pp. 1–23 (1930). "Théorie et application pratique des demi-cellules à trois branches pour filtres électriques," *Ericsson Tech.*, No. 5, pp. 69–99 (1935). For a critical comparison of Serner's and Laurent's methods, *see* G. W. O. Howe, *W. Eng.*, Vol. 14, pp. 227–228 (1937).

† H. Schering and V. Engelhardt, "Eine Frequenzmessbrücke," *Zeits. f. Inst.*, Vol. 40, p. 123 (1920).

If now $S = R$ (100 ohms each), these conditions become

$$\omega C_2 P = 1 \text{ and } 2\omega M = T,$$

which are entirely independent. By fixing M at $1/4\pi$ henry $f = T$, a simple linear frequency bridge.

Serner's, Laurent's, and Schering and Engelhardts' bridges are particular instances of a class of Wheatstone networks in which adjustments of the four branches alone are insufficient to give balance, an auxiliary voltage being injected into the detector circuit to reduce the current in it to zero. This voltage may be derived by mutual inductance from an auxiliary circuit (as above), from the source (Hughes's bridge) or from one of the bridge arms (Dawe's and Hoover's bridge). Alternatively it may be introduced by conductance from an auxiliary network (types of a.c. potentiometer, phase shifters, etc.). The various types have been classified by Geyger.*

55. Churcher's Bridge for Measuring the Losses in Large Synchronous Condensers. Churcher† has described a bridge method for measuring the loss in a large synchronous condenser when delivering full kVA. A wattmeter method is quite unsuitable for this purpose, since the power-factor is very low, while at the same time both the current and the voltage may be very high. The machine to be tested was rated at 10,000 kVA., 11,000 volts, three-phase, star connected, 50 cycles per second, full rated current per phase 525 amperes. It was desired to measure the total loss in the machine both with lagging and leading currents, the loss expected with leading current at 10,000 kVA. being about 250 kW. with power-factor of 0·025. The test was to be made under works conditions using a portable vibration galvanometer.

The over-excited machine is equivalent to an imperfect condenser but cannot easily be measured as such in a Schering bridge, since it is found that an impracticably large standard air condenser is requisite if adequate sensitivity is to be obtained. The arrangement shown in Fig. 173 is used to overcome this difficulty. In series with one phase T_1T_2 of the machine a non-reactive, four-terminal, resistance R_3 (residual inductance l_3) is joined at the star-point end; the resistance must be capable of carrying continuously the full current per phase. Between the terminal T_1 and the potential point P_1 of R_3 is connected a high non-reactive resistance capable of withstanding the phase voltage of the machine in series with the primary of a variable mutual inductor and a low variable non-reactive resistance. By adjustment of R_4 and M the p.d. across the points P_1P_2 may be balanced so that

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* W. Geyger, "Wechselstrom-Messbrücken mit Fremd-Abgleichung durch eine stetig regelbare Gegeninduktivität," *Arch. f. tech. Mess.*, J. 921–10 (Oct., 1936). "Wechselstrom-Messbrücken mit Fremd-Abgleichung durch eine besondere Kompensationsschaltung," *idem.*, J. 921–11 (Nov., 1936).

† B. G. Churcher, "Alternating current bridge methods. Their application to electrical engineering problems, with special reference to the testing of synchronous condensers," *Elecn.*, Vol. 101, pp. 518–520, 545–547 (1928). When the machine is mesh-connected or when star-connected with an inaccessible neutral point, modifications are necessary, *see* B. G. Churcher and C. Dannatt, "An a.c. bridge for power measurement," *M.V. Gaz.*, Vol. 13, pp. 428–430 (1932).

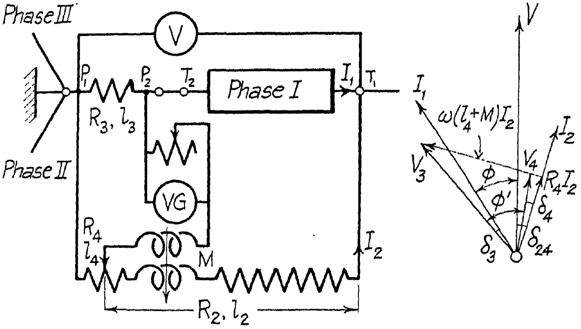

FIG. 173.—CHURCHER'S METHOD FOR MEASURING LOSSES IN LARGE SYNCHRONOUS CONDENSERS

$$(R_3 + j\omega l_3)\mathbf{i}_1 = [R_4 + j\omega(l_4 + M)]\mathbf{i}_2$$

or

$$\mathbf{v}_3 = \mathbf{v}_4 + j\omega M\mathbf{i}_2$$

The current I_2 lags behind V, the voltage across T_1P_1 by a small angle $\delta_{24} = \arctan \omega(l_2 + l_4)/(R_2 + R_4)$; the p.d. V_4 leads on I_2 by a small angle $\delta_4 = \arctan \omega l_4/R_4$; the p.d. V_3 across P_1P_2 leads on I_1 by a small angle $\delta_3 = \arctan \omega l_3/R_3$. The residuals l_2, l_3, l_4 are very small. Now

$$\tan \phi' = \tan\left[\arctan \frac{\omega(l_4 + M)}{R_4} - \delta_4\right]$$

$$= \frac{\omega M}{R_4} \quad 1 + \frac{\omega^2 l_4(l_4 + M)}{R_4{}^2} \qquad \frac{\omega M}{R_4}$$

hence $\phi' \doteqdot \phi_m$ where $\tan \phi_m = \omega M/R_4$. Then

Power per phase $= VI_1 \cos\phi \doteqdot VI_1 \cos(\phi_m + \delta_4 - \delta_3 - \delta_{24})$ watts which can be written, with the aid of the balance condition, very closely as

$$W = \frac{V^2}{R_3(R_2 + R_4)}[R_4{}^2 + \omega^2 M^2]^{\frac{1}{2}} \cos\left[\arctan\frac{\omega M}{R_4} + \delta_4 - \delta_3 - \delta_{24}\right]$$

Neglecting R_4 in comparison with R_2,

$$W = \frac{V^2}{R_2 R_3}[R_4 + \omega M(\delta_3 + \delta_{24} - \delta_4)] \text{ watts ;}$$

this is the total power dissipated in the phase between T_1 and P_1; the actual power in the machine phase is $W - I_1{}^2 R_s$ if R_s is the resistance between P_1 and T_2. In practice, this correction is usually negligible. The measurement should be repeated on each of the three phases to obtain the total losses. Note that M is positive for leading currents and negative for lagging currents.

In the example given by Churcher, R_2 consisted of a 200,000 ohm oil immersed resistance connected in series with the primary of a 20 mH. mutual inductor; R_4 consisted of a 0·1 ohm four-terminal resistance in parallel with a 1 ohm slide wire, the angle δ_4 being quite negligible. The value of δ_{24} did not exceed $\pm$ 0·00015 radian. The resistance R_3 was an oil immersed 0·0002 ohm four-terminal standard with $\delta_3 = 0{\cdot}002$ radian; this is the only important correction term in the expression for W. Settings of R_4 could be repeated within 1 per cent and of M to a greater degree of precision; the whole measurement had an accuracy and sensitivity much superior to that which could be obtained by means of a wattmeter in combination with instrument transformers. It should be pointed out that tests on a synchronous generator are easily made by reversing the connections to P_1 and P_2.

When an earthed neutral point is not available in the apparatus under test, an artificial point is provided by three star-connected high resistors in series with any one of which the $M - R_4$ combination can be connected at will; the common point of the resistors is solidly earthed. The resistor R_3 is omitted, and the current in the three phases measured by three nickel-iron cored current transformers; a resistor of 1 ohm can be connected in the secondary of any one of the transformers. By these means the test circuit is provided with an earth point independently of the machine under investigation, and safety is assured.

56. Potthoff's and Dettmar's High Voltage Bridges. Potthoff* has described a bridge for testing cables and other h.v. insulators with one earthed terminal, a voltage transformer being employed to avoid the need for a special h.v. standard air condenser. The circuit is shown in Fig. 174 (*a*), C_1, ρ_1 being the condenser under test joined in series with the primary winding L_1, R_1 of a fixed value mutual inductor M_1 (76 mH.) specially

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* K. Potthoff, "Messung dielektrischer Verluste bei Drehstrom," *Elekt. Zeits.*, Vol. 52, pp. 474–477 (1931).

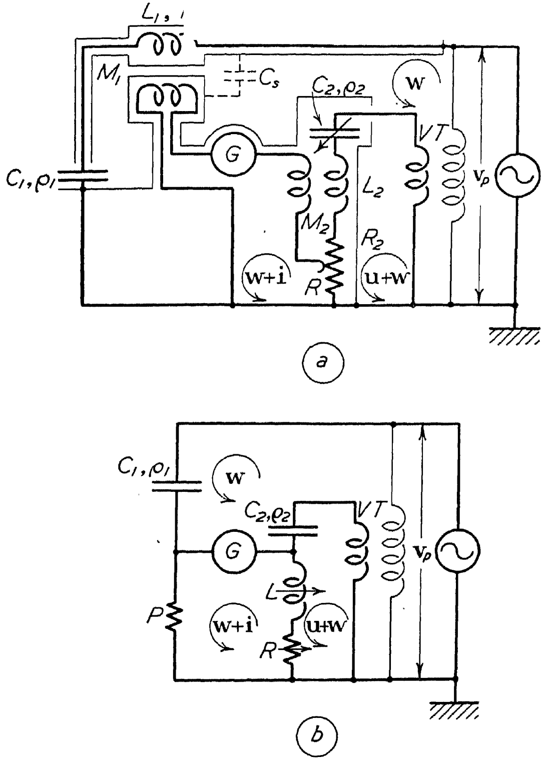

FIG. 174.—POTTHOFF'S AND DETTMAR'S HIGH-VOLTAGE BRIDGES

designed for direct connection to a 75 kV. supply. The primary is a coil inside a paper insulating tube, upon the outside of which the secondary is wound. The considerable capacitance current that would flow between the windings, one at a high voltage and the other earthed, is excluded from the measuring circuit by a system of shields. One shield surrounds the primary and is joined to the h.v. terminal of the supply; the other shield surrounds the secondary and test circuit, so that the current in the intershield capacitance C_s is simply a shunt across the supply. M_2 is a mutual inductance of 19·5 mH.; C_2 is a variable 3-decade mica condenser of loss resistance P_2.

Balance is obtained by variation of C_2 and R. Then if R_2 is the resistance of the winding L_2 together with the resistor upon which the sliding contact is moved, $\mathbf{v}_p$ the primary voltage of the voltage transformer VT (several kV.), $\mathbf{v}_s$ its secondary voltage (about 100 V.), K_v the ratio, and γ the phase-angle of the transformer, then when the detector current $\mathbf{i}$ is zero the following relations hold—

$$\left. R_1 + j\omega L_1 + \rho_1 + \frac{1}{j\omega C_1}\right)\mathbf{w} = \mathbf{v}_p,$$

$$\left(R_2 + j\omega L_2 + \rho_2 + \frac{1}{j\omega C_2}\right)\mathbf{u} = \mathbf{v}_s = -\frac{1}{K_v}\varepsilon^{j\gamma}\,\mathbf{v}_p,$$

$$j\omega M_1\mathbf{w} = (j\omega M_2 - R)\mathbf{u}.$$

Eliminating the currents and voltages gives, after some reduction,

$$-\frac{1}{K_v}\cdot\frac{M_2}{M_1}\cdot\frac{C_2}{C_1}\quad 1 + \frac{jR}{\omega M_2}\;[(1 - \omega^2 L_1 C_1) + j\omega C_1(R_1 + \rho_1)]$$
$$= [(1 - \omega^2 L_2 C_2) + j\omega C_2(R_2 + \rho_2)]$$

It is usually possible to make $\omega^2 L_1 C_1$ and $\omega^2 L_2 C_2$ small in comparison with unity. Also,

$$\varepsilon^{j\gamma} = \cos\gamma + j\sin\gamma \doteqdot 1 + j\tan\gamma,$$

since γ is small in a well-designed voltage transformer. Neglecting products of small terms,

$$-\frac{1}{K_v}\cdot\frac{M_2}{M_1}\cdot\frac{C_1}{C_2}\left\{1 + j\left[\frac{R}{\omega M_2} + \omega C_1(R_1 + \rho_1) + \tan\gamma\right]\right\}$$
$$\doteqdot 1 + j\omega C_2(R_2 + \rho_2)$$

Equating the real components shows that M_2 must be negative, since C_1 is essentially positive, and

$$C_1 \doteqdot \frac{1}{K_v}\cdot\frac{M_2}{M_1}C_2,$$

so that the l.v. standard condenser C_2 is equivalent to a h.v. condenser C_2/K_v. Now equating the quadrature terms, reversing the sign of M_2 and noting that $\omega C_1\rho_1 = \tan\theta_1$ and $\omega C_2\rho_2 = \tan\theta_2$, where θ_1, θ_2 are the loss-angles of C_1 and C_2 gives

$$\tan\theta_1 \doteqdot (R/\omega M_2) - \tan\gamma - \omega C_1 R_1 + \omega C_2 R_2 + \tan\theta_2.$$

The method has been applied by Potthoff to a wide range of measurements and adapted for use in three-phase circuits.

Dettmar's* method differs from the preceding in the use of a variable self-inductor and resistor to compare the currents in the l.v. standard condenser C_2, ρ_2 with that in the l.v. lead of the specimen C_1, ρ_1. It will be seen that the l.v. electrode of C_1 is not directly earthed. The method was particularly applied to tests on laid Hochstädter cables; the lower electrode is then the outermost layer of metallized paper and P is the resistance of the insulation between this and the sheath. When L and R have been regulated so that the detector current **i** is zero, the mesh equations become

$$\left(\rho_1 + \frac{1}{j\omega C_1} + P\right)\mathbf{w} =$$

$$\rho_2 + \quad j\omega C_2 \qquad R + j\omega L\Big)\mathbf{u} = \mathbf{v}_s = -\frac{\dot{}}{K_v}\,\varepsilon^{j\gamma}\,\mathbf{v}_p.$$

$$P\mathbf{w} = (R + j\omega L)\mathbf{u}.$$

Eliminating the currents and voltages makes

$$\frac{C_1}{C_2}\cdot\frac{P}{R}[(1-\omega^2 C_2 L) + j\omega C_2(R+\rho_2)]$$
$$= -\frac{1}{K_v}\,\varepsilon^{j\gamma}\left\{1 - \omega^2\frac{L}{R}C_1(P+\rho_1) + j\omega\left[\frac{L}{R} + C_1(P+\rho_1)\right]\right\}$$

As before, neglecting the products of small quantities,

$$\frac{C_1}{C_2}\cdot\frac{P}{R}[1 + j\omega C_2(R+\rho_2)] \doteqdot -\frac{1}{K_v}\left\{1 + j\left[\frac{\omega L}{R} + \omega C_1(P+\rho_1) + \tan\gamma\right]\right\}$$

Equating real terms, $\quad C_1P/C_2R \doteqdot -1/K_v$

Since the left side is essentially positive the negative sign should be changed to positive,† then

$$C_1 \doteqdot \frac{C_2}{K_v}\cdot\frac{R}{P}$$

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* W. Dettmar, "Brückenmessungen mit Spannungswandler," *Elekt. Zeits.*, Vol. 53, pp. 935–936 (1932). For another method, *see* E. Pugno-Vanoni, "Ponte per la misura delle perdite dielettriche regli isolatori passanti e nei condensatori ad alta tensione," *L'Elettro*, Vol. 17, pp. 37–39 (1930).

† This negative sign is a consequence of the conventional relation between the positive directions of the cyclic mesh currents and the instantaneous directions of the actual currents in the windings of the voltage transformer (*see Instrument Transformers*, p. 35). The practical significance in making a test by this method is to arrange that the unearthed terminals of the primary and secondary are instantaneously of the same polarity.

Equating the quadrature terms and making use of this result,

$$\begin{aligned}\tan\theta_1 &\fallingdotseq \omega C_2 R - \omega C_1 P + \tan\theta_2 - \tan\gamma - (\omega L/R)\\ &\fallingdotseq \omega C_2 R\left(1 - \frac{1}{K_v}\right) + \tan\theta_2 - \tan\gamma - (\omega L/R)\\ &\fallingdotseq \omega C_2 R + \tan\theta_2 - \tan\gamma - (\omega L/R)\end{aligned}$$

which expresses the loss-angle θ, without the need of a knowledge of P.

CHAPTER V

ON THE CHOICE OF A BRIDGE METHOD AND THE PRECAUTIONS TO BE OBSERVED WHEN USING IT

THE object of this chapter is to summarize for use in the laboratory the theoretical and practical information contained in preceding chapters. The subject will be treated under two headings, the first dealing with the choice of suitable apparatus and methods, the second with the precautions which must be observed in practice in order to obtain exact results.

THE CHOICE OF A BRIDGE METHOD

The choice of a method for a given measurement is affected by a number of circumstances, e.g. the available source and detector, the available standards, and the nature of the quantity to be measured. Each of these factors will be considered in turn.

1. The Source of Current. The most suitable source depends upon the nature of the test, the frequency at which it is to be carried out, and on the amount of power required. In general, there is little difficulty in obtaining an adequate amount of power, since only a small number of watts is required in most bridges.

For ordinary rough tests at frequencies of a few hundred cycles per second, the buzzer (p. 200) forms a very convenient source. The frequency is, however, somewhat unsteady and the wave form impure. If a more steady source be desired, several types are available. For a range between about 20 and 1,000 cycles per second, a maintained tuning fork (pp. 207 and 229) can be used. Up to about 500 cycles per second the string interrupter (p. 201) is suitable. The microphone hummer in one of its many forms can also be used; the vibrating bar type (p. 207) extending the range to about 3,000 cycles per second or higher.

All these devices when in action emit a loud note which, when telephones are used to detect balance, becomes a great nuisance to the experimenter listening for the feeble note in the telephones when balance is nearly secured. Such sources should, therefore, be placed at a considerable distance from the bridge, and are best enclosed in a sound-proof box so that the direct note cannot be heard.

Again, some of these sources of current have rather impure wave forms. When telephones are used in the bridge, it becomes very desirable in such cases to choose methods which balance independently of frequency, so that silence occurs for all harmonics of the wave simultaneously, and the balance point is not obscured by the persistence of noise due to overtones. Alternatively, a wave filter (p. 230) may be interposed between the source and the bridge to cut out the undesired harmonics.

For the lowest frequencies, up to about 200 cycles per second, especially when a fair amount of power is required, the most convenient source is an ordinary motor-driven alternator of pure wave form. If means are provided for maintaining constant speed (p. 212), it becomes possible to use low frequency vibration galvanometers in the bridge. Alternators are essential in tests at 50 cycles per second under works conditions, and also in cable testing or other dielectric loss measurements at high voltages. Though small alternators are sometimes used for work at higher frequencies up to the limit of the telephonic range, they are now practically superseded by the triode oscillator.

For all high-class work at frequencies above about 200 or 300 cycles per second the triode oscillator is the best source. It provides an adequate output at a perfectly steady frequency and with a nearly pure wave form. In combination with a vibration galvanometer, or, for the higher acoustic values, a good telephone, it is the source most suitable for precise tests. It is cheap, easy to set up and to maintain. Various types are described on pp. 221 *et seq.* For lower frequencies simple oscillators are not quite so advantageous on account of the large coils and condensers which are necessary; it is usual then to employ the beat-tone oscillator (p. 228).

Whichever source be chosen, it is advisable always to connect it to the bridge *via* a small transformer, p. 232. By providing an earthed shield between the two windings of the transformer, capacitance effects between the bridge and the source are greatly minimized, p. 537. By the choice of a suitable ratio of transformation the internal impedance of the source may be adapted to the impedance of the network and an increase in sensitiveness secured. It is well to arrange the source and its transformer at a distance from the bridge, so that any stray magnetic field therefrom may not upset the balance.

2. **The Detector.** For low frequencies the most convenient detector is the vibration galvanometer. For ordinary laboratory work at frequencies up to a few hundred cycles per second the moving coil type of instrument is usually employed and has a high sensitivity; this instrument is also applicable, even in a portable form, to the requirements of the works test-bed, *see* p. 268. In high voltage testing the moving magnet type of vibration galvanometer with remote controlled tuning possesses many advantages, and is widely used, *see* p. 256. For the higher acoustic values from above 800 cycles per second, the telephone is by far the best detector. The telephone should be chosen to suit the frequency of the test, the type which has a tunable diaphragm being preferable to the fixed pattern.

The vibration galvanometer and telephone have no phase selectivity, i.e. they do not show by their indication whether it is the reactance or the resistance adjustment of the bridge that requires alteration; it should be noted, however, that this defect is much less important than would at first sight appear. The a.c. galvanometer and the separately-excited dynamometer, pp. 247–248, are phase selective and have been used by several workers at low frequencies; the sensitivity of these instruments is high, but their manipulation is somewhat troublesome and they have not been generally adopted.

In many cases, especially in bridges for routine use, a pointer instrument is a great practical advantage, more especially if it can also be made phase selective. These instruments are now much used, and are usually moving coil milliammeters operated by some arrangement of mechanical or copper-oxide rectifiers, as shown on pp. 233 and 236. Alternatively, there are several thermionic detectors (p. 242) giving the same advantages. The copper-oxide rectifier and thermionic detectors are available for use up to very high frequencies.

Modern bridge technique employs the thermionic amplifier (p. 238) as a regular feature, one or more stages of amplification being interposed between the bridge and the detector, with a considerable consequent gain of sensitivity.

At frequencies above about 3,000 cycles per second and particularly at the highest acoustic, carrier-wave and radio frequencies the heterodyne or beat-tone detector is used, see p. 247.

With all detectors the impedance should be chosen to suit that of the bridge. An interbridge transformer is in this case, as with the source, a great advantage, since by adapting the effective impedance of the detector to the network a marked increase of sensitivity can be secured. Again, when telephones are used, precautions should be taken to eliminate capacitance effects between the observer and the telephones by the use of one of the devices described on p. 542. For practical details connected with the use of telephones and vibration galvanometers, *see* pp. 252 and 267.

3. Standards. The choice of a method will be largely guided by the standards which are available in the laboratory. A few remarks on standards in general will, however, assist the experimenter to determine which are the most suitable for a given purpose.

The resistances used in the bridge network should, for work of the highest accuracy at high frequencies, be wound in some way which shall ensure that they are free from residual effects, i.e. they should be as nearly non-reactive as possible. Dial decade boxes reading down to 1 ohm are most convenient (p. 106); the contacts should be good and located by a click, a great advantage when working in a darkened room with a vibration galvanometer, loose plugs being in such circumstances a great nuisance. Below 1 ohm a mercury rheostat or some form of slide wire rheostat is useful for fine adjustment (p. 124). In the case of the latter a little paraffin oil on the wire greatly improves the contact and the ease of setting. For rougher tests at low frequencies, ordinary resistance boxes with bifilar wound coils are frequently quite useful.

Speaking in a general way, the choice of other standards lies between self-inductances, mutual inductances, and condensers. Of these, undoubtedly the most convenient to use is a mutual inductometer. It is the most permanent, is easily and continuously adjusted from zero over a large range of positive or negative values, and is the one most free from corrections, except at high frequencies, when the effects of impurity become important. Suitable types of mutual inductor are described on pp. 147, 150, and 151. Self inductometers are of great service in certain methods, but do not possess the advantages of a good mutual inductometer. The best type is the

Brooks and Weaver* disc pattern described on p. 145. Fixed value self- and mutual-inductance standards are useful for comparison and substitution methods.

Of condensers, a good mica standard is convenient in many cases, but, for accurate work, its frequency and temperature variation must be known and also its phase angle (p. 193). It is worthy of note that at high frequencies a good mica condenser is better than a mutual inductor as a quadrature standard. As pointed out on p. 196, the capacitance of a mica condenser approaches a limit, called the geometric capacitance, while the phase-difference from true quadrature progressively decreases as the frequency is increased. In a mutual inductor, as discussed on pp. 153–161, the effects of impurity become much increased with a rise of frequency, resulting in considerable changes in mutual inductance and large phase defect from quadrature. Hence at high frequencies it is generally best to select bridges containing condenser standards rather than those containing mutual inductors, in order to avoid the necessity of making troublesome corrections.

Air condensers, whether fixed or variable, are almost entirely free from loss, and form nearly perfect quadrature standards. They are used alone in many bridges where their small capacitance is adequate; their construction and properties are explained on pp. 164 and 171. Variable air condensers are also often used as a continuous fine adjustment in conjunction with a decade mica condenser.

In high voltage bridges the standard is often a specially constructed and insulated air condenser with flat or cylindrical electrodes (p. 178). Their considerable bulk is a serious defect, overcome in practice by the use of the more compact compressed gas condenser (p. 182). The same advantage is gained by using a standard mica condenser in the l.v. circuit of a voltage transformer.

4. Self-inductance Measurements. Methods for the measurement of self-inductance are chosen with reference not only to the value of the inductance but to the resistance which accompanies it. For example, an inductance of a few microhenrys may be easily measured when the resistance is of moderate value, but quite special treatment is necessary if the resistance be very large—as in a standard resistance coil—or very small, as in a shunt. Accordingly, the time-constant, L/R, of the coil to be measured must be taken into account in deciding upon the bridge to be used.

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* H. B. Brooks and A. B. Lewis, "Improved continuously variable self- and mutual-inductor," *Bur. Stds. Journal of Res.*, Vol. 19, pp. 493–516 (1937) describes a design with several improvements.

4a. Measurement of Average Inductances. The number of ways of measuring the inductances of average value found in general laboratory testing is very large, but certain methods have outstanding features which recommend them for general use.

Probably the most convenient inductance bridge is the Heaviside-Campbell arrangement, in which a standard mutual inductometer is used (p. 437). The bridge is used preferably with equal ratios and the adjustments are very quickly and conveniently made. The method is particularly free from errors, and it is easy to allow for the leads connecting the test coil to the bridge by taking a preliminary balance with the coil removed. With a mutual inductometer reading up to 0·01 henry, inductances lying between a few microhenrys and 0·02 henry can be measured with considerable precision. The method is particularly valuable, therefore, for tests on coils of low inductance. By the use of a larger inductometer the range can be increased; for example, to 0·2 henry, while keeping all the advantages of an equal ratio bridge.

For larger inductances, Campbell's modifications of Maxwell's bridge on pp. 431 and 433 can be used. In these the ratios are not equal, hence the advantages of the Heaviside-Campbell arrangement—freedom from residual errors—are, in general, lost. These methods give the value of inductance with fair accuracy, but measurements of effective resistance of the coil under test may be considerably in error unless the appropriate corrections are applied.

When a suitable self inductometer is available, Maxwell's method, p. 309, is often very useful. Here, again, an equal ratio bridge should be arranged; hence the coil to be tested should not exceed the value of the available self-inductance standard. With proper arrangement, the method can be made very sensitive and is capable of precise settings.

When the laboratory contains a suitable set of good condensers, ranging, say, from 0·1 to 1 microfarad, Anderson's method forms a very useful and accurate method of measuring inductances over a wide range of values (p. 379). Rosa and Grover have shown (p. 380) that measurements can be made with high precision, since the errors to which the method is liable are, with proper arrangement, very small and easily allowed for. Inductances from a few millihenrys to 1 or 2 henrys are easily measured.

Another useful wide-range bridge in which standard condensers are used is that of Owen (p. 405). This is particularly quick for routine laboratory tests over a large range of inductance. For exact work, however, the method requires some correction for residual effects and for imperfections in the condensers.

4b. Measurement of Large Inductances. It is rather a difficult matter to measure a large inductance, say, of several henrys, especially if the time-constant is large and simultaneous measurement of the effective resistance of the coil is required. In most ordinary bridge methods, e.g. Campbell's methods of pp. 431 and 433, or Maxwell's method of p. 309, it is necessary to use unequal ratios to obtain a suitable range of inductance with available standards of reasonable value. Enormous errors are thereby frequently occasioned in the value of the effective resistance, though the value of the inductance may be obtained with fair accuracy. These defects are removed in Dye's modification of the Heaviside-Campbell equal ratio bridge described on p. 442.

In telephonic work, coils having time-constants lying between a fraction of a second and several seconds are much used, the inductances ranging from several millihenrys upwards. The effective resistance is thus small when compared with the large value of reactance, and special methods are necessary. Shackleton's modification of Maxwell's method, using a standard of self inductance, has been specially developed for such measurements (p. 320). If a good subdivided condenser can be procured, Hay's method of p. 389 and the Reichsanstalt bridge of p. 393 may be used with accurate results. The resonance bridge of p. 396 is also available. In all these methods, the frequency must be constant and known. Wirk's bridge of p. 394 avoids this condition. If tests are made at acoustic frequencies, with telephones to detect balance, a pure wave form is essential. It is important to shield the whole network to make the capacitance effects definite.

4c. Measurement of Iron Cored Coils with Superposed a.c. and d.c. Large inductances, such as are considered here, are usually provided with iron cores. The windings may carry direct current in addition to the a.c., as in telephonic loading coils or in some types of intervalve transformers. Since the superposed d.c. has an important effect on the magnetic properties of the iron core and, therefore, upon the a.c. characteristics, it is essential to test such apparatus when carrying both the d.c. and a.c. simultaneously, each adjusted to the proper strength. Hay's bridge modified in the way described independently by Hartshorn and Landon is particularly suitable for this purpose (p 391).*

4d. Measurement of Small Inductances. The measurement of the inductance of very small coils in which the resistance is not too high is a problem of some difficulty. If a low range mutual inductor is available the Heaviside-Campbell method, p. 437, is useful in general practice; Owen's bridge, p. 405, is also applicable. For coils of a few microhenrys, Butterworth's modification of Anderson's bridge, p. 387, is very valuable, as also is Starr's series resonance bridge (p. 397). In all cases it is necessary to keep residual errors in the bridge down to the minimum by careful arrangement of the apparatus and the application of the necessary corrections.

As the inductance becomes smaller in value, especially with increased values of resistance and consequent small time-constants, it is necessary to adopt one of the special methods described in the following section.

4e. Measurement of the Residual Inductance of Resistance Coils. The resistance coils used in a.c. bridge work are not entirely free from reactance, coils up to about 1,000 ohms having a slight residual inductance which must be taken into account in accurate work. It is necessary, therefore, to be able to measure the very small residual inductance in coils with time-constants of the order of 10^{-7} second or less.

The principle of most methods of measuring such small inductances is the same. The coil to be tested is compared against a standard resistance of approximately equal value, the inductance of which can be calculated. Such standard resistances of calculable inductance usually consist of a pair of parallel wires, or, for the low resistance units, of a wire bent into circular form, as described on page 118. In all the methods, full allowance must be made for any small inductances in the network, and earth capacitances must be corrected

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*Also see H. T. Wilhelm, *Bell Lab. Rec.*, Vol 14, pp. 131–135 (1935).

for. For this reason the bridges are usually worked on the "dummy" balance or substitution method.*

Prerauer† and Wien‡ used Maxwell's bridge of p. 309 to measure an inductance of 500×10^{-9} henry, comparing it with a 1 millihenry calculated standard by means of various auxiliary intermediate standards. The time-constant was 10^{-5} seconds and an accuracy of 1 per cent was secured. Giebe, by using a bifilar bridge network in which all residuals could be calculated, modified Maxwell's method to measure an inductance of 500×10^{-9} henry with a time constant of 10^{-8} second, as described on p. 321. He showed that by introducing relatively large inductances into the ratio branches the corrections due to residuals in the network can be made very small and are capable of experimental determination.

Curtis and Grover, in an extensive investigation of the residuals of resistance coils, have measured inductances of very low time constant by a substitution method with a calculable standard, using the bridges of pp. 316, 376, and 387. Other methods have been developed by Ogawa for the same purpose.§ Young has also introduced a special form of Heaviside-Campbell equal ratio bridge (p. 441).

4f. Measurement of the Inductance of Four-terminal Resistors. Four-terminal resistors, such as are used in the testing of instrument transformers and for other purposes in alternating current laboratory practice, are constructed in such a way as to be very nearly non-reactive. The measurement of the residual inductance is one of some difficulty since the inductance and also the resistance are both small; moreover, being four-terminal resistances with two current and two potential terminals, the measurement cannot be made in the ordinary Wheatstone type of bridge network. Several workers have successfully adapted the Kelvin double bridge for the purpose, the process described by Hartshorn (*see* p. 410) being one of the simplest, most direct, and involving the minimum of corrections; it is capable of high precision, and is suitable for all practical needs. The method requires a calculable four-terminal standard. A simple bridge due to Astbury (p. 412) is independent of such a standard. Arnold (p. 414) has given a direct comparison method for two resistors, using large currents and an auxiliary standard current transformer. Campbell has devised methods involving the use of mutual inductometers, described on p. 443. Other methods, not all using the null or bridge principle, have also been introduced, but these fall outside the scope of the present volume; the interested reader is referred to the *Dictionary of Applied Physics*, Vol. 2, p. 441–442, for further information.

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* For a summary of the various methods, see K. W. Kögler, "Fehlwinkelmessungen an Widerstände mit Nieder-und Tonfrequenz," *Arch. f. tech. Mess.*, V354–1 (Oct., 1935); J. Listray, *Bull. Assoc. Ing. Él. Liège*, Vol. 15, pp. 185–195 (1937).

† O. Prerauer, *Ann. der Phys.*, Vol. 53, pp. 772–784 (1894).

‡ Max Wien, *idem*, pp. 928–947 (1894).

§ K. Ogawa, "Residual Inductance of Resistance Coils," *Journal I.E.E., Japan*, No. 472, pp. 1220–1235, Nov. (1927).

4g. Special Application of Inductance Measurement. Inductance bridges have been given some interesting special applications. Batcheller (p. 398) has used a series resonance bridge to test the uniformity of welded joints by measuring the change of inductance of an iron-cored coil, the magnetic circuit of which is completed by the welded specimen. Faulty welds are readily detected.

Seletzky and Priday* have used the Maxwell bridge of p. 375 to measure the change of inductance of a disconnecting switch as the blade is gradually opened. From the rate of change of inductance with the position of the blade they compute the electro-magnetic force acting on it when carrying a given current and provide a simple experimental check on a formula given by Dwight.†

Laycock‡ makes use of the Heaviside-Campbell (p. 437) and Maxwell (p. 311) bridges to test the uniformity of cable lead sheaths during the extrusion process, by measuring the change in effective inductance of a system of coils placed near the sheath in a suitable holder which can be traversed along and rotated about the cable.

5. Effective Resistance Measurements. When an inductive coil is measured in a bridge network, the balance conditions involve simultaneously the effective self-inductance and resistance of the coil at the frequency of the test. Hence, if the effective inductance and resistance of all the other branches of the network be known that of the coil will be determined.

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* A. C. Seletzky and G. L. Priday, "Bridge measurement of electromagnetic forces," *Elec. Eng.*, Vol. 54, pp. 1149–1152 (1935).

† H. B. Dwight, *Trans. Amer. I.E.E.*, Vol. 39, pp. 1337–1335 (1920).

‡ W. E. Laycock, "An electrical method of determining cable sheath uniformity," *Journal I.E.E.*, Vol. 82, pp. 101–104 (1938).

In practice, the accurate determination of the inductance of a coil presents no serious complication, but it is often very difficult to get an exact measure of the effective resistance, especially at high frequencies, due to the presence of residual inductances and capacitances in the bridge network. For this reason, when effective resistance is to be measured, the utmost care must be taken to make allowance for residuals, since they have more effect on the resistance measurement than on the inductance. The network must be very carefully set up, the standards used having known or calculable residuals. This can be conveniently secured in practice by constructing the bridge on Giebe's bifilar principle, as described on p. 317.

Alternatively, if a coil of known effective resistance is procurable, effective resistance measurements are easily made by measuring this standard coil in the bridge and adjusting the branch residuals until the measured and known values agree at a given frequency. Any other coil can then be accurately measured by the bridge at the same frequency. This principle is very useful in connection with the bridges in which a standard mutual inductometer is used, and has been discussed on p. 437.

For coils of about 0·1 henry the resonance bridge of p. 397 or Hay's method of p. 389 are very useful, the former in particular being capable of considerable sensitiveness.

6. Mutual Inductance Measurements. Mutual inductances are most conveniently measured by comparison with a standard mutual inductometer.* Provided that the unknown does not exceed the standard, the comparison may be made directly by Felici's method (p. 414). The settings can be made with great precision, and the method is remarkably free from errors.

If the unknown is greater than the maximum reading of the inductometer, Maxwell's method (p. 419) is applicable, but the double adjustment for balance is often rather troublesome to effect in practice. Campbell's bridge (p. 422) avoids this difficulty and is applicable to a wide range of measurement.

Mutual inductance between two coils can always be measured as a self-inductance by measuring the self-inductance of the coils in series with the mutual effect assisting and then opposing, as on p. 424. Any suitable method for the measurement of self inductance is then available.

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* For a convenient summary see R. F. Field, "The measurement of mutual inductance," *G.R.Exp.*, Vol. 11, pp. 1–4 (1937).

If a standard condenser is obtainable, mutual inductance can be determined with precision by the modified Carey-Foster bridge (p. 457) or by Campbell's method of p. 465.

Mutual inductances are usually connected so that the primary and secondary windings have a common point, especially in high frequency networks where the effects of impurity are of considerable importance. Tests of such inductances must be made under the conditions of use; the bridges devised by Hartshorn, p. 426, have been specially developed for this purpose, and should always be used for the calibration of mutuals where impurity effects are to be determined.

7. Capacitance and Power-factor Measurements. The choice of method for the measurement of capacitance is governed by the magnitude of the condenser to be tested. With capacitances exceeding about 0·1 μF., the measurements are easy to carry out without special precautions. In testing small condensers, especially at high frequencies, full allowance must be made for the influence of small earth capacitances between the bridge and earth, by arranging suitable shielding and using the Wagner device described on p. 542.

The determination of a power-factor of a condenser involves the measurement of its equivalent shunt or series resistance in addition to its capacitance. Since the power-factor is usually low, the measurement of the equivalent resistance is a matter of some difficulty, owing to the importance of the errors introduced by residuals in the bridge network. As in the measurement of the effective resistance of a coil, considerable care is necessary when setting up the network to ensure that all residuals can be allowed for. If this is not done, the power factor may be widely in error, although the capacitance may be found with considerable precision.

Probably the best way to test the capacitance and equivalent series resistance of a condenser is by the modified Carey Foster method on p. 460, in which a standard mutual inductometer is used. This method is adopted at the National Physical Laboratory for condenser tests, and can be used over a wide range of measurements from a few micro-microfarads upwards. On small condensers and at high frequencies, allowance must be made for residual effects, as shown on p. 461, and it may become important to correct for earth capacitance effects, as discussed on p. 527. A further method using a standard of mutual inductance is the modification of the Heaviside-Campbell equal ratio bridge introduced by Campbell and incorporated in his wide-range capacitance bridge, p. 468.

In most other methods of testing condensers, the unknown is compared with a standard condenser the capacitance and equivalent resistance of which have been accurately determined by comparison with an air condenser and resistances. Curtis and Grover have made an extensive research on the properties of paper and mica condensers by the series resistance (p. 329) and parallel resistance (p. 335) bridges, the reader being referred to these pages for a summary of their work. High precision parallel resistance and Wien shielded bridges are described on pp. 338 and 349 respectively.

The Schering bridge (p. 352) has been shown by Giebe and Zickner to lend itself admirably to the testing of all types of condensers with the highest degree of precision (p. 367).

For the measurement of the direct intercapacitance between two electrodes there are two special methods, that due to Hoch (p. 342) and that of Zickner (p. 343). Both make use of a standard differential air condenser.

7a. Measurement of Small Capacitances. The Schering bridge has been adapted by Hartshorn (p. 368) for the measurement of the very small capacitances between the various parts of a thermionic tube; values below $1\mu\mu$F. can be found with good precision.

Another method for condensers less than $0{\cdot}01\mu$F. is Grover's bridge (p. 403) with inductive ratio arms.

7b. Measurement of Dielectric Losses and Permittivity in Insulating Materials. Hartshorn's form of Schering bridge (p. 369) is the best method for testing the dielectric loss and permittivity of sheet insulating materials. The British Electrical and Allied Industries Research Association has recommended the bridge for use under standard conditions (see B.S.I. Specification No. 234/1925, for example) and apparatus in accordance with their requirements is made by Messrs. H. Tinsley & Co. and by the Cambridge Instrument Co. It is important to provide suitable guard-ring electrodes between which to clamp the specimen, some details being given on p. 370 and in the references there cited.

For oil testing the Schering bridge has been suitably adapted by Balsbaugh and others (p. 370); attention has been given to the complete shielding of the bridge and to the design of containers for the oil sample.

7c. Measurement of Dielectric Losses at High Voltages. One of the most important measurements in practice is the determination of the dielectric losses in samples of insulating materials, in porcelain insulators, and in cables. In such tests the capacitances are not very large, and have to be measured at low frequencies, 50 cycles per second or lower, and at high voltages, up to several hundred kilovolts; the dielectric losses may be quite considerable. For this purpose the Wien two-condenser bridge of p. 344 has been widely used, in its simplest form by Hanauer and by Monasch (p. 346) or in modifications due to Rosen (p. 347), and to Atkinson (p. 348).

This bridge has now been superseded by the three-condenser method due to Schering, p. 352. The Schering bridge has great technical advantages over most other methods for the measurement of dielectric losses at high voltages; it is now regarded as the standard method by investigators all over the world. The bridge is easily adapted to measure small or large capacitances (p. 354), and the specimen may be earthed (p. 363) or not. It is widely used for routine testing of single-core and three-core cables during manufacture or after laying, and for works tests on insulators of every description.*

High-voltage bridges making use of a standard mutual inductor have been described by Dawes and Hoover and by Geyger; these methods (pp. 478–480) give excellent results but have not been widely adopted. A few bridges containing self inductances, e.g. Butman's adaptation of the series inductance bridge of Rosa and Grover (p. 403), have also been occasionally used.

All the methods cited require the use of a standard high-voltage condenser. This is avoided in the methods due to Potthoff (p. 488) and Dettmarr (p. 489) in which a standard mica condenser is coupled to the bridge through a voltage transformer of appropriate ratio.

7d. Measurement of the Residual Capacitance of Resistance Coils. Resistance coils over 1,000 ohms have usually a small residual self capacitance, which must be known in accurate work. Since the self capacitance of a coil can be represented by a condenser in parallel with the coil, a bridge which is suitable for the measurement of a small condenser in parallel with a relatively high resistance will serve also to measure the self capacitance of a resistance coil.

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* Additional information on the Schering bridge and comparison with other methods will be found in; Vassillière-Arlhac, "Méthodes de mesure des pertes dans les diéléctriques," *Bull. Soc. Franç. des Élecns.*, Vol. 1, pp. 1105–1124 (1931). R. F. Field, "An audio-frequency Schering bridge," *G.R.Exp.*, Vol. 11, pp. 1–5 (Dec., 1936). H. W. Bousman, "Insulation power factor," *Gen. Elec. Rev.*, Vol. 40, pp. 522–524 (1937). L. N. Bramley, "Schering bridge technique," *Dist. of Elec.*, Vol. 10, pp. 2363–2365, 2399–2402, 2418–2419 (1938).

The procedure is the same as for the measurement of residual inductance of low resistance coils ; the coil is compared by a method of substitution with a parallel wire standard, of which the residual can be calculated. Allowance must be made for earth capacitances, especially between the parallel wire standard and earth, the reader being referred to the papers of Wagner and Wertheimer, and of Curtis and Grover, for further details. (*See* pp. 334 and 321.) The Anderson bridge has been used by Taylor and Williams (p. 387) and in a modified form by Hay (p. 351). A particularly versatile form of Heaviside equal ratio bridge due to Young (p. 441) covers a range of resistors up to 10 megohms at frequencies up to 100 kilocycles per second. For tests on shielded resistors, where the shield potential must be maintained at a definite potential relative to the resistor, the modified parallel resistance bridge of Berberich (p. 340) is applicable.

7e. Measurements on Electrolytes. The measurement of the conductivity of an electrolyte is one of the greatest importance in physical chemistry, since it gives valuable information concerning the molecular electrical properties, such as degree of dissociation, of such conducting solutions. The measurement is also important in general chemistry, e.g. in the determination of titration end-points, basicity of acids, etc., and particularly as a control test in various branches of technical chemistry, such as sugar refining, tanning, food inspection, textile and paper making, etc. It is, moreover, a measurement frequently required in physiology, pathology, and bacteriology, as mentioned in the next section.

The earliest experiments on the conductivity of electrolytes are those of Kohlrausch and others, the electrolyte sample being treated as a resistance and an attempt made to measure it by means of a simple slide-wire bridge, using an induction coil as a source of current to minimize the effects of polarization together with a telephone to find the point of balance (*see* page 3 for example). It is not possible to secure silence in the telephone for the following reason.

An electrolytic cell is not a simple ohmic resistance, since it conducts by electrolytic dissociation. Moreover, the electrodes have an appreciable capacitance. Hence, it is impossible to balance an electrolytic cell against resistances alone. The cell behaves as if it were a resistance in combination with a small capacitance, and should be measured as such. By connecting it in a suitable a.c. bridge network and making the proper adjustments, it is possible to secure exact balance of the cell against standard resistances in combination with self or mutual inductances or condensers. This has been done by numerous workers, one of the earliest being Max Wien,[*] who used networks containing condensers and self inductances.

Extensive researches have been made by a variety of methods. Haworth has used a resonance bridge (p. 396) and a form of Heaviside equal ratio bridge (p. 466). In America important work has been done by Washburn and others,[†] using principally the series resonance bridge and either the series or parallel resistance capacitance bridges (pp. 329 and 335) with suitable shielding.

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[*] Max Wien, "Uber Widerstandsmessung mit Hilfe den Telephons," *Ann. der Phys.*, Vol. 47, p. 626 (1892). "Uber die Polarisation bei Wechselstromen," *Ann. der Phys.*, Vol. 58, p. 37 (1896).

[†] E. W. Washburn and J. E. Bell, "An improved apparatus for measuring the conductivity of electrolytes," *Journal Amer. Chem. Soc.*, Vol. 35, pp. 177–184 (1913). W. A. Taylor and S. F. Acree, "Studies in the measurement of electrical conductivity of solutions at different frequencies. V. Investigations on the use of the Vreeland oscillator and other sources of current for conductivity measurements," *ibid.*, Vol. 38, pp. 2396–2403 (1916); "VI. Investigations on bridge methods, resistances, cells, capacities, inductances, phase relations, precision of measurements, and a comparison of the resistances obtained by the use of inductance and capacity bridges," *ibid.*, Vol. 38, pp. 2403–2415 (1916). E. W. Washburn, "The measurement of electrolytic conductivity. I. The theory of the design of conductivity cells," *ibid.*, Vol. 38, pp. 2431–2460 (1916). E. W. Washburn and K. Parker, "The measurement of electrolytic conductivity. II. The telephone receiver as an indicating instrument for use with the alternating current bridge," *ibid.*, Vol. 39, pp. 235–245 (1917). L. H. Adams and R. E. Hall, "Application of the thermionic amplifier to conductivity measurements," *ibid.*, Vol. 41, pp. 1515–1525 (1919). J. L. R. Morgan and O. M. Lammert, "The design and use of conductance cells for non-aqueous solutions," *ibid.*, Vol. 45, pp. 1692–1705 (1923); "Factors influencing the accuracy of measurements of the electrical conductance of liquids and solutions. II. A discussion of the bridge assembly for the measurement of electrical conductance, with particular reference to the Vreeland oscillator as a source of current of constant frequency," *ibid.*, Vol. 48, pp. 1220–1233 (1926). G. Jones and R. C. Josephs, "The measurement of the conductance of electrolytes. I. An experimental and theoretical study of principles of design of the Wheatstone bridge for use with alternating currents and an improved form of direct reading alternating current bridge," *ibid.*, Vol. 50, pp. 1049–1092 (1928); G. Jones and G. M. Bollinger, "II. Improvements in the oscillator and detector," *ibid.*, Vol. 51, pp. 2407–2416 (1929). T. Shedlovsky, "A screened bridge for the measurement of electrolytic conductance. I. Theory of capacity errors. II. Description of the bridge," *ibid.*, Vol. 52, pp. 1793–1805 (1930). P. H. Dike, "A bridge for the measurement of the conductance of electrolytes," *Rev. Sci. Insts.*, Vol. 2, pp. 379–395 (1931) (Describes a Jones and Joseph's shielded bridge of Leeds and Northrup design). Many of these papers contain valuable bibliographies of other publications. A good summary is given by J. Krönert, "Wechselstrombrücken. Messbrücken zur Bestimmung der Leitfähigkeit von Flüssigkeiten," *Arch. f. tech. Mess.*, J.921–9 (Feb., 1933).

In addition to the conductivity the dielectric constant of conducting liquids is of considerable physical interest. This quantity can be found at once from the effective capacitance of an electrolytic cell; or it may form the subject of a special investigation, as, for example, in the work of Harris,* who used a variety of types of a.c. bridge.

An interesting technical application of these measurements is the apparatus due to Carsten and Walter† for the continuous testing of the moisture content in paper during manufacture. The paper passes over a metal plate with 132 holes, forming one electrode; the second electrode consists of 132 metal pins insulated in the holes and joined in parallel. This condenser forms one arm of an equal ratio, four-condenser bridge. The detector is an amplifier applying the out-of-balance current to a phase-selective rectifier and galvanometer, arranged to be sensitive to the capacitance adjustment of the bridge. Changes in the moisture content affects the capacitance balance and causes an indication on the galvanometer.

Large capacitances are frequently obtained in practice from electrolytic condensers, which require an auxilary d.c. forming voltage during use and testing. Suitable bridges are the series resistance method (p. 334), the parallel resistance method (p. 335), and the Schering bridge (p. 356).

7f. Medical and Biological Applications of a.c. Bridges. A further interesting application of a.c. bridge technique to a problem closely connected with the testing of conductivity and capacitance of electrolytes is in the investigation of the "psycho-galvanic reflex." If two electrodes are attached to two points on the body, say on the palm and back of one hand, it is found that large changes in the resistance and capacitance of the skin and subcutaneous tissues measured between the electrodes take place if a stimulus be applied to the subject. These changes are of the greatest physiological and psychological interest, and have been studied by several workers, notably by Gildemeister* and by Thouless. Both workers used a tuned-arm or resonance bridge.

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* H. Harris, "The measurement of the dielectric constants of liquids," *Journal Chem. Soc.*, Vol. 127, pp. 1049–1069 (1925).

† H. Carsten and C. H. Walter, "Feuchtigkeitsmessungen an Lufttrockenen Papierbahnen mit dem Siccometer," *Siemens Zeits.*, Vol. 11, pp. 267–272 (1931); *Elekt. Zeits.*, Vol. 54, p. 109 (1933).

Since 1791, when Galvani first showed that living matter possesses electrical properties, there have been few attempts to correlate these electrical properties with the pathological condition of the tissues. In 1881 Vigoreaux made measurements of the d.c. resistance of the body in relation to activity of the thyroid gland, but difficulties due to polarization caused the experiments to be abandoned until they were repeated with alternating current by Gildemeister and shown to be a valuable method of diagnosis in this and other biochemical conditions. Brazier† has used a shielded parallel-resistance capacitance bridge (p. 335) with a Wagner earthing device to measure the loss angle of the tissues at 9,000 cycles per second, and has shown a definite diagnostic relation to exist between this angle and the state of the thyroid gland in cases of thyrotoxicosis. These results have been further investigated by Horton‡ and others who show that there is a marked difference between the behaviour of the internal and external tissues; measurement of the impedance of the internal tissues offers considerable promise as a diagnostic indicator.

In addition to their application in medical practice, a.c. bridges are being increasingly used in general biological research for the investigation of tissue changes under various chemical treatments, of the structure of blood vessels, of heat dosage in diathermy, etc. For this purpose measurements are made over a wide range of frequency from the audio to the radio region (30 to 10^7 cycles per second), and the requirements have been met by Grover's method (p. 404) for comparing two condensers using inductive ratio arms. The equipment is fully shielded.§

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* M. Gildemeister, "Ueber elektrischen Widerstand, Kapazität und Polarisation der Haut," *Arch. f. d. ges. Physiologie*," Vol. 176 (1919). See Report of the British Association Meeting at Glasgow, 1928.

† M. A. B. Brazier, "A method for the investigation of the impedance of the human body to an alternating current," *Journal I.E.E.*, Vol. 73, pp. 204–209 (1933).

‡ J. W. Horton and A. C. van Ravenswaay, "Electrical impedance of the human body," *Journal F. Inst.*, Vol. 220, pp. 557–572 (1935). J. W. Horton, "Measurement of the impedance of the human body," *G. R. Exp.*, Vol. 10, pp. 1–4 (Feb. 1936). Also see *Endocrinology*, pp. 72–80 (Jan., 1936).

§ J. A. Stratton, "A high frequency bridge," *Journal Opt. Soc. Amer.*, Vol. 13, pp. 471–494 (1926). A. Hemingway and J. F. McClendon, "An a.c. Wheatstone bridge for audio and radio frequency measurements," *Physics*, Vol. 2, pp. 396–402 (1932). K. S. Cole and H. J. Curtis, "Wheatstone bridge and electrolytic resistor for impedance measurements over a wide frequency range," *Rev. Sci. Insts.*, Vol. 8, pp. 333–339 (1937).

8. Bridge Tests on the Characteristics of Telegraph and Telephone Cables. High-speed telegraph cables with continuous loading are now in use, and their testing during manufacture and after installation is an important matter. It is known that the transmission speed of such a loaded cable is limited by its attenuation, so that the performance can be determined in terms of its a.c. characteristics. Smith* has shown how the desired a.c. constants may be measured during the process of manufacture by using the Maxwell self-inductance bridge (p. 311) and the Heaviside equal-ratio bridge (p. 437) for inductance and effective resistance tests, while capacitance and leakance measurements are made by the parallel resistance and Wien bridges of pp. 335 and 344.

The analogous problem of measuring the transmission characteristics, i.e. the attenuation, wave-length constant, and characteristic impedance, for long telephone cables has been investigated by Rosen,† who has shown how to express the usual open and closed impedance measurements by simplified expressions directly in terms of impedance bridge settings. See also pp. 321, 335, and 349.

9. Localization of Faults on h.v. Cables. Urmston‡ has shown that the a.c. bridge forms an excellent method for the location of the more difficult kinds of faults on h.v. power cables. For this purpose the Wien bridge was used for capacitive impedances and the Heaviside equal-ratio method for inductive impedances, full details of the necessary measurements being given, together with charts enabling the fault to be localized from the test results. A portable Wien bridge has also been developed by Beck and Shotter (p. 335) for the same purpose.

10. Use of Bridge Methods in Geophysical Prospecting. An interesting application of bridge methods has been made to the localization of metalliferous deposits in the earth. In one process§ alternating current at 500 cycles per second is passed into the ground by distant electrodes. Three exploring electrodes, A, D, B, are set into the ground, D being midway between A and B, forming one of the detector branch points; the potential drops between the points AD and DB being compared by two bridge arms AC, CB, which are adjusted until a telephone between C and D is silent. These arms may each consist of a variable resistance and variable condenser; or one may be a variable resistance while the other is a fixed resistance shunted by a variable condenser. A traverse is taken along a series of parallel straight lines, from the results of which the systems of surface equipotential and equiphase lines may be plotted out; the buried metallic deposit can be localized by the distortion of these contours In another method instead of probes driven into the ground two flat coils may be placed at the positions A, B, the e.m.f.s induced in them being balanced by a condenser bridge as before. In yet another method,* originally used to locate unexploded bombs buried in ground that was to be tilled, a Felici mutual inductance bridge is used. Two primary coils, A and B, have nearly equal mutual inductances with respect to a secondary coil C, all being wound on a portable frame with their common axis vertical. In series with A and B is connected the primary of a variable mutual inductor *M* and a source of 500 cycle alternating current, its secondary being in series with C and a slide wire resistance joined at one end to a common point of the primary. A telephone joins the end of *M* and the slider, adjustments being made to balance the mutuals A and B to give silence. If now the portable coils are placed over a region where immediately beneath there is some buried metal object the field of the coils is distorted and the balance disturbed. All these methods and others described in the literature have been used with considerable success.

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* E. W. Smith, "Alternating current tests on high-speed telegraph cables," *Journal I.E.E.*, Vol. 68, pp. 447–474 (1930).

† A Rosen, "Measurements on long telephone lines by the 'open and closed' method," *Journal I.E.E.*, Vol. 68, pp. 499–503 (1930).

‡ J. Urmston, "The electrical high pressure testing of cables and the localization of faults," *Journal I.E.E.*, Vol. 69, pp. 983–1003 (1931).

§ A. B. B. Edge, "Geoelectric prospecting by a.c. bridge methods," *Nature*, Vol. 127, pp. 37–39 (1931). Also see "Principles and Practice of Geophysical Prospecting" (Imperial Geophysical Experimental Survey); D. C. Gall, "Some experiments relating to geophysical prospecting," *Journal Sci. Insts.*, Vol. 8, pp. 305–313 (1931); J. Bruckshaw, "Electrical methods of geophysical prospecting," *Journal I.E.E.*, Vol. 73, pp. 521–533 (1933).

11. Measurement of Small Movements. Bridge methods have been used by several workers to provide a record of small mechanical movements, e.g. in measurements of pressure and length changes, deformation of loaded structures, displacement of cutting tools, etc. Mershon† uses a magnetic method consisting of a hybrid coil arrangement like Fig. 146 with source and detector interchanged. One side of the bridge contains an iron-cored coil with a variable air gap which is changed by the movement to be measured; the other side contains a similar dummy coil for zero balancing. The detector is an oscillograph or other deflectional instrument, about 5 mm. on the scale corresponding with an air-gap change of 1/1000 in., a magnification of about 150. The apparatus is used for routine gauging of mass-produced details.

Other methods are electric in their fundamental principle, the small movement changing the capacitance of a special air condenser; the bridge unbalance is amplified and applied either to an oscillograph or to a rectifier and pointer instrument. Schulze and Zickner use for this purpose a type of Schering bridge (p. 371), as also does Nissen.‡

12. Frequency Measurements. In theory, any alternating current bridge which depends for balance upon the frequency of the source, may be used for the measurement of frequency. In practice, it is found that some of these bridges are more useful than others, owing to the greater ease with which they can be set up and balanced, and also to the range of frequency which can be measured by them.§

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* T. Theodorsen, "Instrument for detecting metallic bodies buried in the earth," *Journal F. Inst.*, Vol. 210, pp. 311–326 (1930).

† A. V. Mershon, "Electric micrometer," *Gen. Elec. Rev.*, Vol. 29, pp. 815–817 (1926); "Precision measurements of mechanical dimensions by electrical measuring devices," *ibid.*, Vol. 35, pp. 139–144 (1932).

‡ H. F. Nissen, "Aufzeichnung kleiner Kapazitätsänderungen mit Hilfe einer Brückenanordnung," *Arch. f. tech. Mess.*, V. 3531–2 (Jan., 1934).

§ A useful summary of methods is given by J. Krönert, "Frequenz Messbrücken," *Arch. f. tech. Mess.*, J. 921–8 (Aug., 1932).

The choice of a frequency bridge will depend upon the range of frequency that is to be measured, and upon the standards available for the assembly of the network. The bridge should consume little power and thus be as purely reactive as possible. Since any a.c. bridge requires, in general, two adjustments for balance, it is most convenient to select as frequency bridges those methods in which (*a*) one adjustment appears only in the first balance condition, and the other adjustment only in the second, i.e. in which there is no interference; and (*b*) only one of the two conditions of balance is a function of frequency. It is possible in such cases to ensure that the condition independent of frequency is satisfied once for all; balance for frequency is then obtainable by a single adjustment. Many quite useful bridges do not, however, satisfy both criteria.

The table on p. 515 gives particulars of the most useful bridges, including with those described in detail in Chapter IV certain new methods introduced by Sacerdote* and illustrated in Fig. 175.

The first bridge, Fig. 175 (*a*), is a generalized Maxwell method of p. 311; the balance conditions are

$$\omega^2 L_2 L_3 \doteq QR - SP \text{ and } L_1 S = L_3 Q + L_2 R.$$

Fixing L_1, L_2, L_3, Q, R and S the second condition can be satisfied independently of the first. The first condition then balances by P alone and gives

$$f^2 = (QR - SP)/4\pi^2 L_2 L_3.$$

With $L_1 = 0{\cdot}1$ H, $L_2 = 0{\cdot}01$ H, $L_3 = 0{\cdot}01$ H, $Q = 500$ ohms, $R = 500$ ohms and $S = 100$ ohms, $f = 7{,}960\sqrt{(1 - 4P10^{-4})}$ cycles per second, giving a range from 7,960 to 0 for a variation of P between zero and 2,500 ohms. Reducing R to 50 and S to 10 ohms gives

$$f = 796\sqrt{(1 - 4P10^{-3})}.$$

A capacitance variant suitable for high frequencies is shown in Fig. 175 (*b*), the balance conditions being

$$\omega^2 C_1 C_3 = 1/(QR - SP) \text{ and } S/C_1 = (Q/C_3) + (R/C_2),$$

which yields a single adjustment method if C_1, C_2, C_3, Q, R, and S are fixed to enable the second condition to be satisfied. The range of frequency runs from a lower limit, defined by $\omega_0{}^2 = 1/QRC_1C_3$ when $P = 0$, to infinity as P approaches the value QR/S.

four-condenser bridge shown in Fig. 175 (*c*) balances when

$$C_3 = 1 - (C_2 C_3/C_1 C_4) \text{ and } P/C_4 = (R/C_2) + (Q/C_3),$$

a single adjustment bridge. The frequency range is from

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[*] “Ponti per la misura della frequenza,” *Alta Freq.*, Vol. 3,

zero, when $C_1 = C_2C_3/C_4$, to an upper limit given by $\omega_\infty{}^2 = 1/QRC_2C_3$ as C_1 approaches infinity.

Finally, Fig. 175 (d) shows a generalized Maxwell bridge (p. 375) for which

$$\omega^2L_2CSR = (QR - SP) \text{ and } Q = \frac{1}{C}[L_1/R) - (L_2/S)].$$

This can be balanced by adjustment of P only. The frequency range is given by the upper limit $\omega_0{}^2 = Q/L_2CS$ for $P = 0$ and zero for $P = QR/S$.

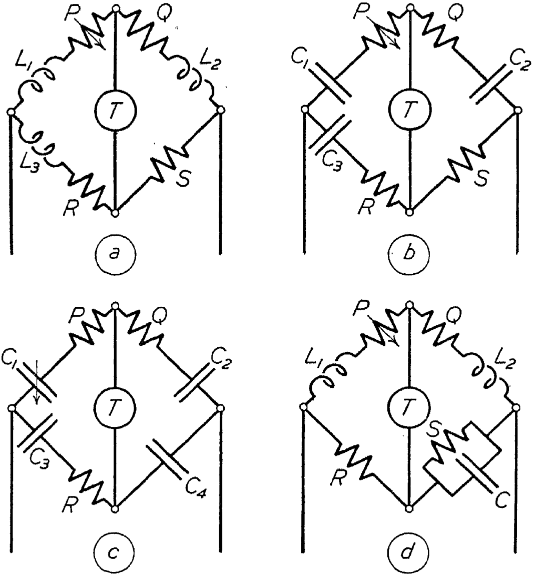

Fig. 175.—Frequency Bridges Due to Sacerdote

Classification. If x is the adjustment of the bridge entering into the frequency balance condition we can write for the *adjustment characteristic* of the bridge, f = function (x). The nature of the function depends on the arrangement of the network and involves explicitly the fixed values of the constants in some or all of the branches of the bridge. Reference to the table will show that there are eleven characteristics covering the frequency bridges used in practice.

(i) $f = ax$ is the linear frequency bridge.*

(ii) $f = a\sqrt{(x^2 - b^2)}$ gives a hyperbolic characteristic approaching the line ax as asymptote. The frequency is zero for $x = b$, and necessarily $x > b$. For large values of x the method is an approximate linear frequency bridge.

(iii) $f = a\sqrt{(x - b)}$ again necessitates $x > b$ and gives a parabolic characteristic.

(iv) $f = a\sqrt{(b - x)}$ which is again a parabolic curve with $x < b$. The whole frequency range is comprised between the values $a\sqrt{b}$ (for $x = 0$) and zero (for $x = b$). By contrast bridges (i), (ii), and (iii) have an infinite frequency range from zero. It does not follow, however, that the whole of this range is practically available; in inductive bridges the effect of eddy currents and in capacitance bridges the influence of earth capacitances may seriously curtail the available upper limit. These characteristics (i) to (iv) are shown for $a = b = 1$ in Fig. 176 (*a*).

(v) $f = a/x$ is a rectangular hyperbola. In the inverse form $T = x/a$; such bridges are linear period bridges.

(vi) $f = a/\sqrt{x}$ is a very common type of characteristic, found in all resonance bridges and in some others in which the square of the frequency is inversely proportional to the adjustment.

(vii) $f = a/\sqrt{(x + b)}$ is the preceding curve moved to the left by an amount b. In the only representative of this class x may become negative but cannot exceed $- b$.

(viii) $f = a/\sqrt{(b - x)}$ is characteristic (vii) reflected in the vertical axis. In the only bridge of this kind x can only be positive and cannot, therefore, exceed b. The frequency runs from $a/\sqrt{b}$ to infinity as x goes from 0 to b. In bridges of types (v) to (viii) the upper limit of frequency is infinite and zero frequency is unattainable, since in (v) to (vii) this corresponds with an infinite value of x. The curves are shown in Fig. 176 (*b*) for $a = b = 1$.

(ix) $f = a\sqrt{[(1/x) - b]}$ approaches the characteristic $a/\sqrt{x}$ when x is small, and gives infinite frequency for $x = 0$. It has the advantage over the resonance type of characteristic of giving zero frequency, when $x = 1/b$. Hence x must be less than b.

(x) $f = a\sqrt{[b - (1/x)]}$ gives zero frequency for $x = 1/b$; moreover x must always exceed b. As x increases, the curve becomes asymptotic to the constant frequency $a\sqrt{b}$.

(xi) $f = a\sqrt{\left[b - \frac{1}{x + (1/b)}\right]}$ is the preceding curve moved back to the origin, so that $f = 0$ for $x = 0$. The curves for (ix), (x), and (xi) are included also in Fig. 176 (*b*).

Because of its simple construction and adjustment the most commonly used bridge is No. 18; balance cannot be secured, however, unless the condenser is free from loss, although this defect can be overcome by somewhat complicating the network. The range is limited in the upward direction by the impurity effects in the mutual inductor, but when this is allowed for by Butterworth's method, No. 19, the bridge can be used with precision at high frequencies. For low frequencies the method is unsuitable on account of the large mutual inductor and

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* a and b are constants. x is usually positive, unless stated otherwise.

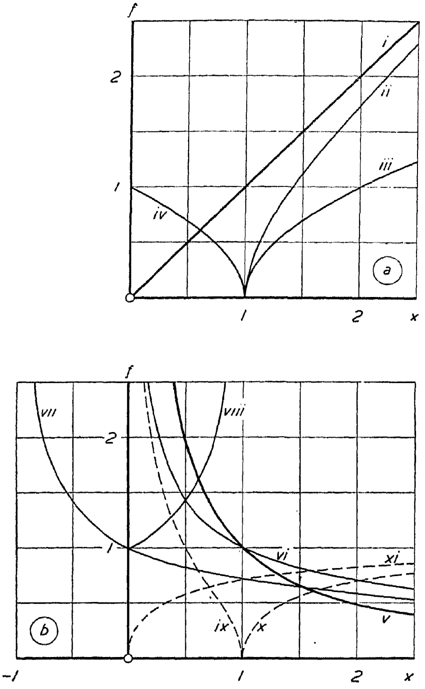

FIG. 176.—ADJUSTMENT CHARACTERISTICS FOR FREQUENCY BRIDGES

(i) $f = ax$. (ii) $f = a\sqrt{(x^2 - b^2)}$. (iii) $f = a\sqrt{(x - b)}$.
(iv) $f = a\sqrt{(b - x)}$. (v) $f = a/x$. (vi) $f = a/\sqrt{x}$.
(vii) $f = a/\sqrt{(x + b)}$. (viii) $f = a/\sqrt{(b - x)}$. (ix) $f = a\sqrt{[(1/x) - b]}$.
(x) $f = a\sqrt{[b - (1/x)]}$. (xi) $f = a\sqrt{\left[b - \dfrac{1}{x + (1/b)}\right]}$.

All plotted for $a = b = 1$.

condenser required, though this may be avoided by using one of Chiba's modifications, No. 20.

For general laboratory practice the most useful methods are Nos. 1 to 6, 10, 14, 15, and 21, all of these covering a long range

	Author	Page	Fig.	Network Contains	Balance Adjusted by	Frequency Relation	Range	Remarks
1	Kurokawa and Hoashi.	326	113	R and L	Two resistances	$f = a\sqrt{(x^2 - b^2)}$	Few cycles to 2,000	Skin effect limits range upward
2	Sacerdote	511	175*a*	R and L	One resistance	$f = a\sqrt{(b - x)}$	0 to 8,000	
3	Cone-Mathes, Kurokawa and Hoashi	350	115	R and C	Two resistances	$f = a/x$	Infinite period range	Better than preceding at high frequency.
3*a*	Robinson, Field	351	115	R and C	Three resistances Two resistances	$f = ax$	0 to 2,500 20 to 20,000	
4	Sacerdote	511	175*b*	R and C	One resistance	$f = a/\sqrt{(b - x)}$		
5	Sacerdote	511	175*c*	R and C	One resistance	$f = a\sqrt{[b - (1/x)]}$		
6	Kurokawa and Hoashi	392	133	R, L, and C	Two resistances	$f = ax$	25 to 5,000	
7	(Tuned arm)	398	137	R, L, and C	R and L	$f = a/\sqrt{x}$	300 to 2,000	
8	Sacerdote	512	175*d*	R, L, and C	One resistance	$f = a\sqrt{(b - x)}$		
9	Pegler	456	140*b*	R, L, and C	Two resistances	$f = ax$	Audio range	
10	Campbell	446	158	R, L, and M	M only	$f = a/\sqrt{x}$	18 to 4,000	
11*a* 11*b*	Butterworth, Dunand	450 451	160*a*	R, L, and M	One resistance Two resistances	$f = a\sqrt{(x - b)}$ $f = ax$	Infinite	
12	Campbell	451	160*b*	R, L, and M	One resistance	$f = a\sqrt{(x - b)}$	Infinite	
13	Hughes-Campbell	453	161	R, L, and M	R and M	$f = a\sqrt{[(1/x) - b]}$	10 to 120	Specially for low frequencies.
14	Kurokawa	455	162	R, L, and M	Three resistances varied simultaneously	$f = ax$	Few cycles to 5,000	Upward range limited by self capacitance of coils.
15	Sase and Mutô	455	163	R, L, and M	Two resistances simultaneously	$f = ax$	30 to 5,000	
16	Campbell	466	167*b*	R, L, M, and C	R and M	$f = a/\sqrt{x}$	Low frequencies	
17	Davies	470	168	R, L, M, and C	One resistance	$f = a\sqrt{\left[b - \frac{1}{x + (1/b)}\right]}$	Audio range	
18	Campbell	470	169	R, L, M, and C	M	$f = a/\sqrt{x}$	300 to 1,000	Requires perfect condenser and pure mutual.
19	Butterworth	474	192	R, L, M, and C	R, M, C		High telephonic frequencies	Compensates 18 for loss and impurity.
20	Chiba	558	193	R, L, M, and C	R and M	$f^2 = a/x$	Low frequencies	Modifies 18 for low frequencies.
21	Kennelly and Velander	475	170	R, L, M, and C	R or L and M	$f = a/\sqrt{(x + b)}$	400 to 3,200	Range easily extended lower or higher.
22	Schering and Engelhardt	483	172*c*	R, L, M, and C	Two resistances	$f = ax$	Audio range	

with considerable freedom from residual troubles and other errors, at the same time being rapid in adjustment and direct reading. For very low frequencies Nos. 1, 13, and 16 are particularly suitable.

For high frequencies it is usually best to use condenser frequency bridges in preference to those containing coils, since the quadrature errors of condensers diminish while those of coils increase, in consequence of the importance of self-capacitance and skin-effect, as the frequency increases. Thus, while the well-known tuned arm bridge No. 7 or the bridges Nos. 1 and 2 are useful within the limits specified their range is restricted in the upward direction by these secondary effects. For high frequencies Nos. 3 to 5 are very suitable, and No. 21 can readily be made so since it contains very small coils.

With very high frequencies a condenser bridge is convenient. The condensers should be air condensers, and one, at least, should be adjustable. Suitable shielding arrangements should be provided. For example, let a series resistance bridge be set up, as in Fig. 114, and balanced; then $P/R = Q/S = C_2/C_1$. Now without alteration of the rest of the bridge, let the resistance P be put in parallel with C_1, as in Wien's bridge in Fig. 115. Re-balancing by alteration of these to values P' and C'_1 gives $(C'_1/C_2) = (S/Q) - (R/P')$ and $C'_1 C_2 = 1/\omega^2 P'R$. From these and the former balance is found $\omega^2 = 1/\omega C_1 C'_1 PP'$. In practice, some trouble can be avoided by setting $P = 1/\omega C_1$ approximately; then $P' = 2P$ and $C'_1 = \frac{1}{2} C_1$. Hence, if balance be obtained for the series arrangement with this value of P, it will be possible to obtain balance in the parallel arrangement with the same adjustable condenser. It may be advantageous to replace the resistance ratios by condensers as in Fleming and Dykes bridge, Fig. 129.

12a. Special Uses of Frequency Bridges. When a frequency bridge is balanced, no current of the frequency expressed by the balance conditions can flow in the detector circuit. Hence, if the wave-form of the source be impure, the harmonic for which the network is adjusted will be totally suppressed from the detector circuit. A frequency bridge acts, therefore, as a wave-form sifter, suppressing any desired frequency component; such a use of a frequency bridge is often of service in dealing with one troublesome harmonic in the wave-form of a source of alternating current, *see* p. 231. The choice of a suitable sifter circuit may be made from the table just given; Campbell's method No. 18, being one of the simplest, is one of the most commonly used for this purpose, but it may be replaced by any other which enables the given frequency to be dealt with.

This filtering or sifting characteristic of a frequency bridge can be utilized as a harmonic analyser. If the bridge be set for the fundamental of a given voltage wave the detector current will be composed entirely of higher harmonics, and will be a measure of the harmonic residual of the applied voltage wave. Belfils and Chiodi have used this method to test the purity of wave-shape of alternating current machinery (p. 399). Wolff has employed it in the investigation of fidelity of response in loud speakers; both workers use the series resonance bridge, No. 7. The parallel resonance bridge, p. 403, has been used by Wagner and by Linckh for voltage wave analysis; and also by Greig and Parton* to investigate flux distortion in a.c. testing of iron.

13. Determination of the Ohm in Absolute Measure. The International ohm is defined as the resistance of a mercury column of constant cross-section, 106·300 cm. in length and 14·4521 grammes in mass. When this definition was prepared in 1908 it was intended that the International ohm should have a value nearly equal to 1 True ohm, i.e. to 10^9 c.g.s. electromagnetic units of resistance; measurements by absolute methods have established, however, that 1 International ohm is actually about 1·0005 True ohm. The standard method for determination of the ohm in absolute measure is the Lorenz apparatus, originally used at the National Physical Laboratory by F. E. Smith. A.c. bridge methods giving equal, and possibly even higher, precision have been used successfully at the Physikalisch-Technische Reichsanstalt (P.T.R.), the Bureau of Standards (B.S.) and the National Physical Laboratory (N.P.L.).

The bridge methods depend upon the use of an absolute standard of self or mutual inductance, i.e. one which can be calculated in true henrys (10^9 cm.) from its measured linear dimensions. Such an inductor is then compared in an a.c. bridge with resistance and time, enabling resistance to be expressed in True ohms (10^9 cm. per second). A simple Wheatstone bridge gives the value of resistance in International ohms.

The P.T.R. method used by Grüneisen and Giebe† in 1914 and described by them in 1920 uses a standard self-inductor (p. 129) wound on a porcelain former. The inductor is accurately measured in terms of length standards, and its true (or c.g.s.) inductance is computed from its dimensions. Using an intermediary capacitance, the ratio L/C is found in terms of resistance by a Maxwell bridge, Fig. 130 (p. 375). The capacitance is found in terms of resistance and time by a Maxwell charge and discharge bridge. From the two bridge measurements C is eliminated and L expressed in terms of International ohms and time. Since the ratio of the true to the International ohm is the same as the corresponding ratio for the henry, the International ohm has been found by these measurements in terms of the true ohm. The Bureau of Standards* has repeated this method with all modern precautions to ensure the highest precision. Three inductors were used, wound respectively upon porcelain, pyrex glass, and fused quartz. Time signals from the U.S. Naval Observatory were used to calibrate a piezo-electric oscillator, which in turn maintains a tuning fork contact make-and-break in the condenser bridge at a frequency of 100 cycles per second with a precision of at least 1 part in 10^6. Special precautions were taken in the construction and metrology of the coils, and specially accurate formulae for the computation of their inductance were developed. The final result is believed to be accurate within 20 parts in 10^6.

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* J. Greig and J. E. Parton, "Flux distortion in iron testing," *Engg.*, Vol. 144, pp. 439–441 (1937).

† E. Grüneisen and E. Giebe, "Eine neue Bestimmung der absoluten elektrischen Widerstandseinheit," *Ann. der Phys.*, Vol. 63, pp. 179–200 (1920).

The N.P.L. use a method due to Campbell in which two mutual inductors are employed, see Fig. 158 (p. 447). The experimental procedure has been sketched on p. 448. Hartshorn and Astbury† have made a very complete study of the method, allowing for all possible sources of error, and have shown that remarkably high precision can be obtained; their paper should be consulted for full details. The results are accurate to 15 parts in 10^6, and there is excellent agreement with the Lorenz method.‡

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* H. L. Curtis, C. Moon and C. M. Sparks, "An absolute determination of the ohm," *Bur. Stds. Journal of Res.*, Vol. 16, pp. 1–82 (1936).

† L. Hartshorn and N. F. Astbury, "The absolute measurement of resistance by the method of Albert Campbell," *Phil. Trans. Roy. Soc.*, Vol. 236, pp. 423–471 (1937). The mutual inductors are compared with Campbell's absolute mutual inductor, p. 139 and Fig. 53 (*d*). For details of a reconstruction of this instrument see N. F. Astbury, "The primary standard of mutual inductance of the National Physical Laboratory," *Phil. Mag.*, Vol. 25, pp. 290–317 (1938). Improvements in construction, metrology, calculation, and use indicate that this standard is now usable to an accuracy of 1 part in 10^6.

‡ For absolute methods useful for teaching purposes see H. R. Nettleton and E. G. Balls, "Two simple methods of absolute measurement of electrical resistance in terms inductance and frequency," *Proc. Phys. Soc.*, Vol. 45, pp. 545–554 (1933).

Closely related is the question of measuring self-inductance or capacitance in terms of frequency and resistance. The balance conditions for the Wien-Dolezalek bridge of Fig. 113 (p. 323) enable the ratio and the product of two self inductances to be expressed in terms of resistance and frequency, whence either inductance can be found. Another method, also due to Wien, is shown in Fig. 138 (pp. 400–401); it gives the ratio and product of a self inductance and a capacitance in a similar way. Other methods make use of two bridges, one to give the product and the other the ratio of the self inductance and capacitance. Boner uses the series resonance and parallel resonance bridges (p. 402). Ferguson uses the series resonance and Owen bridges for the same purpose.

14. Bridge Methods for Iron Loss Measurements. Bridge methods are of great value in the magnetic testing of iron and ferrous alloys, especially for the measurement of losses in alternating fields of low value where the usual wattmeter method presents considerable difficulty. Campbell devised an excellent method in 1910, described on p. 445, and later used it to investigate the properties of silicon-iron plates at low and at telephonic frequencies. By putting a condenser in series with the detector in Fig. 157 it is possible to superpose d.c. magnetization in the primary winding, and to test the combined effect of a.c. and d.c. fields upon the losses and the permeability. His method has been used by Webb and Ford* and also by Hughes,† who has made tests on nickel-iron alloys. Hughes has also used a simple Maxwell self-inductance bridge of Fig. 110 (p. 311); the same method has also been employed by Legg.‡ An important series of tests is recorded by Dannatt. Using a single strip specimen he was able to measure the loss by a simple adaptation of the *M*-*R* pair as an a.c. potentiometer.§ With ring specimens measurements of permeability and losses were made on four different materials over a frequency range from zero to 3×10^6 cycles per second, tests up to 50,000 cycles per second being made by bridge methods.‖ From 5,000 to 50,000 cycles per second the circuit was that of Fig. 135 (p. 393); up to 5,000 cycles per second the Maxwell bridge of Fig. 130 (p. 375) was used. In both cases suitable shielding and a Wagner earthing device are provided.

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* C. E. Webb and L. H. Ford, "Alternating current permeability and the bridge method of magnetic testing," *Journal I.E.E.*, Vol. 76, pp. 185–194 (1935).

† E. Hughes, "Magnetic characteristics of nickel-iron alloys with alternating magnetizing forces," *Journal I.E.E.*, Vol. 79, pp. 213–223 (1936).

‡ V. E. Legg, "Magnetic measurements at low flux densities using the alternating current bridge," *Bell Syst. Tech. J.*, Vol. 15, pp. 39–62 (1936).

§ C. Dannatt, "Energy testing of magnetic materials utilizing a single strip specimen," *Journal Sci. Inst.*, Vol. 10, pp. 276–285 (1933).

‖ C. Dannatt, "The variation of the magnetic properties of ferromagnetic laminae with frequency," *Journal I.E.E.*, Vol. 79, pp. 667–680 (1936).

The Maxwell bridge has also been used by Goldstein to measure the iron losses in transformers, see p. 376. Hohle* has adapted the same bridge as a substitute for the wattmeter in laboratory measurements using the Epstein square.

An interesting application of Grover's bridge of Fig. 140 (*a*) (p. 404) has been made by Sanford† in a magnetic comparator, in which the hysteresis loss in a standard bar and that in a test bar are compared as a criterion of relative hardness. The method has been used in particular for the inspection of tool-resisting prison bars.

15. A.C. Bridges in the Works Test. The alternating current bridge principle has been increasingly applied in measurements made at commercial frequencies of 25 to 60 cycles per second on the works test-bed; such measurements are usually employed in preference to wattmeter methods for the determination of power losses at very low power-factors. For the measurement of losses in apparatus such as large oil switches, where the volt drop is small while the current may be several thousand amperes, Churcher and Dannatt's use of the mutual inductance-resistance pair is applicable, Fig. 159 and p. 448. When the voltage is high and the current small, as in the testing of the dielectric losses in high-voltage cables and insulators under normal working conditions the Schering bridge, p. 352, is the best arrangement, and now forms part of the standard equipment in the high-voltage testing laboratories installed by manufacturers of cables, switchgear, porcelain insulators, and other high tension apparatus; the reader may refer in this connection to Section 7*c*. If the voltage is high and the current considerable, as in the case of synchronous condensers and other rotating machinery, Churcher has devised a special bridge, Fig. 173 and p. 484, which enables the losses to be measured with much higher precision than is otherwise possible. Using instrument transformers Kuriyama has adapted the Maxwell *M–L* bridge to measure the inductance of the windings of alternators and transformers and the losses in them, Fig. 151 and p. 432. There would appear to be a promising line of development in adapting the bridge technique to measurement problems arising in heavy electrical engineering.

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* W. Hohle, "Messung der Eisenverluste im Epsteinapparat mit der Wechselstrombrücke," *Arch. f. Elekt.*, Vol. 25, pp. 813–825 (1931).

† R. L. Sanford, "An alternating current magnetic comparator, and the testing of tool-resisting prison bars," *Bur. Stds. Journal of Res.*, Vol. 16, pp. 563–574 (1936).

15a. Bridge Measurement of Cable-sheath Losses. The measurement of additional losses in the lead sheath and armouring of power cables is difficult to perform satisfactorily by wattmeter methods, but has been very successfully carried out by appropriate bridge methods. The general principle is first to measure the d.c. resistance of the cable core, so that the loss in the cable without a.c. increments can be calculated for a given current; alternating current is then passed through the cable core and the effective resistance, which represents, in addition to the core-loss, the incremental skin effect losses in the core and the sheath and armour losses, is measured by an a.c. bridge method. Vogel* has used a type of a.c. Kelvin double bridge, and has made measurements under single-phase and three-phase conditions of working with single-core cables. Brockbank and Webb† have used a Maxwell bridge (Fig. 130 on page 375), in which L, P is the cable sample, Q a four-terminal low resistance about equal to P, capable of carrying the normal current of the cable, R a variable resistor up to 10,000 ohms, S a fixed resistor of about 4,000 ohms, and C a 2μF. variable condenser. The method works well provided the current carried by the cable is not too large, when it is necessary to use a water-cooled resistor for Q since the energy wasted in it becomes very considerable. This defect has been overcome by Potthoff,‡ who uses a resistor suitable for 5 amperes on the secondary of a current transformer, the primary of which occupies the arm CB. These papers give a useful bibliography of the important theoretical and experimental work relative to cable losses.

PRECAUTIONS

Assuming that a suitable method of measurement has been chosen and that the source and detector are given, there are several practical precautions which must be observed in obtaining accurate results. The general principle is to reduce the amount of experimental error to a minimum, and, in cases where this is not possible, to make allowance for errors unavoidably introduced.

General. Before connecting up the network, the necessary apparatus should be collected together and a diagram made to show the various instruments in their correct relative positions. By this means an orderly arrangement of the connecting wires can be made, and it is only by some such systematic procedure that residual errors can be reduced.

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* W. Vogel, "Magnetische Messungen an Einleiter-Hochspannungskabeln," *Elekt. Zeits.*, Vol. 48, pp. 1361–1363 (1927).

† R. A. Brockbank and J. K. Webb, "Sheath and armour losses in single-core cables for single-phase and three-phase transmission," *Journal I.E.E.*, Vol. 67, pp. 337–358 (1929). For a valuable bibliography *see* also O. R. Schurig, H. P. Kühni and F. H. Buller, "Losses in armoured single conductor lead-covered cables," *Trans. Amer. I.E.E.*, Vol. 48, pp. 417–435 (1929).

‡ K Potthoff, "Methode zur Bestimmung der zusätzlichen Verluste in den Bleimänteln und Bewehrungen von Einleiterkabeln," *Arch. f. Elekt.*, Vol. 23, pp. 441–446 (1930).

Leads. The leads should be carefully laid out in such a way that no loops or long lengths enclosing magnetic flux are produced, with consequent stray inductance errors. Twisted leads, such as bifilar flex, are often useful for bridge connections, but may introduce considerable residual capacitance. The correct arrangement of leads is a problem requiring careful judgment, according to the type of measurement which is being attempted.

In measuring an inductance, the leads connecting it to the bridge should be a metre or more in length so that the coil is well removed from the bridge. With a small inductance the leads should be close together so that their inductance is very small, and may be conveniently composed of twisted bifilar flex. With a large inductance the self capacitance of the leads is more important than their inductance, so they should be spaced relatively farther apart.

In measuring a condenser it is important to keep the lead capacitance as low as possible.* For this reason the leads should not be too close together, and should be made of fine wire; the wire should be bare, to eliminate the additional capacitance and dielectric loss that would be introduced by cotton or silk insulation.

In very precise inductance or capacitance measurements it is best to follow the practice, introduced by Giebe, of constructing a carefully balanced, symmetrical bifilar network with the leads encased in metal tubes to shield them from mutual electromagnetic action; or to design a completely shielded bridge in the manner discussed later.

Arrangement of Apparatus. The various resistance boxes, inductometers, etc., forming the network should be arranged so that all the necessary balancing adjustments may be made by the observer without changing his position relative to the bridge. High insulation resistance may be ensured by mounting the various pieces of apparatus upon blocks of paraffin wax; this is especially necessary in the case of high impedance bridges. In work at high frequencies or at high voltages the influence of capacitance effects between the bridge arms and from the entire network to earth must be allowed for by the provision of adequate shielding devices; this subject is more fully discussed in later sections of this chapter.

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* R. F. Field, "Connection errors in capacitance measurements," *G. R. Exp.*, Vol. 12, pp. 1–4 (Jan. 1938).

A matter requiring attention is the reduction of inductive interference between the various portions of the network to a negligible amount. For example, in a bridge containing two or three self-inductances the effect may be avoided by placing the coils at some little distance from the rest of the network with their axes lying in planes mutually at right angles; a similar arrangement is effective for mutual inductances. The problem is simplified by the use of astatically wound coils in inductance standards. One of the most troublesome instances of direct inductive action is that between the source, the inductive parts of the network and the detector. When the detector is a moving coil vibration galvanometer direct inductive action has usually a negligible influence; with a moving magnet galvanometer the effect may be much more troublesome, but is avoided in practice by enclosing the instrument in a laminated iron shield. The effect is most important, however, when a telephone is used. Direct influence of the source can be reduced by removing it to a considerable distance from the bridge, by astatic construction and by magnetic shielding. Direct action of the bridge network on the telephone cannot easily be eliminated by these means. The following process may be, therefore, adopted.* Balance the bridge as nearly as possible; then disconnect the telephone from the network and turn it into such a position as will cause it to become as silent as possible. The telephone must now be kept in this position, reconnected in the bridge and the balance amended. If a single receiver is used it is best to support it in a stand and to listen through stethoscope ear tubes. If held in the hand, an insulating handle should be provided; head sets should be well insulated, and the observer should turn himself into the position of least interference. The advantages of a loud speaker may be referred to, see p. 253.

Procedure. Having satisfactorily connected up the bridge network so that the leads are properly disposed, all contacts are good, and undesired mutual action between the separate branches is eliminated, the following systematic procedure may be advantageously adopted—

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* *See*, for example, D. A. Oliver, "The elimination of magnetic induction in the telephone detector in refined alternating current measurements," *Journal Sci. Insts.*, Vol. 3, pp. 122–123 (1926). If the effect is slight, the mean of two balances with the telephone reversed in the bridge will eliminate the effect.

(i) When possible, choose an equal ratio bridge. Residual errors are thereby rendered very much less important and can be allowed for with ease by taking two balances, the second with the ratios interchanged in the network. The average is the corrected value.

(ii) Make the two balancing adjustments successively, altering first one and then the other until balance is secured. Read all scales and observe temperatures and frequency. The frequency should be maintained constant and must always be recorded, even when using bridges in which the balance conditions do not explicitly involve the frequency. This is especially necessary when measurements of inductance and effective resistance are made on coils containing a large amount of wire, with consequent frequency effects due to self capacitance or to skin effect, or on coils provided with iron cores.

(iii) Reverse connections to the source to detect and, if necessary, to allow for capacitance effects and direct inductive action between the source and the bridge. Re-balance as in (ii).

(iv) Repeat the balances with the ratio branches reversed in the case when they are equal.

(v) When measuring an inductance, balancing should be repeated when the coil is turned through 180° so that inductive action of stray magnetic fields can be eliminated.

(vi) Make allowance for the leads connecting the apparatus to be measured into the bridge. In the case of inductance measurements, the leads should be short-circuited at the terminals of the coil and their inductance measured. When measuring a condenser the leads should be disconnected from the condenser terminals, taking care to disturb their position as little as possible, and a measurement of their capacitance obtained.

(vii) Most bridge errors can be eliminated by the process of substitution or dummy balance. The bridge is first balanced in the usual way. The apparatus under test is then removed, and is replaced by a standard as nearly equal to it as possible, so that balance can be restored by *slight* adjustments of one branch of the network.

16. Stray Field Effects in A.C. Bridges. In the theory of a.c. bridge networks it is usually assumed that the branches are composed of definitely located impedances, between some of which electric or magnetic coupling may have been intentionally arranged. In practice, however, there may be introduced unintentional electric and magnetic couplings between various components of the network, represented respectively by the leakage conductances and capacitances joining the branches to one another or connecting the branches to earth, and by stray magnetic fields. Consequently, although the branch balance conditions may be satisfied, current may still pass through the detector, and the true balance conditions must involve the undesired coupling effects; it is necessary, therefore, in precise work to have means by which the influences of stray electric and magnetic fields can .be reduced to a minimum, and their effect on the bridge balance made definite and calculable. It will be realized that the electric and magnetic fields are the components of the total electromagnetic field of the network and are, strictly speaking, not independent. It is justifiable to consider them independently in most practical instances, since one component or the other in specified parts of the circuit becomes of preponderant importance; for example, the stray magnetic field of a coil has more influence than its electric field, while the stray electric field of a condenser is of greater interest than its very small associated magnetic field. Hence we may distinguish separately between undesirable magnetic and electrostatic, or better electric, couplings, and consider the methods available for dealing with each of them. It is clear that both kinds of coupling can be reduced by separation of the component apparatus, but there are serious practical limitations to this treatment of the problem. Firstly, the increased length of connecting leads will increase the possibility of residual errors arising from the inductance and capacitance of the connections; secondly, while spacing-out of the apparatus has an important reducing effect on inter-branch mutual inductance and capacitance couplings it has practically no influence on earth capacitances. Since, as will be shown later, earth capacitances constitute one of the most troublesome of stray-field effects, especially at high frequencies or with high voltages, it is clear that some other solution must be found for the problem.

Magnetic Coupling. Magnetic coupling, or the mutual induction effect between various branches of a network, is only of importance in bridges containing two or more wound self or mutual inductances, especially if the source and detector are connected to the bridge through transformers. The effect can usually be made small by setting the coils at some little distance from each other with their planes mutually perpendicular; still better results will be obtained if the coils are astatically wound, or arranged with the windings in toroidal form, or provided with high permeability cores. The combined use of toroidal windings on permeable cores gives great freedom from stray fields, and is recommended for inter-bridge transformers; the principle has also been applied, using modern nickel-iron core materials, to standard inductances. In very precise work, even when these expedients are used, the small residual stray field may still be troublesome, but its effect is easily eliminated by the use of magnetic shielding; two principal methods are available. The stray field can be short-circuited by the use of a shield of high permeability material. Alternatively, air-core coils may be magnetically shielded by the action of eddy currents induced in a non-magnetic shield; such a shield usually consists of a copper box in which the coil is placed, and will be most effective at high frequencies. In general, there is usually little difficulty in reducing the magnetic coupling between inductive apparatus practically to zero by judicious arrangement of the circuit and the proper use of permeable or eddy-current shielding. It may be pointed out that such magnetic effects are almost entirely absent from condenser bridges, where they are limited to direct influence of the source upon the detector; the use of properly wound and magnetically shielded interbridge transformers removes the trouble here. It is usually best, unless an elaborate system of magnetic shields can be permitted, to limit the number of coils in the branches of the network to a single one, so that inter-branch undesired mutual effects are zero, and to use toroïdal, iron-cored, inter-bridge transformers.

Another type of magnetic interference is not uncommon in works testing, where other tests are being made at the same frequency as the bridge. For example, a load test on a large motor may produce a sufficiently strong field seriously to affect a Schering bridge test of the losses in a bushing, both being carried out at the standard frequency of 50 cycles per second; the interference may be especially troublesome when a tuned moving magnet type of vibration galvanometer is used. Two remedies are possible: either (i) enclose the galvanometer in a magnetic shield, preferably of nickel iron; or (ii) operate the bridge at a slightly different frequency, e.g. 52 cycles per second. The second is a simple and very effective way of eliminating trouble due to external magnetic fields.

Electric Coupling. While it is possible, by use of the artifices described, to reduce the magnetic interactions between parts of a bridge network to zero, it is never possible entirely to eliminate electric couplings, either between component pieces of apparatus or to surrounding earthed objects; all that can be done is to make their influence a minimum. Two principal methods are in general use for minimizing the influence of electric couplings on the bridge balance, namely, shielding or screening and the use of an earthing device; these will first be considered in general terms before discussing their practical details.

In a shielded network every piece of apparatus is enclosed in a separate metallic shield, which is connected to earth or to some suitable point in the network; the source and detector are electrically screened from the bridge by the use of shielded interbridge transformers, and some portion of the bridge is usually earthed. The purpose of the system of shielding is to make the inter- and earth-admittances definite in magnitude, independent of the position of the network or of the observer, and at the same time to modify the distribution and location of these admittances, so that their effect on the accuracy of the bridge balance is as small as possible. The electrically shielded bridge was introduced as long ago as 1904 by G. A. Campbell,* and has been brought to a high degree of perfection by other workers in the Bell Telephone Laboratories of New York, notably by J. G. Ferguson;† the process was adopted in 1911 by Giebe in precision bridge work at the Reichsanstalt, and is now a standard feature of a.c. bridge technique all over the world.

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* G. A. Campbell, "The shielded balance," *Elec. World,* Vol. 43, pp. 647-649 (1904). Also see *Elec. World,* Vol. 44, pp. 728–729 (1904).

† J. G. Ferguson, "Shielding in high frequency measurements," *Journal Amer. I.E.E.*, Vol. 48, pp. 517–521; *Bell Syst. Tech. J.*, Vol. 8, pp. 560–575 (1929); "Shielding for electric circuits," *Bell Lab. Rec.* Vol. 10, pp. 88–92 (1931). Useful information is also given by J. H. Morecroft and A. Turner, "The shielding of electric and magnetic fields," *Proc. Inst. Rad. Eng.*, Vol. 13, pp. 497–505 (1925), A. Rosen, *Journal Sci. Insts.*, Vol. 81, pp. 142–143 (1931); T. Walcher, "Elektrostatische Abschirmung von Wechselstrom-Messbrücken in Tonfrequenzbereich," *Arch. f. tech. Mess.*, J025–1 (Aug. 1936).

A completely shielded bridge is an instrument of high precision, and is almost entirely free from electric coupling errors; the presence of the shields, however, necessarily introduces considerable constructional and manipulative complications. Electric coupling effects can be very satisfactorily controlled in many cases by the use of an earthing device, such as the well-known artifice introduced in 1911 by Wagner; such a device is often combined with a simple system of shields, and constitutes an arrangement which can be made to give very precise results.

The electric coupling takes two forms, from branch to branch of the network and from each branch to earth; of these the second is of more importance than the first, since the inter-branch admittances can usually be made very small by adequate spacing of the apparatus and careful layout of the network. Consideration will, therefore, be restricted to earth admittances, each of which is compounded of a leakage conductance to earth, and a susceptance due to earth capacitance. By the use of high-grade insulators, such as ebonite, glass, or quartz, in the construction of the apparatus, the insulation resistance can be made very high, so that the leakage conductance can generally be neglected. The earth admittances are, therefore, almost entirely capacitive. In consequence of the presence of these earth capacitances the network consists not only of the visible branches, which appear in the usual balance conditions, but contains other branches, the earth capacitances connecting various points in the network to earth, and hence various parts of the bridge with each other. Although the earth capacitances are usually very small their effect may become important when their admittance is relatively large, as at high frequencies, or when parts of the bridge network are at high potentials relatively to earth; in both cases the earth capacitances may carry currents comparable with those flowing in the bridge arms, and cause profound errors. Unless these earth capacitances are allowed for, by shielding, by the use of a properly chosen earth point, or by a combination of these processes, the balance condition of bridges used at high frequencies or at high voltages may be seriously invalidated; indeed, without taking account of the earth capacitances by some such device it may not be possible to attain balance at all.

17. Electric Shielding of Bridge Elements. The general principles of shielding have been fully discussed in the papers cited on p. 527; the conclusions will be summarized here. In the case of a resistance or an inductance, shown in Fig. 177 (*a*), there is distributed capacitance to earth. The amount of this capacitance will depend upon the proximity of the impedance to earthed objects, and will vary with any change in the position of the piece of apparatus with respect to its surroundings.

Consequently the potentials of the terminals A, B of such an unshielded impedance are indefinite, the currents at the two terminals are different, and the effective impedance between the terminals is an uncertain quantity. If the impedance be enclosed in a metal shield, usually consisting of a closed thin copper box, the impedance will now have distributed capacitance

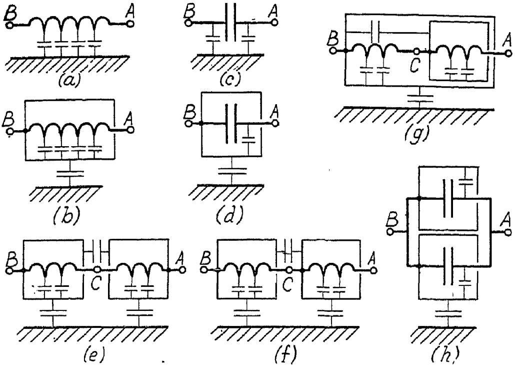

FIG. 177. ILLUSTRATING THE SHIELDING OF BRIDGE ELEMENTS

of fixed value to the shield only, while the shield itself has an indefinite capacitance to earth. Since the capacitance to shield is now a definite quantity, the terminal to terminal impedance of the enclosed element is also definite, and will be independent of the location of the shielded apparatus. Such a shield is termed a " floating " shield. If the shield be joined to one terminal of the apparatus, as in Fig. 177 (*b*), it is clear that all the earth capacitance current associated with the shielded element leaves the circuit at that point. If, in particular, this terminal can be brought to earth potential all indefinite earth capacitances are eliminated. A similar argument applies when the piece of apparatus is a condenser, as Fig. 177 (*c*) and (*d*) will show; here the effect of shielding is to add a constant amount, represented by the capacitance to shield of one set of the condenser plates, to the inter-electrode capacitance; the variable shield-earth capacitance is eliminated if B is earthed.

When two shielded impedances are joined in series, as in Fig. 177 (*e*), one shield may be joined to *A* and one to *B*. Then there will be a capacitance between the two shields, variable in amount with changes in the relative positions of the two pieces of apparatus. Again, no matter whether *A* or *B* is earthed, there will always be one shield-earth capacitance which will be of indefinite magnitude. If the shields be arranged as in Fig. 177 (*f*) the inter-shield capacitance will appear as a variable admittance in parallel with the impedance *BC*. Now if the shield of *B* is made large enough to enclose that of *A*, as in Fig. 177 (*g*), the inter-shield capacitance becomes fixed in value and shunts the impedance *BC*; there is now only one variable shield-earth capacitance, and the effect of this can be reduced to zero if *B* can be kept at earth potential. If *BC* is a condenser the capacitive shunt is merely a small addition to the inter-electrode capacitance. If *BC* is a resistance, the capacitive shunt will increase the residual phase-angle; while if *BC* is an inductance the shunt will increase the variation of effective inductance and resistance with frequency. This principle can be extended to include any number of elements in series, the effect being to place capacitive shunts across all the elements but one, while the only remaining indefinite capacitance effect can be removed by earthing the outermost shield. Decade resistors and inductors for use in precise work at high frequencies are usually provided with such multiple shields, examples being given by G. A. Campbell and by Ferguson in the places cited on p. 527.*

Impedances in parallel should be individually shielded, the shields being all connected to a common point, preferably to an earthed terminal. An example of two parallel-connected, shielded condensers is shown in Fig. 177 (*h*).

18. Theory of Earth Effects in Bridge Elements. It is important to work out the effect of earth admittance upon the properties of bridge elements, particularly inductors and resistors, whether shielded or not. Since the insulation of bridge elements is exceptionally good, the leakage conductance to earth is usually negligible; the earth admittance is, therefore, predominantly capacitive. A number of important practical cases will be considered here.

Unshielded Elements in Series. As a simple example consider the case of two unshielded elements, z_A and z_B, of negligible earth admittance, and let them be joined in series at a massive terminal *T* which has an earth impedance z_E as shown in Fig. 178 (*a*). Using the star-to-mesh transformation of Equation 11*a* on p. 58, construct the equivalent mesh shown in Fig. 178 (*b*), the components of which are*

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* See also R. Tamm, "Über die Genauigkeit von Wechselstrommessbrücken," *Zeits, f. tech. Phys.*, Vol. 14, pp. 472–474 (1933).

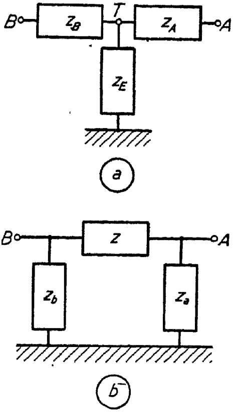

FIG. 178. UNSHIELDED ELEMENTS IN SERIES

$$z = z_A + z_B + (z_A z_B/z_E), z_a = z_E + z_A + (z_A z_E/z_B)$$

and $$z_b = z_E + z_B + (z_B z_E/z_A).$$

If the elements are two resistances $z_A = R_A$ and $z_B = R_B$ joined at a terminal of earth capacitance C_E, i.e. $z_E = 1/j\omega C_E$, then

$$z = R_A + R_B + j\omega C_E R_A R_B,$$

$$z_a = R_A + \left(1 + \frac{R_A}{R_B}\right) j\omega C_E$$

and $$z_b = R_B + \left(1 + \frac{R_B}{R_A}\right)\frac{}{j\omega C_E}.$$

Hence the effect of the earth capacitance is to add a residual inductance of amount $C_E R_A R_B$ in series with the total resistance $R_A + R_B$, and to earth the terminals A, B, through a series combination of resistance and capacitance. In particular, if

$$R_A = R_B = R/2 \text{ then } z = R + j\omega(C_E R^2/4)$$

and $z_a = z_b = (R/2) + (2/j\omega C_E)$; i.e. the residual inductance is $C_E R^2/4$ and the earth branches each consist of half the resistance and half the capacitance in series. In a numerical example $R_A = R_B = R/2 = 1000$ ohms and $C_E = 1\mu\mu$F. (approximately the capacitance of a sphere of 1 cm. radius); then the residual amounts to 1μH., and the time-constant of the resistor is 1/2000 microsecond.

Now let $z_A = R + j\omega L$ and $z_B = 1/j\omega C$, as in the tuned arm of a resonance bridge; then the effect of earth capacitance C_E at their junction is to make

$$z = R\left(1 + \frac{C_E}{C}\right) + j\omega L\left(1 + \frac{C_E}{C}\right) + \frac{1}{j\omega C}$$

for the inter-terminal impedance operator instead of $R + j\omega L + (1/j\omega C)$. Hence the resistance of the arm is increased by $C_E R/C$ and its inductance by $C_E L/C$.

Symmetrical Shielded Element. Consider a symmetrical impedance, e.g. a uniform resistor or inductor, enclosed within a shield which is assumed to be earthed, as in Fig. 179 (*a*). Let ρ, λ, γ, $\varkappa$ be the resistance, inductance, leakance, and earth capacitance per unit length of the impedance; R, L, G, and C being the total values. Suppose B to be maintained at a lower potential than A and let a sinusoidal potential difference be impressed between the terminals.* Then at any point distant x from B let $\mathbf{i}$ be the vector of current and $\mathbf{v}$ that of potential relative to the shield. The usual differential conditions are

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* S. Butterworth, "Notes on earth capacity effects in alternating current bridges," *Proc. Phys. Soc.*, Vol. 34, pp. 8–15 (1921).

$$\partial\mathbf{v}/\partial x = (\rho + j\omega\lambda)\,\mathbf{i} \text{ and } \partial\mathbf{i}/\partial x = (\gamma + j\omega\varkappa)\mathbf{v},$$

from which $$\partial^2\mathbf{v}/\partial x^2 = (\rho + j\omega\lambda)(\gamma + j\omega\varkappa)\mathbf{v} = a^2\mathbf{v}$$

where $$a = \sqrt{[(\rho + j\omega\lambda)(\gamma + j\omega\varkappa)]}$$

The solution for the distribution of potential and current is

$$\mathbf{v} = \mathbf{A}\cosh ax + \mathbf{B}\sinh ax,$$

$$\mathbf{i} = \frac{1}{z_C}[\mathbf{A}\sinh ax + \mathbf{B}\cosh ax],$$

where $\mathbf{A}$ and $\mathbf{B}$ are harmonic vectors derived from the terminal conditions

and $$z_C = \sqrt{[(R + j\omega L)/(G + j\omega C)]}$$

is the characteristic or surge impedance of the element.

With terminal B insulated the terminal conditions are $\mathbf{i} = 0$ when $x = 0$ and $\mathbf{v} = \mathbf{v}_A$ when $x = l$, leading to

$$\mathbf{v} = \mathbf{v}_A \cosh ax/\cosh al \text{ and } \mathbf{i} = (\mathbf{v}_A/z_C)(\sinh ax/\cosh al)$$

At terminal A, $x = l$ which gives

$$\mathbf{v} = \mathbf{v}_A \text{ and } \mathbf{i} = \mathbf{i}_A = \mathbf{v}_A/[z_c \coth al].$$

Defining the impedance with B insulated as the ratio of the p.d. between A and the shield to the current entering A under this condition,

$$z_i = z_c \coth al = z_c \coth p$$

where $$p = al = \sqrt{[(R + j\omega L)(G + j\omega C)]}.$$

With terminal B earthed, the conditions are $\mathbf{v} = 0$ when $x = 0$ and $\mathbf{v} = \mathbf{v}_A$ when $x = l$, giving

$$\mathbf{v} = \mathbf{v}_A \sinh ax/\sinh al \text{ and } \mathbf{i} = (\mathbf{v}_A/z_c)(\cosh ax/\sinh al).$$

At terminal A, $x = l$ and

$$\mathbf{v} = \mathbf{v}_A \text{ with } \mathbf{i} = \mathbf{i}_A = \mathbf{v}_A/[z_c \tanh al].$$

giving $$z_e = z_c \tanh p.$$

The shielded element is a three-terminal impedance and may be represented, therefore, with regard to the currents and voltages *at its terminals* by an arrangement of three lumped impedances. Since the element has been assumed to be symmetrically disposed within the shield, so that z_i and z_e are the same when measured at B as they are at A, it follows that the equivalent network must also be symmetrical and will take either of the forms shown in Figs. 179 (*b*) or (*c*). These are the well-known T and Π equivalents used in transmission line theory. In Fig. 179(*b*) insulating and earthing B gives the two relations.

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* Current passing to earth from the shield terminal S will, of course, return to the network which contains the element *via* the earth admittances of other portions of the network.

$$z_T + z_S = z_c \coth p \text{ and } z_T + [z_T z_S/(z_T + z_S)] = z_c \tanh p;$$

from which it is easy to deduce that

$$z_T = z_c \tanh (p/2) \text{ and } z_S = z_c/\sinh p.$$

Converting the star into the more useful mesh of Fig. 179 (*c*) by the theorem on p. 58 gives

$$z_\pi = z_c \sinh p \text{ and } z_V = z_c \coth (p/2).$$

When p is small enough we may write

$$z_\pi = z_c \sinh p \doteqdot z_c\left(p + \frac{p^3}{6}\right)$$

$$\doteqdot [R - \tfrac{1}{3}\omega^2 LCR + \tfrac{1}{6}G(R^2 - \omega^2 L^2)] + j\omega\,[L + \tfrac{1}{3}\,LRG + \tfrac{1}{6}C(R^2 - \omega^2 L^2)]$$

for the general case. The most important example is that of a shielded resistor in which G is negligible, and both L and C are residual quantities small enough to justify neglect of their squares and products. In such a case

$$z_\pi \doteqdot R + j\omega[L + (CR^2/6)];$$

i.e. the distributed capacitance increases the residual inductance of a resistor by $CR^2/6$; compare the effect of centrally-situated earth capacitance given on p. 531.

Unsymmetrical Shielded Element. The preceding example covers the case of a single resistor or inductor with a shield, all the parameters being uniformly distributed and the element symmetrical with respect to its terminals. In practice, however, two or more such shielded elements may be joined in series to form one arm of a bridge, as in Fig. 177 (*e*), (*f*), and (*g*); the commonest case is that of a decade resistor, each shielded decade of which forms a symmetrical shielded unit. In such cases imagine each element replaced by its equivalent Π as in Fig. 180 (*a*); which is, in turn, equivalent to Fig. 180 (*b*) by combining the earth admittances y_3 and y_4 at the common terminal. Now convert the star of impedances z_1, z_2 and $1/(y_3 + y_4)$ into its equivalent mesh, and combine the earth admittances with those at A and B in Fig. 180 (*b*). The result is shown symbolically in Fig. 180 (*c*), which is an unsymmetrical Π circuit identical with Fig. 178 (*b*).

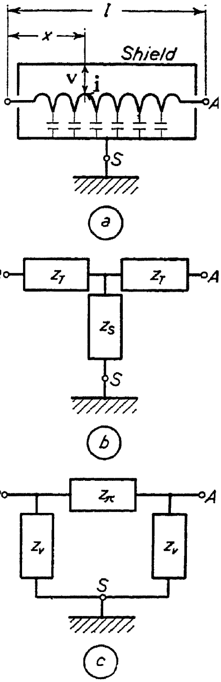

FIG. 179. SYMMETRICAL SHIELDED ELEMENT

The process can obviously be extended* to any number of elements and leads to an important conclusion: any combination of bridge elements is equivalent to an impedance joining the two ultimate terminals of the combination, together with dissimilar earth admittances connecting those terminals to earth. In other words, earth admittance effects in a bridge arm are in general equivalent to unequal lumped earth admittances at the terminals of the arm, a theorem of great practical importance.

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* As discussed by A. E. Kennelly, *The Application of Hyperbolic Functions to Electrical Engineering Problems*, pp. 159–161 (Univ. of London Press, 1912).

Impedances of a Shielded Element. The impedance of any element of a circuit is usually defined as the ratio of the voltage across the element to the current through it, but this definition is ambiguous when the element has distributed earth capacitance, since the current is not the same at all points in the element. Consider a shielded impedance, as in Fig. 181 (*a*), and suppose the shield to be earthed, as is very common in practice. The current entering at *B* and that leaving at *A* will differ by the amount of the shield current which flows to earth at *S* and returns to the network at some other earthed point such as *P*. The currents $\mathbf{i}_A$ and $\mathbf{i}_B$ depend not only upon the potential difference $\mathbf{v}_A - \mathbf{v}_B$ between the terminals but also on the potentials $\mathbf{v}_A$ and $\mathbf{v}_B$ of the terminals, since these determine the magnitude of the shield current $\mathbf{i}_S$. Consequently, the impedance of the element can only have a definite value if some convention be adopted which imposes a definite potential upon either *A* or *B*;* such conditions are chosen as are usually realized in bridge circuits.

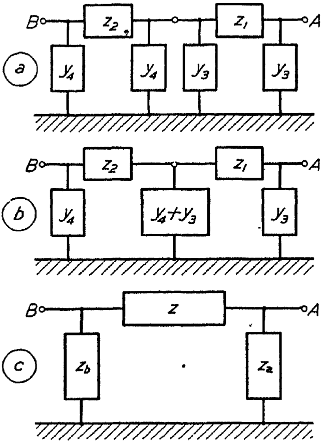

FIG. 180. SHIELDED ELEMENTS IN SERIES

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* L. Hartshorn and R. M. Wilmotte, "Note on shielded non-inductive resistances," *Journal Sci. Insts.*, Vol. 4, pp. 33–37 (1926). L. Hartshorn, "Standards of phase angle," *World Power*, Vol. 8, pp. 171–180, 234–240 (1927). R. M. Wilmotte, "A general theorem on screened impedances," *Phil. Mag.*, Vol. 6, pp. 788–795 (1928).

(i) Let A be maintained at earth potential but not necessarily directly earthed. Then $\mathbf{i}_B = \mathbf{i}_A + \mathbf{i}_S$. If $\mathbf{v}$ is the p.d. between A and B there are two possible values of the impedance, according as $\mathbf{i}_A$ or $\mathbf{i}_B$ is used, the former omitting and the latter including the shield current. These are

$$z_{AO} = \mathbf{v}/\mathbf{i}_A \text{ and } z_{AS} = \mathbf{v}/\mathbf{i}_B = \mathbf{v}/(\mathbf{i}_A + \mathbf{i}_S).$$

(ii) Now maintain B at earth potential and apply the same p.d. between the terminals. Since the impedance is, in general, unsymmetrical the current distribution will be different and we can write $\mathbf{i}_A' = \mathbf{i}_B' + \mathbf{i}_S'$. The impedances are now

$$z_{BO} = \mathbf{v}/\mathbf{i}_B' \text{ and } z_{BS} = \mathbf{v}/\mathbf{i}_A' = \mathbf{v}/(\mathbf{i}_B' + \mathbf{i}_S').$$

There are, apparently, four impedances but it is easy to prove that two of them are identical. In Fig. 181 (*b*) the equivalent T network is shown, A being at earth potential. The shield current is $\mathbf{i}_S = z_A\mathbf{i}_A/z_S$ and

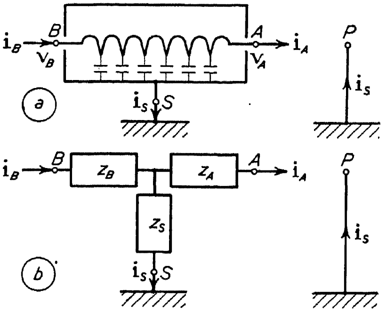

FIG. 181. THE IMPEDANCES OF A SHIELDED ELEMENT

$$\mathbf{i}_B = \mathbf{i}_A + \mathbf{i}_S = \mathbf{i}_A\,[1 + (z_A/z_S)].$$

Then $$z_A\,\mathbf{i}_A + z_B\,\mathbf{i}_B = \mathbf{i}_A\,[z_A + z_B + (z_A z_B/z_S)]$$

and $$z_{AO} = \mathbf{v}/\mathbf{i}_A = z_A + z_B + (z_A z_B/z_S).$$

This expression is symmetrical in z_A and z_B, and therefore $z_{AO} = z_{BO} = z_O$ say.

Hence a shielded impedance, such as a resistor, has three values when one of its terminals is maintained at earth potential, namely, z_{AS} and z_{BS} when the current considered includes the shield current and z_O when it does not. The values z_{AS}, z_{BS} are in practice equivalent to the impedances when either A or B is joined to the shield and the current is measured at the opposite end. In high resistors used for bridge work the three impedances should be known and the correct one used according to circumstances; examples will be given on p. 546.

19. The Principles of Shielding in Bridge Networks. The object of shielding is to make the earth admittances of bridge elements definite in amount and situation, so that any error they may introduce is known. In combining elements to form a bridge network the fundamental principle is to arrange the shielding in such a way that any residual admittance is put into such a position that it may either be balanced out or make a negligible error in the measurements.

When pieces of shielded apparatus are combined to form a network it is usually impossible to earth all the shields; in such cases it is necessary to determine the effect of the shield-earth capacitances on the action of the network. Consider first a bridge network of four shielded impedances with the source and detector omitted as in Fig. 182 (*a*); the total resultant earth capacitance of the shields has the effect of linking *A* to *B* via the earth, and clearly cannot affect the bridge balance. If source and detector are added, as in Fig. 182 (*b*), these details will have distributed earth capacitance; in the one case from the windings of the source to earth, in the other from the detector, which is usually a telephone in high frequency work, to the earthed observer; the earth capacitance effects are no longer confined between *A* and *B*. If source and detector are shielded and the shields are joined to *A* and *D* respectively, variable shield-earth capacitances are introduced linking *A* to *B* due to the source shield, and joining *D* to *A* and to *B* in consequence of the detector shield. The former has no effect on the balance; the influence of the latter can be minimized by the use of a second shield joined to *A*, which causes the capacitance effect to be reduced to a shunt across *AD*. If in addition the corner *A* is earthed the total effect of the earth admittances is reduced to a definite shunt across *AD* and a capacitance from *B* to earth.

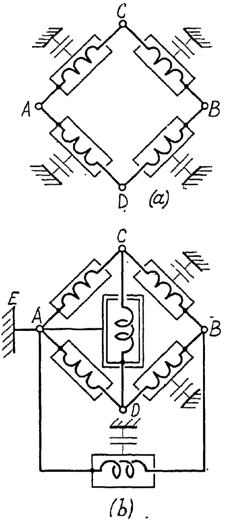

FIG. 182. ILLUSTRATING THE SHIELDING OF A BRIDGE NETWORK

It is assumed in this discussion that all four impedances are shielded and that one branch point is earthed. Frequently in practice the impedance to be measured is not shielded, and must be tested with its terminals at specified potentials relative to earth. The system described will not succeed in such general cases in removing the effects of earth capacitance; each set of specified conditions must then be considered on its own merits. Examples will be given in Section 25.

20. Earth Admittance in Interbridge Transformers. It is impracticable to shield the actual source and the detector; these are, therefore, connected to the bridge branch points through interbridge transformers of appropriate ratio (see p. 232) to which suitable shields are applied. An ideal transformer would have no distributed earth capacitance either on its primary or its secondary winding and no intercapacitance between its windings. In actual cases all these undesirable capacitances are present, and the system of shielding is designed to reduce, and if possible to eliminate, any error they may introduce into the measurements. When substitution methods are used the effects of stray capacitances generally cancel out, but it is desirable even then to keep them as small as possible in order to reduce second order corrections and to remove any practical difficulties in securing balance.* The type of shielded transformer to be used will depend upon the potential conditions imposed at the bridge branch points. In practice it is very common for one branch point to be maintained at earth potential, either by direct earthing or otherwise; we shall confine the present discussion to this case, but other examples will be referred to in Section 25.

Secondary Capacitance. Consider a bridge network consisting of impedances z_1, z_2, z_3, z_4 earthed at the branch point A in Fig. 183 (a); the shields of the impedances are omitted and would be arranged as in Fig. 182. Apply a transformer to the points C, D to link the network either with source or detector, and suppose the distributed earth capacitance of the secondary or bridge winding together with its connecting leads to be represented by terminal capacitances C_C, C_D joining the branch points to earth; primary earth capacitance and interwinding capacitances are for the present ignored. Capacitances C_C and C_D shunt z_1 and z_4 respectively and will cause an error which must be eliminated. When the bridge is unsymmetrical, i.e. when z_1, z_2, z_3 and z_4 are in the general case unequal and dissimilar, the Wagner earthing device described in Section 22 is a complete solution. When the bridge is symmetrical the use of an earthing condenser is permissible as is shown below.

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* M. Reed, "Unbalance in circuits," *Phil. Mag.*, Vol. 8, pp. 341–353 (1929). T. Walcher, "Über die Verwendung von Ein-und Ausgangstransformatoren bei genauen Brückenmessungen," *E.u.M.*, Vol. 51, pp. 397–401 (1933). R. F. Field, "A shielded transformer for bridge circuit use," *G.R.Exp.*, Vol. 8, pp. 5–8 (Apr. 1934). "Shielded transformers for impedance bridges," *ibid.*, Vol. 10, pp. 1–4 (Oct. 1935). W. G. Webster, "Shielded transformers for balanced circuit measurements," *ibid.*, Vol. 12, pp. 6–7 (Jan. 1938).

In many practical bridges two arms are equal, similar impedances; these are the so-called "ratio arms" and are usually either equal resistances or equal fixed condensers. Let $z_2 = z_3$ be the fixed arms; then $z_1 = z_4$, one being the unknown (say z_1) and the other a similar type of standard. Suppose $C_C > C_D$ and earth D through a condenser C_E. Then when the bridge is balanced

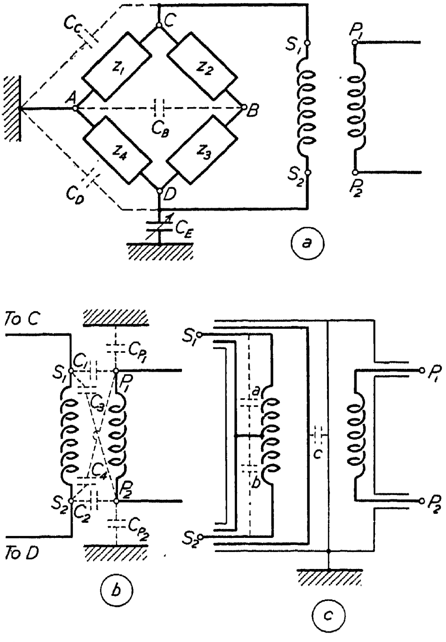

FIG. 183. CAPACITANCE EFFECTS IN INTERBRIDGE TRANSFORMERS

$$(1/z_1) + j\omega C_C = (1/z_4) + jw\,(C_D + C_E).$$

If $z_1 = z_4$ is the correct balance condition, then

$$C_E = C_C - C_D$$

will eliminate the effect of unbalanced secondary earth capacitances. A preliminary setting can be made by disconnecting z_1 and z_4 from the bridge at A. Then reconnect, balance by adjustment of z_4 and check the earth capacitance balance by reversing the secondary connections to the bridge; if correct there will be no disturbance of the balance.

Primary and Inter-capacitances. As shown in Fig. 183 (*b*), the primary winding has terminal earth capacitances C_{p1}, C_{p2} and these would have no affect on the bridge if they existed alone. They are, however, connected to the bridge in a complex way by the distributed intercapacitance between the two windings, this being represented in the usual way (see Fig. 62 (*b*) on p. 154) by inter-terminal direct- and cross-capacitances. Their effect on the bridge balance will, in general, depend upon the potentials of the primary terminals and cannot be eliminated, except in certain special cases, without the use of shields. If, for example, P_1 is earthed directly C_1 is thrown in parallel with z_1 and C_4 with z_4; and in an equal ratio bridge they may be balanced out by C_E.

Shielded Transformer. A transformer for use in an equal ratio bridge will be satisfactory if no dissymmetry is introduced either by earthing a primary terminal or by reversing the secondary terminals connected to the bridge; this can be ensured by the system of shields shown in Fig. 183 (*c*). The secondary and its leads are enclosed in a shield joined to the mid-point of the winding. The terminal capacitances *a*, *b* to this shield can be made equal by adjusting the amount of insulation between the two ends of the winding and the shield; they are, therefore, self-balanced across the secondary winding and have no effect on the bridge. Surrounding this is an earthed shield, with respect to which the inner shield and mid-point of the winding have capacitance *c*. As the mid-point is balanced in potential half-way between that at C and D, and as A is earthed, there is no p.d. across this inter-shield capacitance, which also has no effect. The earthed shield also encloses the primary, and capacitance between them is similarly balanced.

Double-shielded transformers may be applied in general to any type of bridge, no matter what the conditions of earthing or of symmetry may be. It is shown on p. 536 that when one corner is earthed it is only essential for one transformer to have double shielding, namely, that connected to the unearthed diagonal of the bridge. Whether this be the source or detector branch can only be decided by the special circumstances of the case; the reader is referred to the examples mentioned in Section 25 and to the detailed discussion of these bridges given in the text and in the relevant references.

21. Theory of Earth Admittances in Bridges. The four branches of a bridge network, the source and the detector all have distributed capacitance with respect to earth; the purpose of a system of shields is to make the earth capacitances definite in their location, magnitudes, and effects. It is convenient, first, to consider the influence of the source and detector earth capacitances independently of those associated with the balancing branches; two distinct effects can then be analysed.

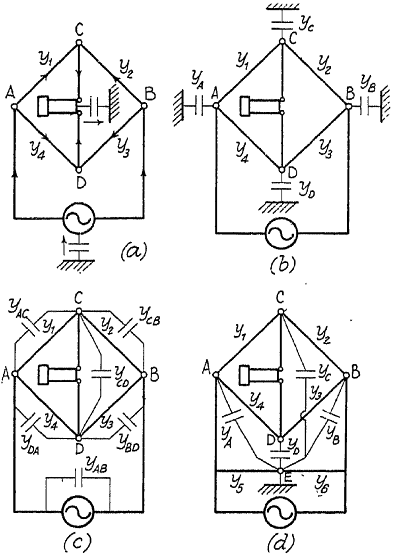

FIG. 184. ILLUSTRATING THE THEORY OF EARTH ADMITTANCES IN BRIDGE NETWORKS

"*Head Effect.*" Referring to Fig. 184 (a), the source and detector earth capacitances are shown approximately as single condensers. In consequence of the differences of potential between the bridge branch-points and earth, parasitic currents may flow in the bridge as shown in the diagram; these currents are supplementary to those circulating in the network because of the p.d. between A and B which, for simplicity, have been omitted from the diagram. The parasitic currents necessarily upset the balance, and cannot be eliminated by adjustments of the branch admittances y_1, y_2, y_3, y_4 alone. The "head effect," so called because the connection to earth of the telephone earth admittance is *via* the head of the earthed observer, can only be eliminated if C and D are permanently at zero potential; the currents will then vanish.

Branch Capacitance Effect. The earth capacitances of the four branches, whether shielded or not, will carry currents in consequence of any difference of potential existing between the branch points and earth; these earth currents constitute a second parasitic system superposed in the network and modify the balance in a way now to be determined.

Theory of Earth Capacitances. It is assumed that all earth capacitance effects are sufficiently represented by condensers joining the four branch points to earth.* The earth admittances y_A, y_B, y_C, y_D joining A, B, C, D to earth in Fig. 184 (*b*) form a star-connected system which can be replaced by a pair-connected system of six admittances, y_{AB}, y_{CB}, etc., in Fig. 184 (*c*) by use of Rosen's transformation stated on page 61. Then, y_{AB}, y_{CD} shunt the source and detector respectively, and can have no influence on the balance; the others appear in parallel with the branches. If $\Delta = y_A + y_B + y_C + y_D$ balance will occur if

$$\left(y_1 + \frac{y_A y_C}{\Delta}\right)\left(y_3 + \frac{y_B y_D}{\Delta}\right) = \left(y_2 + \frac{y_B y_C}{\Delta}\right)\left(y_4 + \frac{y_A y_D}{\Delta}\right)$$

since $y_{AC} = y_A y_C/\Delta$, etc. This may be written as

$$(y_1 y_3 - y_2 y_4) + \frac{1}{\Delta}(y_3 y_A y_C + y_1 y_B y_D - y_2 y_A y_D - y_4 y_B y_C) = 0.$$

If the presence of the earth admittances is to have no effect on the balance, so that we may take $y_1 y_3 = y_2 y_4$† then

$$y_1 y_B y_D + y_3 y_A y_C - y_2 y_A y_D - y_4 y_B y_C = 0.$$

This condition may be fulfilled two ways. Thus first writing

$$y_C y_3\left(y_A - y_B\frac{y_4}{y_3}\right) = y_D y_2\left(y_A - y_B\frac{y_1}{y_2}\right)$$

and remembering that $y_1/y_2 =$ gives

$$y_C/y_D = y_2/y_3 = y_1/y_4 \qquad . \ (a)$$

---

* This will be true when the network is shielded (see p. 534). It will be a close approximation to the effects of distributed capacitance when the branches are unshielded, and each consists of a single impedance. When a branch is composed of impedances in series the assumption is not true unless adequate shielding is provided (see p. 530).

† Note that this balance condition is based on defining the admittances in relation to the current which flows in them at their point of junction with the detector branch, since this current will be common to two successive branches of the bridge. This is especially important when the branches contain shielded impedances with considerable earth capacitance (e.g. high resistances) and specified conditions relating to the potentials of the terminals of the impedance relative to its shield, the latter being usually earthed. (See pp. 534 and 546 for theory and examples.)

Again, $$y_A y_3\left(y_C - y_D\frac{y_2}{y_3}\right) = y_B y_4\left(y_C - y_D\frac{y_1}{y_4}\right)$$

or since $y_1/y_4 = y_2/y_3$,

$$y_B = y_4/y_3 = y_1/y_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$$

These equations (*a*) and (*b*) express a theorem due to Carvallo*—Earth admittances will have no effect on the balance condition if the two earth admittances at one opposite pair of branch points, such as *AB* or *CD*, are in the same ratio as the admittances of the pairs of branches shunted by the chosen diagonal. The necessary adjustment can be effected by joining suitable adjustable admittances to earth from the appropriate pair of branch points in the way introduced by G. A. Campbell and his co-workers (for examples *see* p. 548). It should be noted that this adjustment does not necessarily remove the "head effect"; hence it is usually combined with system of shielding for source and detector, if not for the other elements also. Alternatively, an entirely different earthing device, now to be described may be used.

22. The Wagner Earthing Device. The Wagner earthing device† is an ingenious arrangement originally introduced for the elimination of the "head effect" in bridges; actually, as will be shown later, the device removes entirely *all* earth capacitance effects in bridges of any ratio. The arrangement is shown applied to a four-branch or Wheatstone network in Figs. 184 (*d*) and 185. Referring to Fig. 185 if $z_1z_3 = z_2z_4$ be satisfied this merely implies that the branch points *CD* are instantaneously at the *same* potential and not necessarily at *zero* potential. If the detector be a telephone on the head of an earthed observer, it follows that current will still flow in the instrument, even when the balance condition is fulfilled, *via* the earth capacitances from *C* and *D* to the observer's head. To eliminate the effect, two auxiliary branches, z_5, z_6, capable of balancing the impedances z_1, z_2, are joined across the alternator and earthed at their common point *E*. The earth capacitances from *A* and *B* to *E* merely shunt z_5, z_6. To eliminate the capacitances between *C*, *D*, and *E*, let the bridge be balanced as nearly as possible so that minimum sound occurs in the telephone when joined between *C* and *D*. Now transfer the telephone into the auxiliary network between *C* and *E*, balancing by means of z_5, z_6. Then *C* and *E* are nearly at the same potential, the latter being earthed. Reverting to the original bridge, balance can now be more nearly secured, and repetition of the process finally results in exact balance. Then *C*, *D*, *E* are at the same potential, namely, zero, and the " head effect " to the earthed observer is eliminated.*

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* J. Carvallo, "Quelques considérations de l'ordre pratique concernant l'emploi du pont de Wheatstone en courant alternatif," *Rev. Gén. de l'Él.*, Vol. 17, pp. 337–349 (1925).

† K. W. Wagner, "Zur Messung dielektrischer Verluste mit der Wechselstrombrücke," *Elekt. Zeits.*, Vol. 32, pp. 1001–1002 (1911). Also see *idem*, Vol. 33, pp. 635–637 (1912).

An instance in which the head effect is of great importance is the Carey Foster bridge (*see* p. 457). D. W. Dye† has shown how the Wagner earth may be adapted to this case, as in Fig. 186. The main bridge is first approximately balanced, with the auxiliary telephone switch open, by successive adjustments of *M* and *S*. The main telephone is then disconnected from the branch point *C*, while the second telephone is joined to *E*. Balance in the auxiliary bridge is then obtained approximately by adjustments of *L′* (which is about equal to the inductance of *a*), *R′*, and, if necessary, of *Q′* also. The

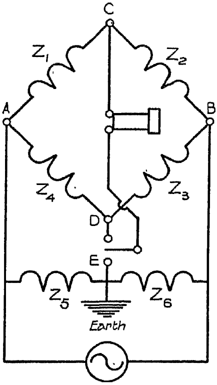

FIG. 185. WAGNER EARTHING DEVICE IN WHEATSTONE NETWORK

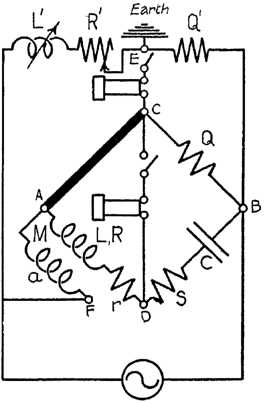

FIG. 186. WAGNER EARTHING DEVICE IN CAREY FOSTER BRIDGE

telephone connection in the main bridge is then replaced on the point C, the auxiliary being removed from E, and the main bridge balance is corrected finally to zero by adjustments of M and S. Thus the points C, D, and E are brought to zero potential. (Alternatively, a single telephone and change-over switch can be used.)

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* It is sometimes possible in bridges of the Wheatstone type with resistance ratios to connect the earthing device so that z_5, z_6 are also resistances.

† D. W. Dye, "The 'Wagner' earth connection," *Elecn.*, Vol. 87, pp. 55–56 (1921); also G. E. Moore, *Elecn.*, Vol. 86, pp. 744–745 (1921). The earthing device is easily applied to the conjugate Carey Foster bridge of Fig. 171 (*b*) on p. 479. In this, z_5 is a condenser in series with a resistance; z_6 is a resistance. The detector must be disconnected at the *C* branch point when transferring it to the auxiliary bridge.

The Wagner earthing device can also be adapted to mutual inductance bridges of the Heaviside-Campbell type (Figs. 150, 152, 153, and 155) of any ratio in the way shown by Hartshorn.* In Fig. 187 the auxiliary branches are an adjustable self-inductance and resistance for z_5 and a resistance for z_6. A third resistor W in series with the primary of the mutual inductor is sometimes required to make the auxiliary balance possible.

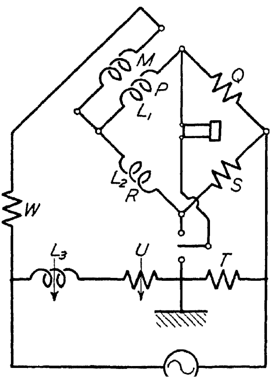

FIG. 187. WAGNER EARTHING DEVICE IN HEAVISIDE-CAMPBELL BRIDGES

Theory—In Fig. 184 (*d*), the points C, D and E are reduced to the same potential by successive adjustment of the main bridge admittances y_1, y_2, y_3, y_4, and an auxiliary bridge, say, y_1, y_2, y_5, y_6. Then since E is earthed, y_C and y_D carry no current, and the "head effect" is zero. Also y_A, y_B shunt y_5 and y_6 respectively, and are, therefore, removed outside the bridge proper; hence all earth admittance effects are eliminated from the main bridge. The conditions for balance in the main and auxiliary networks are—

$$y_1y_3 = y_2y_4, \text{ and } y_2(y_5 + y_A) = y_1(y_6 + y_B).$$

Although the earth admittances are entirely eliminated from the main bridge when the simultaneous perfect balance of the main and auxiliary bridges has been finally secured, the partial imperfect balances successively obtained during the balancing process and the rate at which they converge upon the final balance necessarily depend on the values of the earth admittances. According to the amount and nature of the admittances, successive adjustments of the main and auxiliary bridges usually converge upon simultaneous perfect balance with more or less rapidity. Cases are known, however, where successive trials diverge from simultaneous balance, i.e. get progressively worse, or in the intermediate case remain stationary. In such instances convergence can be secured by joining one of the detector branch points to earth through an appropriate admittance, the amount and nature of which can easily be found by trial (see paper by Dye and Jones cited on p. 371); the same artifice can be used to improve convergence when this is slow. In most cases, however, the Wagner earthing device operates without difficulty and with rapid convergence.

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* L. Hartshorn, "The measurement of current and voltage in alternating current bridges," *Journal Sci. Insts.*, Vol. 3, pp. 87–89 (1925).

Ogawa,[*] to whom the complete theory of the earthing device in a.c. bridges is due, has pointed out that it is sometimes difficult to make the proper choice of the earthing admittances y_5, y_6; these will depend not only on the type of bridge but on the condition of the source. If the source be joined to the bridge through a transformer in which there is no capacitance between primary and secondary and in which the capacitances to earth of the two halves of the secondary are carefully balanced so that the mid-point of the secondary is sensibly at earth potential, little difficulty will be encountered in operating the Wagner earthing device. These conditions can be satisfied by proper winding

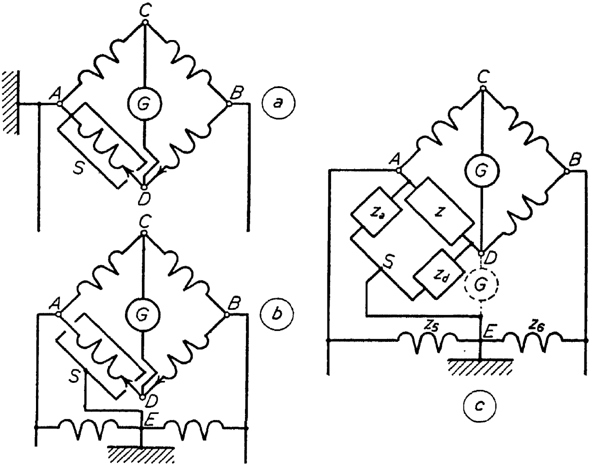

FIG. 188. BALANCING CURRENT IN A SHIELDED BRIDGE ARM

of the two halves of the secondary and interposing an earthed shield between the secondary and primary as discussed on p. 539; the whole construction should be toroidal and the entire transformer enclosed in an iron magnetic shield. Ogawa recommends that the earthing device should not contain self inductances, if possible, so that magnetic interference with the rest of the network is avoided.

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[*] K. Ogawa, "Earth capacity effects in alternating current bridges," *Journal I.E.E. Japan*, No. 475, pp. 115–131, Feb. (1928) (in Japanese). "General theory and earthing device of alternating current bridges," *Elect. Lab. Min. Comm. Tokyo Res.*, No. 254, pp. 1–45, April (1929) (in English). "Further researches on alternating current bridges with perfect earthing device," *ibid.*, No. 277, pp. 1–22, Feb. (1930) (in English). "Accidental errors in alternating current bridges," *Journal I.E.E. Japan*, Vol. 53, p. 29, pp. 311–314 (Apr. 1933) (in Japanese, English abstract). "Universal alternating current bridges," *ibid.*, vol. 53, pp. 42–43, 436–442 (June 1933) (in Japanese, English abstract).

Balancing Current in a Shielded Bridge Arm. When a shielded impedance, such as a decade resistor, is used in a bridge arm it is important to use the correct value of impedance in the balance condition $z_1 z_3 = z_2 z_4$. This is defined in relation to the currents at the detector branch points, since these are common to two adjacent branches.

In Fig. 188 (*a*) the branch *AD* contains a shielded impedance, the shield and the point *A* being earthed. The current at the branch point *D* includes the shield current and the impedance to be used is, therefore, z_{AS} as defined on p. 535.

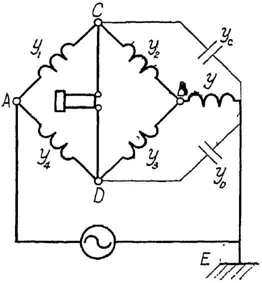

FIG. 189. BUTTERWORTH'S EARTHING DEVICE

In Fig. 188 (*b*) a Wagner earth is provided, so that the points *C*, *D*, and *E* are at earth potential when balance is secured, *E* and *S* being directly earthed. The current at *D*, which is at earth potential but not directly earthed, does not include the shield current; the effective impedance is z_0 as defined on p. 535.

These arrangements provide a simple means of measuring the effective impedances of a shielded arm in terms of a calculable standard, as Hartshorn and Wilmotte have shown (*loc. cit. ante*).

An alternative view of the matter is given by Bartlett* by replacing the impedance with its earthed shield by its equivalent Π circuit, as in Fig. 188 (*c*) which is equivalent to Fig. 188 (*b*). Here z_a shunts z_5; z_d joins *D* to *E* and has no p.d. across it. Hence *z* alone enters the bridge balance and is the "architrave" of the equivalent Π; its value is easily expressed for symmetrical and unsymmetrical impedances by the theory on pp. 530 to 534.

23. Other Earthing Devices. Butterworth has described an earthing device, shown in Fig. 189, which has some practical advantages over the Wagner arrangement, and which deserves to be more widely used. In this diagram y is an auxiliary admittance composed of the two constituents of a Wagner earth joined in parallel. Earth capacitance from A shunts the source; that from B is in parallel with y, and may be regarded as part of it; hence only y_C, y_D require consideration. Two balances are made (i) with y infinite, i.e. the earthed terminal of the source joined to B; (ii) with y finite. Balance is secured by adjustment of y_2 giving values y_2', y_2'' respectively. Converting the star y, y_2, y_3 into a mesh the two balance conditions are easily seen to be

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* A. C. Bartlett, "Note on the theory of screened impedances in a.c. bridges with the Wagner earth," *Journal Sci. Insts.*, Vol. 6, pp. 277–280 (1929).

$$\frac{y_1}{y} \quad \frac{y_2' + y_C}{\quad + y_D} = \left[y_C + \frac{y_2''y}{y + y_3 + y_2''}\right] \Big/ \left[y_D + \frac{y_3 y}{y + y_3 + y_2''}\right]$$

$$= \frac{y_2'' + \mu y_C}{y_3 + \mu y_D}$$

where $\mu = 1 + \dfrac{y_2'' + y_3}{y}$. Eliminating y_C/y_D gives

$$\frac{y_1}{y_4} = \frac{(y_2'' - \mu\, y_2')}{(1 - \mu) y_3}$$

Compared with the Wagner method only two adjustments are necessary instead of a series, and an estimate may be made of the magnitude of earth capacitance errors; the body of the observer must, however, be kept in the same position relative to the bridge for both balances.

Other earthing arrangements are sometimes used, but usually have the purpose of approximate rather than exact elimination of earth capacitance effects. Thus if two branches meeting in a source branch-point have a very high impedance in comparison with the other two, the effects of earth capacitances on the balance and on the "head effect" will be very small if the source branch-point of the low impedance pair is earthed and appropriate shields are used. This is the process applied by Monasch to the Wien condenser bridge (*see* page 346), and adapted to a number of similar bridges; e.g. the Schering bridge discussed on pages 352 *et. seq.*

24. Measurement of Current and Voltage in a.c. Bridges. It is frequently required to measure an impedance, such as a telephone loading coil, when a definite current is flowing through it. The inclusion of an ammeter in series with the impedance under test is clearly not permissible and, in any case, the current is usually too small to be measured accurately by means of portable instruments. A possible method is to increase the total current supplied by the source, measuring this by means of an ammeter, and applying a shunt across the bridge; if the shunt be known, and also the impedance of the whole bridge network, the current passing through the given impedance may be calculated. Hartshorn* has shown that the difficulty of knowing the network impedance can be overcome in bridges which have resistance ratios by using the Wagner earthing device as a shunt. In Fig. 190 let $z_2 = Q$, $z_3 = S$, while $z_6 = T$ say; when both main and auxiliary bridges are balanced the currents will be i_C, i_D and i respectively in $z_1 + z_2$,

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* L. Hartshorn, "The measurement of current and voltage in alternating current bridges," *Journal Sci. Insts.*, Vol. 3, pp. 87–89 (1925).

$z_3 + z_4$, $z_5 + z_6$ respectively. These currents are all in phase, and the common p.d. over z_2, z_3, and z_6 will be $Qi_C = Si_D = Ti$. The current supplied by the source I can be measured by an ammeter and $I = i_C + i_D + i = i_C\left(1 + \frac{Q}{S} + \frac{Q}{T}\right)$ from which i_C, the current in the test impedance z_1 is known. If, as is often the case, the bridge has equal ratios $S = Q$ and $I = i_C\left(2 + \frac{Q}{T}\right)$; then if $T = Q/8$, $I = 10i_C$, and the desired current is 1/10 of the ammeter reading. In a like manner, if z_1 is to be measured with a definite p.d. across it a voltmeter joined across z_5 will give the desired voltage when main and auxiliary balances have been obtained.

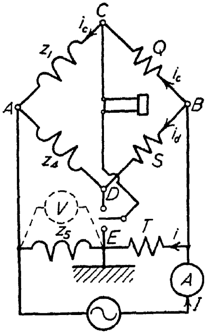

Fig. 190. Use of Wagner Earth to Measure Current and Voltage in Bridge Arm

25. Examples of Shielded Bridges. Electric shielding of bridge networks was introduced by G. A. Campbell in 1904 with the object of increasing the accuracy of measurements at about 2,000 cycles per second, the highest frequency then considered important in telephonic research. Subsequent experience has shown that the principles then stated hold good at still higher frequencies; indeed it is only by the use of adequate shielding that bridge methods can be satisfactorily used at carrier frequencies (of the order of a hundred or so kilocycles per second) and ultimately at radio frequencies. The main purpose of shielding is to make the earth- and inter-admittances of the bridge arms definite in amount and location. The addition of the shields certainly increases the magnitudes of these admittances, and may add new admittances to the network, but their definiteness enables their effect on the measurements to be strictly controlled.

Shielded bridges are of two main classes, according to the way of dealing with the stray admittances. In the G. A. Campbell type the undesired admittances of the shielded arms are, by suitable arrangement and connection of the shields, put outside the bridge-proper whenever this is possible; any residual admittance which cannot be dealt with in this way is thrown in parallel with another bridge arm in such a way that it can be balanced out by an admittance, either stray or intentional, across another arm. Such bridges are usually of unity ratio, and can be arranged to give a variety of conditions for the potentials of the test impedance terminals. The Campbell type of bridge is largely an American product and in the American Telegraph and Telephone Co. laboratories and elsewhere has been brought to a high degree of perfection. In the Wagner type of bridge, widely used in Great Britain and Europe generally, the earth admittances are made definite by shields and are then eliminated from the bridge balance by the Wagner earthing device or its equivalent; the bridge ratio may have any desired value, but is preferably unity for reasons of symmetry. In both classes of bridge it is usual to use a substitution method, balancing first with the unknown impedance in the network and then substituting for it a known standard impedance such that balance can be restored by small readjustments of the balancing arms. This method constitutes what is probably the most precise way of using a shielded bridge (see p. 524). It is not usually possible to shield the unknown impedance, but in many bridges this difficulty is removed by providing a shielded compartment in which the unknown can be tested. Many examples of both kinds of bridge have been mentioned in the text and will now be briefly classified.

AUDIO AND CARRIER FREQUENCIES. (i) *Inductive Impedance.* Bridges for the measurement of inductive impedance are three in number. The Giebe bifilar arrangement of Maxwell's method for comparing self-inductances, p. 317 and Fig. 111, is suitable up to 3,000 cycles per second; Shackleton's arrangement of the same method, p. 320 and Fig. 112, extends the range to 50,000 cycles per second. The Owen bridge arranged by Ferguson, p. 408 and Fig. 141, measures self-inductance and effective resistance at 3,000 cycles per second, and in a form due to Bartlett, p. 409, is available up to 50,000 cycles per second.

(ii) *Capacitive Impedance.* Shielded bridges for measurement of capacitance and dielectric losses are more numerous. Ford and Reynolds, p. 338 and Fig. 116 (*a*), use the parallel resistance method up to 1,000 cycles per second; from 1,000 to 12,000 cycles per second, these workers have designed a Wien bridge, p. 349 and Fig. 121. Sommerman, p. 350, uses an elaborate form of the same method up to 10,000 cycles per second. The Schering bridge is much favoured either in the form due to Giebe and Zickner, p. 367 and Fig. 128 (*a*), or as described by Hartshorn, p. 368 and Fig. 128 (*b*). Various adaptations have been made to dielectric material and oil testing, the shielding system being very fully worked out by Balsbaugh, Berberich and others, p. 370. Tinsley and Co.* make a fully shielded, self-contained bridge for 1,000 cycles per second with a shielded compartment for the test object.

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* H. Tinsley & Co., "Precision audio-frequency capacity and power-factor bridge," *Journal Sci. Insts.*, Vol. 12, pp. 328–329 (1935).

(iii) *Universal Arrangements.* Many shielded bridges have been designed to be used in a variety of ways. Walcher* describes an arrangement to measure capacitances by the parallel resistance method and inductances by the Owen method. Dike† has used a bridge with equal, doubly-shielded, resistance ratio arms and shielded transformers, on the plan of the Shackleton bridge; by use of a separate standard inductance or condenser, inductive or capacitive impedances can be measured up to 3,000 cycles per second. A similar purpose is served by specially shielded ratio boxes devised by Zammatoro‡ and also by Behr and Williams;§ these papers give details of the shielding system.

RADIO FREQUENCIES. Bridge measurements at radio frequencies fall outside the proper scope of this volume, on account of the special nature of the technique involved; but certain cogent reasons make it necessary to give a brief account of such h.f. bridges. The great success of shielded bridges at the higher acoustic and carrier frequencies has attracted many workers to adapt them to the radio range. Two features are outstanding (i) the provision of elaborate shielding systems and careful layout; (ii) the use of ratio and standard branches with the lowest possible residuals. The second point limits the bridge arms to air condensers and low reactance resistors, either wire wound or electrolytic; inductors, whether self or mutual, do not constitute such satisfactory standards, though exception may be made in the case of specially designed self-inductors such as those of Griffiths. Detectors are almost always some form of heterodyne receiver with triode amplifier and telephone. R.f. bridges have been used for three main purposes: (i) to measure effective resistance, inductance, and capacitance of radio apparatus and to investigate the properties of insulating materials; (ii) in physical chemistry, to measure the dielectric constants of liquids free from polarization, following the lead given by Nernst in 1897; (iii) in biological research. Many bridges, especially for the last-named purpose, are designed to work continuously from low acoustic up to radio frequencies.

Bridges for capacitance measurement using resistance ratio branches are represented by the Ford and Reynolds parallel-resistance bridge (p. 338) used up to $1{\cdot}5 \times 10^6$ cycles per second, and the series-resistance method of Burke.‖ For biological tests over a wide range from 30 to

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* T. Walcher, "Eine kombinierte Wechselstrommessbrücke. Ein Beitrag zur genauen Messung von Kapazitäten und Selbstinduktivitäten mit abschirmten Messeinrichtungen," *E.u.M.*, Vol. 50, pp. 518–523 (1932). Also see R. F. Field, *G.R.Exp.*, Vol. 6, pp. 1–4 (Jan. 1932) for a bridge capable of a large number of rearrangements.

† P. H. Dike, "A precision audio-frequency bridge," *Rev. Sci. Insts.*, Vol. I, pp. 744–748 (1930).

‡ S. J. Zammatoro, "Portable balance unit for a.c. precision bridge," *Bell Lab. Rec.*, Vol. 10, pp. 173–177 (1932).

§ L. Behr and A. J. Williams, "The Campbell-Shackleton shielded ratio box," *Proc. Inst. Rad. Eng.*, Vol. 20, pp. 969–988 (1932).

‖ C. T. Burke, "Bridge methods for measurements at radio frequencies," *G.R.Exp.*, Vol. 7, pp. 1–6 (July 1932). Of historical interest is M. D. Hart, "A bridge method for determining dielectric losses at wireless frequencies," *Journal I.E.E.*, Vol. 61, pp. 697–700 (1923). For a bridge for routine testing of radio components see J. M. Unk, "Messung von Hochfrequenzbestandteilen der Radioapparate bei Massenherstellung," *Schweiz. Elekt. Ver. Bull.*, Vol.

10^7 cycles per second, Cole and Curtis* have also used the series-resistance method, but with electrolytic resistors (p. 508). The same bridge has also been used by Küpfmüller† with the substitution of capacitance ratio arms, thus making a four-condenser bridge. An outstanding bridge is that developed at the N.P.L. by Dye and Jones, see p. 371; a Schering network with Wagner earth is used up to $1{\cdot}2 \times 10^6$ cycles per second. Stratton‡ and later Hemingway and McClendon§ eliminate the effect of ratio-arm residuals by the inductive ratio bridge of Grover, Fig. 140 (*a*) on p. 403; see also p. 508. Both these bridges are used for biological research over a wide frequency range.

Turning to inductance bridges the Giebe bifilar bridge has been used at 10^6 cycles per second by Kreielsheimer and by Krüger in tests of skin effect and permeability of wires, see p. 318. The simple bridge of Watson‖ is now only of historical interest as a method for comparing inductances, using a Maxwell bridge with condenser ratios. Boner uses the series and parallel resonance bridges to measure inductances and capacitances at frequencies up to 500 kilocycles per second, see p. 402. Finally the Campbell filter bridge has been modified by Eckersley to measure condenser losses and by Jackson to measure effective resistance, see p. 475; both have troubles due to impurity in the mutual inductor at high frequencies. Fortescue and Mole¶ have used a series resonance shielded bridge to measure effective resistance at frequencies up to 10×10^6 cycles per second, and give full constructional details. A useful account of r.f. bridges is given in Electrical Research Association Report, Ref. L/T 56 (1933), by T. I. Jones.

HIGH VOLTAGES. Shielding is necessary to control earth capacitance effects in bridges used at high voltages, even at ordinary supply frequencies. Full details will be found in Section 7*c*.

25, pp. 561–568 (1934). A bridge of the Shackleton type with equal resistance ratios has been described by V. Balasubramaniam, "The Wheatstone bridge for radio frequencies," *Electrotechnics*, No. 10, pp. 82–84 (1937). Also see C. H. Young, "A 5-megacycle impedence bridge," *Bell. Lab. Rec.*, Vol. 15, pp. 261–265 (1937).

* K. S. Cole and H. J. Curtis, "Wheatstone bridge and electrolytic resistor for impedance measurements over a wide frequency range," *Rev. Sci. Insts.*, Vol. 8, pp. 333–339 (1937).

† K. Küpfmüller, "Ueber eine technische Hochfrequenzmessbrücke," *E.N.T.*, Vol. 2, pp. 263–270 (1925).

‡ J. A. Stratton, "A high frequency bridge," *Journal Opt. Soc. Amer.*, Vol. 13, pp. 471–494 (1926).

§ A. Hemingway and J. F. McClendon, "An a.c. Wheatstone bridge for audio and radio frequency measurements," *Physics.*, Vol. 2, pp. 396–402 (1932).

‖ C. J. Watson, "A simple means of measuring inductances," *Elecn.*, Vol. 62, pp. 809–810 (1909).

¶ C. L. Fortescue and G. Mole, "A resonance bridge for use at frequencies up to 10 megacycles per sec.," *Journal I.E.E.*, Vol. 82, pp. 687–692 (1938). In the discussion, Dr. Hartshorn points out that the junction of the coil and condenser in a tuned arm may attain a high potential and cause earth capacitance troubles.

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THE following is a list of books in which the subject of alternating current bridge measurements receives some treatment. The list is not in any way complete, but is merely intended to assist the student requiring further reading.

1. R. G. Allen, *Principles and Practice of Electrical Testing*, Chapter X, pp. 190–203 (1919).
2. A. Campbell and E. C. Childs, *The Measurement of Inductance, Capacitance and Frequency*, pp. i-xxiv, 1-488 (1935).
3. *A Dictionary of Applied Physics*, edited by Sir Richard Glazebrook, Vol. 2, *see* the following articles: "Capacity, electrical, and its measurement," pp. 103–144; "Inductance, the measurement of," pp. 392–445; "Vibration galvanometers," pp. 960–980 (1922).
4. P. Dunsheath, *High Voltage Cables. Theory and Practice of their Design and Operation*, pp. i–xii, 1–161 (1929). In particular see pp. 55–59, dealing with the Schering bridge; pp. 118–119, where the bridge measurement of cable-sheath losses is discussed and useful references are given.
5. J. A. Fleming, *The Principles of Electric Wave Telegraphy and Telephony*, 3rd Edn., Chapter II, pp. 129–248 (1916).
6. E. Gérard, *Mesures Electriques*, 4th Edn., Chapter XIII, pp. 328–338, and Chapter XVII, pp. 398–419 (1912).
7. A. Gray, *Absolute Measurements in Electricity and Magnetism*, 2nd Edn., Chapter XIV, pp. 528–592 (1921).
8. B. Hague, *Instrument Transformers. Their theory, characteristics and testing*, particularly Chapter IX, pp. 327–371, dealing with low four-terminal resistors and very high value shielded resistors; Chapter X, pp. 374–378 dealing with h.v. capacitors (1936).
9. *Handbuch der Physik*, edited by H. Geiger and K. Scheel, Vol. 16, *Apparate und Messmethoden für Elektrizität und Magnetismus.* The following chapters give an excellent summary of modern German practice in bridge measurements: Chapter X, *Schwingungsinstrumente*, by H. Schering, pp. 304–322, dealing with the vibration galvanometer. Chapter XVIII, *Kondensatoren und Induktivitätsspulen*, by E. Giebe, pp. 470–494. Chapter XIX, *Messungen von Kapazitaten und Induktivitaten*, by E. Giebe, pp. 495–539. Chapter XXIV, *Elektrochemische Messungen*, by E. Baars, pp. 594–623 (1927).
10. W. Jaeger, *Elektrische Messtechnik*, in particular *see* pp. 370–384 (1917).
11. J. Krönert, *Messbrücken und Kompensatoren, Band I, Theoretische Grundlagen*, pp. 1–282 (1935).

12. F. A. Laws, *Electrical Measurements*, 2nd Edition, Chapter VII, pp. 345–480 (1938).
13. H. W. Malcolm, *The Theory of the Submarine Telegraph and Telephone Cable*, Chapter VII, pp. 187–227 (1917).
14. W. H. Nottage, *The Calculation and Measurement of Inductance and Capacity*, 2nd Edition, pp. i–viii, 1–224 (1924).
15. E. Orlich, *Kapazität und Induktivität, ihre Begriffsbestimmung, Berechnung und Messung*, pp. 1–294 (1909).
16. D. Owen, *Alternating Current Measurements at Audio and Radio Frequencies*, pp. i–vii, 1–120 (1937).
17. G. Zickner, *Über Kondensator und ihre Eichung*, pp. 1–60 (1928).

APPENDIX I

EXAMPLES OF STAR-MESH TRANSFORMATIONS IN BRIDGE NETWORKS

Maxwell's Bridge for Comparison of Self with Mutual Inductance. Consider the case of Maxwell's bridge shown in Fig. 150 (*a*) on page 429, assuming the resistance W to be used to adjust for balance. Convert the mesh formed of R, S, and W into a star-connected set of impedances z_A, z_B, z_D joined to the points A, B, D. The transformed network is shown in Fig. 191, which is seen to be of the four branch type, in which $z_1 = P + j\omega L$, $z_2 = Q$, $z_3 = z_B$, $z_4 = z_A$, and $m_{16} = j\omega M$. Inserting these values in the equations given on page 71, $\alpha = 0$, $\beta = 0$, $\gamma = -j\omega M$, $\delta = 0$, so that for balance

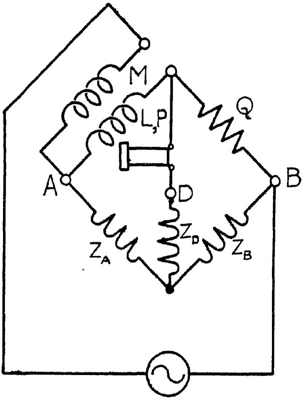

FIG. 191. MESH-STAR TRANSFORMATION OF MAXWELL'S BRIDGE (*see* Fig. 150 (*a*))

$$z_B(P + j\omega L) - z_A Q + j\omega M(Q + z_B) = 0.$$

Applying Equations (10*a*) on page 48,

$$z_A = \frac{RW}{S + R + W} \text{ and } z_B = \frac{SW}{S + R + W},$$

hence,

$$SW(P + j\omega L) - RWQ + j\omega M[SW + Q(S + R + W)] = 0;$$

whence, by equating components,

$$SP = QR$$

and $$L = -\left\{1 + \frac{Q}{S} + \frac{Q(S + R)}{WS}\right\}M,$$

as quoted on page 430.

Butterworth's Modification of the Campbell Frequency Bridge. The details of this bridge have been discussed on page 474, the network being shown in Fig. 192 (*a*). This example is rendered more complex by the fact that the mesh joining the points *U*, *V*, *W* contains an impure mutual inductance. Reduction of the network to a simpler type can be made in two stages, as follows—

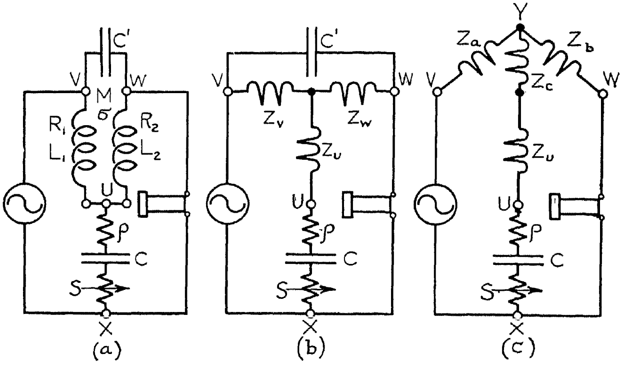

FIG. 192. TRANSFORMATION OF BUTTERWORTH'S MODIFICATION OF THE CAMPBELL FREQUENCY BRIDGE

(i) By means of the transformation given on page 70, substitute a star system of impedances z_U, z_V, z_W for the two coils of the mutual inductance, the mutual operator for which is $m = -(\sigma + j\omega M)$, since the mutual inductance is impure and must be negative for balance (*see* p. 470); then,

$$z_U = (\sigma + j\omega M),$$
$$z_V = R_1 - \sigma + j\omega(L_1 - M),$$
$$z_W = R_2 - \sigma + j\omega(L_2 - M),$$

as in Fig. 192 (*b*).

(ii) Now by means of Equation (10*a*), replace the mesh of impedances z_V, z_W, $1/j\omega C'$ by their equivalent star z_a, z_b, z_c, as in Fig. 192 (*c*). Since z_a and z_b lie in circuit with the source and detector respectively, for balance the impedance of the common branch XY must be zero, that is,

$$z_c + z_U + \rho + S + \frac{1}{j\omega C} = 0.$$

Substituting for $z_c = \dfrac{z_W z_V}{z_W + z_V + \frac{}{j\omega C'}}$ and for z_U,

$$\frac{z_W z_V}{z_W + z_V - \dfrac{j}{\omega C'}} + \left[\sigma + \rho + S + j\left(\omega M - \frac{1}{\omega C}\right)\right] = 0$$

Since σ and ρ are small and at balance $\omega M - \dfrac{1}{\omega C}$ is nearly zero, the bracketed expression is small compared with z_W and z_V; to the first order of small quantities

$$j\omega C' z_W z_V + \left[\sigma + \rho + S + j\left(\omega M - \frac{1}{\omega C}\right)\right] = 0,$$

since $1/j\omega C'$ is large. Inserting the values for z_V and z_W, separating the two components, and neglecting $\sigma C'$ in comparison with R_1 and R_2 gives

$$\omega^2 MC = 1 - \omega^2 CC'[R_1R_2 - \omega^2(L_1 - M)(L_2 - M)],$$

and $$\sigma + \rho + S = \omega^2 C'[R_1(L_2 - M) + R_2(L_1 - M)],$$

as quoted on page 474.

Chiba's Modifications of the Campbell Frequency Bridge.—Chiba has suggested several modifications of Campbell's frequency bridge to adapt it for low frequencies. Two of these are shown in Fig. 193, and these will serve as further useful examples of circuit transformations. Referring to the diagrams Figs. 193 (*a*) and (*b*) it will be seen that in both cases the simple condenser of the original bridge, Fig. 169 (*a*), is replaced by a mesh connection of impedances joining the points A, B, C. By use of the mesh-star transformation the three mesh-connected impedances may be replaced by z_A, z_B, z_C joined in star as in Fig. 193 (*c*). Of these, z_B goes in series with the alternator, z_C in series with the detector, leaving only z_A to be considered in the balance condition. We shall suppose the mutual inductance to be free from impurity at the low frequencies to which the bridges are applied.

In Fig. 193 (*a*) let C_1, C_2, C_3 be perfect condensers. Then

$$z_A = \frac{1}{j\omega C_1} \cdot \frac{1}{j\omega C_3} \Big/ \left(\frac{1}{j\omega C_1} + \frac{1}{j\omega C_2} + \frac{1}{j\omega C_3} \right)$$

$$1/j\omega C_1 C_3 \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right)$$

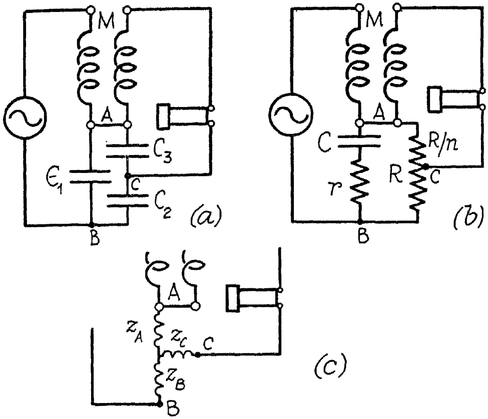

FIG. 193. CHIBA'S MODIFICATIONS OF THE CAMPBELL FREQUENCY BRIDGE

For zero current in the detector M must be adjusted until

$$j\omega M - z_A = 0$$

or

$$\omega^2 = -1/MC_1C_3\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right.$$

Hence M must be negative for balance. Now write

$$C_3 = (n-1)\,C_2$$

then numerically

$$\omega^2 = 1/M(nC_1 + C_3)$$

thus, by suitable choice of n, any desired low value of ω may be measured.

Turning now to Fig. 193 (*b*), the condenser C is joined in series with r; C need not be perfect, in which case r includes its series loss resistance. In the transformed network

$$= \frac{R}{n}\left(r + \frac{1}{j\omega C}\right) \Big/ \left(R + r + \frac{1}{j\omega C}\right)$$

Balance is secured when

$$j\omega M - z_{\text{A}} = 0$$

or

$$j\omega M\left(R + r + \frac{1}{j\omega C}\right) = \frac{R}{n}\left(r - \frac{j}{\omega C}\right)$$

Separating components,

$$r = nM/CR$$

and

$$\omega^2 = -\,1/nCM\left(1 + \frac{r}{R}\right)$$

Here again, M must be negative and any desired low frequency may be measured by choosing a suitably large value for n. It will be seen, however, that a negative M would necessitate a negative r; hence, true balance of this bridge is impossible, since it is not possible to satisfy both balance conditions simultaneously. As a rule r is small in comparison with R, so that, numerically, $\omega^2 \doteqdot 1/nCM$. when minimum indication has been obtained.

APPENDIX II

PERIODICAL LITERATURE

A BRIEF analysis of the considerable quantity of periodical literature consulted during the preparation of this book will be of interest. Publications have been examined under ninety-seven titles distributed geographically as follows: Great Britain, 31; U.S.A., 21; Germany, 15; France, 12; Italy and Japan, 4 each; Belgium, 3; Sweden, 2; Austria, Holland, India, Switzerland, and U.S.S.R., 1 each.

A total of 741 articles dealing with the theory, apparatus, construction and use of a.c. bridges has been referred to in writing the text. This number excludes (i) a few articles published before 1880; (ii) articles dealing with d.c. and ballistic bridges; (iii) articles describing low-value, four-terminal resistors and high-value shielded voltage dividers which, though occasionally used in bridge circuits, have other primary purposes and are examined in the companion volume on *Instrument Transformers*.

Of the 741 references published between 1880 and 1937, both years inclusive, 203 (27·5 per cent) appeared by the end of 1913 and 238 (32 per cent) by the conclusion of the World War. No less than 503 articles, or 68 per cent of the total, have been published since the War. The Third Edition of this book analysed the literature down to the end of 1931; since then 216 papers, or 29·1 per cent, have appeared to the end of 1937 and receive notice in the present edition. In other words, the amount of literature during the last six years exceeds that during the thirty-four years before the War, and is about 43 per cent of the post-war output. These figures testify to the enormous development of the subject, especially in recent years, shown by the wide range of applications in which a.c. bridge methods are now used, as a reference to Chapter V will verify.

The development is strikingly illustrated by Fig. 194, which gives the annual output of periodical literature since 1880, and correlates it with some of the more important developments in technique. The first stimulus, given by the development of mutual inductors and of vibration galvanometers by

A. Campbell, occurred about 1907, but was followed by the general decline in scientific activity during the War. The second stimulus was undoubtedly the introduction of the Schering bridge in 1921 and the consequent extension of

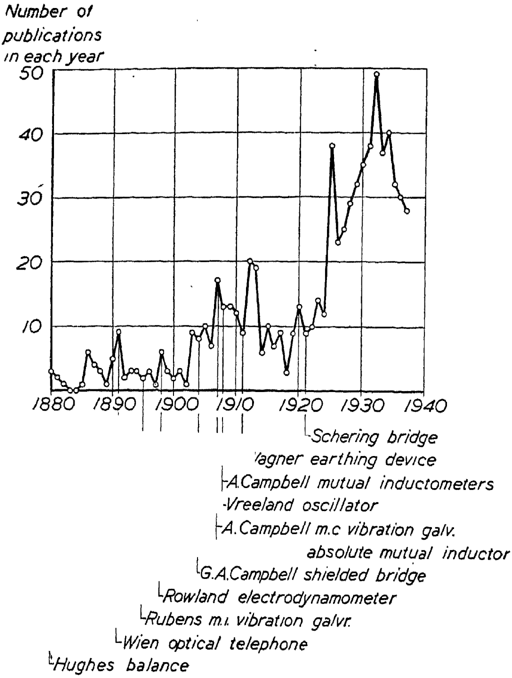

FIG. 194. ANNUAL OUTPUT OF PERIODICAL LITERATURE ON A.C. BRIDGE METHODS

bridge methods into high voltage measurements and works testing, leading up to a peak in 1932. Since then there has been a steady fall in the amount of published work on bridge methods, but the annual total still stands at a fairly high figure.

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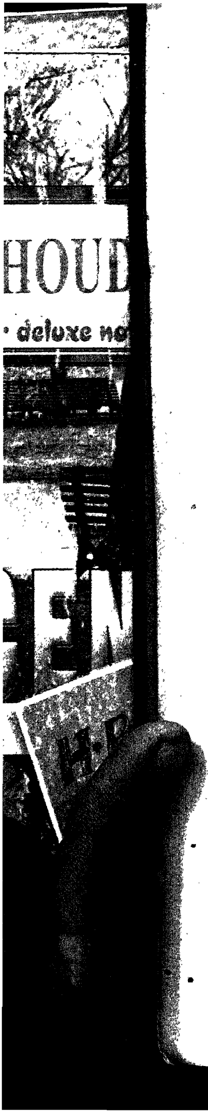
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